DIRECTION ESTIMATION USING CONJUGATE CYCLIC CROSS-CORRELATION : MORE SIGNALS THAN SENSORS

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ABSTRACT

We consider the problem of estimating the directions of arrival of multiple communication signals arriving at an uniform linear array. By considering the conjugate cyclic cross-correlations of the sensor outputs and using a Bayesian framework, we propose a direction finding algorithm that allows us to estimate the directions of arrival for a more signals than sensors scenario. The algorithm does not need any training sequence and only requires *a priori* knowledge of the cyclic frequencies and the number of sources. It is also possible to estimate the directions of arrival in a multi-path fading environment under certain conditions.

1. INTRODUCTION

In the last few years, due to the advances in mobile communications and the limitation of the available spectrum, there has been an interest in incorporating knowledge of the directions of the signals to enhance the desired signal in the presence of interferers. The objective of direction finding is the estimation of the directions of arrival (DOAs) of signals of interest (SOIs) at the sensor array. Various algorithms, mainly subspace based methods, such as MUSIC and ESPRIT, have been proposed for the estimation of the DOAs.

Communication signals are cyclostationary due to periodicity corresponding to their pulse rates or carrier frequencies. Compared with conventional correlation based methods, algorithms that exploit this cyclostationarity yield better parameter estimates in the presence of interferers that do not exhibit the same cyclic features . The cyclic-selectivity property allows the cyclostationary algorithms to discriminate in favour of the signals of interest in the presence of interfering signals. Cyclic direction finding methods have been proposed, that incorporate the cyclostationary characteristic to improve the performance. A cyclic-MUSIC method was proposed in [1, 6] and a least-squares method based on a LP model of the sensor outputs was presented in [5].

In this paper, we propose a direction finding algorithm based on the conjugate cyclic cross-correlation function of the sensor outputs. By considering the estimation error of the sample conjugate cyclic cross-correlation function (CCCF) and by using a Bayesian estimation criterion, we are able to obtain the DOAs of the signals. The DOAs can be estimated even when there are more signals than sensors. Cyclic algorithms based on cyclic-MUSIC were presented in [4] using cyclic cumulants, and in [2] using minimum-redundancy linear arrays for a more signals than sensors scenario. In this paper, we show that by using only second order statistics and an uniform linear array we can estimate the DOAs of signals in a Bayesian framework. The algorithm does not depend on the noise distribution as long as the noise is wide-sense stationary. The algorithm does not use any training data and only requires information about the cyclic frequencies. It is also possible to estimate the DOAs in a multi-path fading environment under some conditions.

2. PROBLEM STATEMENT

We consider the narrow-band signal model of L signals impinging on an array of M sensors:

$$\mathbf{x(t)} = \mathbf{A}(\Theta)\mathbf{s(t)} + \mathbf{n(t)} \tag{1}$$

where

$$\mathbf{x(t)} = [\mathbf{x_1(t)}, .., \mathbf{x_M(t)}]^T \quad \text{are the observations} \tag{2}$$
$$\mathbf{s(t)} = [\mathbf{s_1(t)}, .., \mathbf{s_L(t)}]^T \quad \text{are the signals} \tag{3}$$
$$\mathbf{n(t)} = [\mathbf{n_1(t)}, .., \mathbf{n_M(t)}]^T \quad \text{is the noise vector} \tag{4}$$
$$\mathbf{A}(\Theta) = [\mathbf{a}(\theta_\mathbf{1}), .., \mathbf{a}(\theta_\mathbf{L})] \tag{5}$$

The steering vector $\mathbf{a}(\theta_l)$ is:

$$\mathbf{a}(\theta_l) = [1, e^{-j2\pi\psi_l}, .., e^{-j2\pi(M-1)\psi_l}]^T \tag{6}$$
$$\psi_l = \frac{d}{\lambda}\sin(\theta_l)$$

where λ is the wavelength of the signals, d is the inter-sensor spacing and θ_l is the direction of arrival (DOA) of the l^{th} signal.

The source signals are second order cyclostationary; here we consider them to be digitally modulated communication signals:

$$s_l(t) = \sum_k A_l b_l(k) p_l(t - kT_l - \epsilon_l) e^{j2\pi f_{ol} t} \quad (7)$$

where A_l, ϵ_l, f_{ol} are respectively the constant amplitude, the delay and the frequency offset of the l^{th} signal and $p_l(t)$ is the shaping pulse.

Our aim is to estimate the directions of arrival $\Theta = \{\theta_l\}_{l=1}^{L}$ from N samples of $\mathbf{x(t)}$. We make the following assumptions:

(A1) The source signals are zero mean, statistically independent, second order cyclostationary processes.

(A2) The symbols $b_l(k)$ are zero-mean, stationary, uncorrelated and uniformly distributed over a finite alphabet.

(A3) The additive noise $n(t)$ is a wide-sense stationary process.

(A4) The source signals and the noise are statistically independent.

(A5) $\mathbf{x(t)}$ satisfy the *mixing condition*.

3. CYCLIC CORRELATION MODEL

Our algorithm is based on the CCCF of the sensor outputs. Most signals (e.g : BPSK, MSK, CPFSK) are conjugate cyclostationary at twice their frequency offset (or carrier frequency). In this paper, we assume that this frequency, which is related to the cyclic frequency α by $\alpha = 2f_{ol}$, is known or has been estimated. The frequency offsets of the multiple signals can be estimated from the samples of one of the sensor outputs using the blind frequency offset estimation algorithm we presented in [3].

We consider the scenario where the L signals share the same frequency offset $f_{ol} = f_o$. If they have different frequency offsets, then the algorithms that are presented in the following sections could be used with the appropriate cyclic frequencies. We assume we know L.

The conjugate cyclic cross-correlation of two signals $y(t)$, $z(t)$ at a cyclic frequency α and a lag τ is defined by:

$$R_{yz}^{\alpha}(\tau) = < y(t) z(t+\tau) e^{-j2\pi\alpha t} > \quad (8)$$

We oversample the received signals and form the CCCF between the sensor outputs from sensors u, v. At lag τ and $\alpha = 2f_o$, the CCCF is given by:

$$R_{uv}^{\alpha}(\tau) = \sum_{l=1}^{L} R_{s_l}^{\alpha}(\tau) e^{-j2\pi(u+v-2)\psi_l} + R_n^{\alpha}(\tau) \quad (9)$$

Since $R_n^{\alpha}(\tau) = 0$ for $\alpha \neq 0$, we can write:

$$R_{uv}^{\alpha}(\tau) = \sum_{l=1}^{L} R_{s_l}^{\alpha}(\tau) e^{-j2\pi(u+v-2)\psi_l} \quad (10)$$

In practice, we would only have a finite number of samples and $R_{uv}^{\alpha}(\tau)$ has to be estimated from these samples:

$$\hat{R}_{uv}^{\alpha}(\tau) = \frac{1}{N} \sum_{n=0}^{N-1} x_u(n) x_v(n+\tau) e^{-j2\pi\alpha n} \quad (11)$$

Under assumption (A5), $\hat{R}_{uv}^{\alpha}(\tau)$ converges in the mean square sense:

$$R_{uv}^{\alpha}(\tau) = \lim_{N\to\infty} E\{\hat{R}_{uv}^{\alpha}(\tau)\} \quad (12)$$

and $[\hat{R}_{uv}^{\alpha}(\tau) - R_{uv}^{\alpha}(\tau)]$ is asymptotically complex normal.

Hence we can write at a lag τ:

$$\hat{R}_{uv}^{\alpha}(\tau) = \sum_{l=1}^{L} R_{s_l}^{\alpha}(\tau) e^{-j2\pi(u+v-2)\psi_l} + e(\tau) \quad (13)$$

where $e(\tau)$ is the complex Gaussian estimation error.

For a M sensor array, $(u + v - 2) \in \{0, .., 2M - 2\}$. Considering appropriate combinations of the sensor outputs, such that $(u + v - 2) = 0, .., 2M - 2$ is in increasing order, and stacking them, we get the following structure:

$$\mathbf{r_{uv}}(\tau) = \mathbf{B}(\Theta)\mathbf{r_s}(\tau) + \mathbf{e}(\tau) \quad (14)$$

where

$$\begin{aligned}
\mathbf{r_{uv}}(\tau) &= [\hat{\mathbf{R}}_{\mathbf{1,1}}^{\alpha}(\tau), .., \hat{\mathbf{R}}_{\mathbf{M,M}}^{\alpha}(\tau)]^T && (15)\\
\mathbf{B}(\Theta) &= [\mathbf{b}(\theta_1), .., \mathbf{b}(\theta_L)] && (16)\\
\mathbf{b}(\theta_l) &= [1, e^{-j2\pi\psi_l}, .., e^{-j2\pi(2M-2)\psi_l}]^T &&\\
\mathbf{r_s}(\tau) &= [\mathbf{R}_{\mathbf{s_1}}^{\alpha}(\tau), .., \mathbf{R}_{\mathbf{s_L}}^{\alpha}(\tau)]^T && (17)
\end{aligned}$$

This is similar to the time domain model (1), but now the matrix $\mathbf{B}(\Theta)$ is $(2M-1) \times L$ and thus we can estimate $L < (2M-1)$ DOAs by utilising the CCCF of the sensor outputs and using a Bayesian approach.

It is also possible to consider multiple lags instead of just one lag. Multiple lags lead to improved performance than using just one lag at the cost of increased computation. Multiple lags were also used in [6] to improve the performance of the cyclic-MUSIC method. Stacking the CCCF corresponding to multiple lags we obtain:

$$\mathbf{d} = \mathbf{G}(\Theta)\mathbf{h} + \mathbf{e} \quad (18)$$

where

$$\begin{aligned}
\mathbf{d} &= [\mathbf{r_{uv}}(0), .., \mathbf{r_{uv}}(\Gamma - 1)]^T && (19)\\
\mathbf{h} &= [\mathbf{r_s}(0), .., \mathbf{r_s}(\Gamma - 1)]^T && (20)\\
\mathbf{G}(\Theta) &= \mathbf{B}(\Theta) \otimes \mathbf{I}_{\Gamma\times\Gamma} && (21)\\
\mathbf{e} &\sim \mathbf{N}(\mathbf{0}, \Sigma) && (22)
\end{aligned}$$

4. DIRECTION ESTIMATION

Since $\mathbf{e}$ is complex Gaussian we can write the likelihood as:

$$p(\mathbf{d}|\mathbf{h},\Theta,\Sigma,I) = (2\pi)^{-N/2}|\Sigma|^{-1/2}\exp[-\frac{1}{2}(\mathbf{d}-\mathbf{Gh})'\Sigma^{-1}(\mathbf{d}-\mathbf{Gh})] \quad (23)$$

where Σ is the covariance matrix.

We assume $\Sigma = \sigma^2\mathbf{I}$ for computational simplicity and thus the likelihood can be written as:

$$p(\mathbf{d}|\mathbf{h},\Theta,\sigma,I) = (2\pi\sigma^2)^{-N/2}\exp[-\frac{(\mathbf{d}-\mathbf{Gh})'(\mathbf{d}-\mathbf{Gh})}{2\sigma^2}] \quad (24)$$

Alternatively, we could whiten the conjugate cyclic cross-correlation outputs and then use the whitened data.

Using Bayes' theorem, with uniform priors for Θ and $\mathbf{h}$ and Jeffrey's prior for σ, the posterior probability density is given by:

$$p(\mathbf{h},\Theta,\sigma|\mathbf{d},I) = (2\pi\sigma^2)^{-N/2}\exp[-\frac{(\mathbf{d}-\mathbf{Gh})'(\mathbf{d}-\mathbf{Gh})}{2\sigma^2}]\frac{1}{\sigma} \quad (25)$$

Integrating out the nuisance parameters, the marginal posterior probability density is:

$$p(\Theta|\mathbf{d},I) = \frac{(\mathbf{d}'\mathbf{d}-\mathbf{d}'\mathbf{G}(\mathbf{G}'\mathbf{G})^{-1}\mathbf{G}'\mathbf{d})^{-(\Gamma-L)/2}}{\sqrt{\det(\mathbf{G}'\mathbf{G})}} \quad (26)$$

The Maximum A Posteriori (MAP) estimate of Θ is obtained as:

$$\hat{\Theta} = \max_{\Theta}[p(\Theta|\mathbf{d},I)] \quad (27)$$

This requires a multi-dimensional search of the posterior probability density to find Θ. We use an iterative method, where the L-dimensional search is split into a series of L one-dimensional searches.

5. MULTI-PATH ENVIRONMENT

In the previous sections, we presented an algorithm for DOA estimation of multiple uncorrelated signals. In this section, we consider multiple signals arriving at an antenna array via a multi-path fading environment. We consider the narrow-band, multi-path model and the m^{th} sensor output is given by:

$$x_m(t) = \sum_k\sum_{l=1}^{L}\sum_{q=1}^{Q_l} a(\theta_{l,q})\beta_{l,q}(t)b_l(k)p_l(t-kT_l-\epsilon_{l,q})e^{j2\pi f_o t} \quad (28)$$

where Q_l is the number of multi-paths corresponding to the l^{th} source signals, $\beta_{l,q}$ is the complex attenuation factor, which is assumed to be constant over a symbol period and $\epsilon_{l,q}$ is the constant delay associated with the q^{th} path of the l^{th} source. Considering the M sensor outputs, we can represent the model in the form as in (1) with:

$$\mathbf{A} = [\mathbf{a}(\theta_{1,1}),..,\mathbf{a}(\theta_{L,Q})] \quad (29)$$

$$\mathbf{a}(\theta_{\mathbf{l},\mathbf{q}}) = [1,..,e^{-j2\pi(M-1)\psi_{l,q}}]^T \quad (30)$$

$$\mathbf{s}(\mathbf{t}) = [\mathbf{s_{1,1}(t)},..,\mathbf{s_{L,Q_L}(t)}] \quad (31)$$

$$s_{l,q}(t) = \sum_k \beta_{l,q}(t)b_l(k)p_l(t-kT_l-\epsilon_{q,l})e^{j2\pi f_o t}$$

As before, utilising the CCCF between sensor outputs and considering appropriate combinations of the sensor outputs, such that $(u+v-2) = 0,..,2M-2$ and stacking them in increasing order, we obtain a conjugate cyclic cross-correlation model representation of the problem similar to (14):

$$\mathbf{r_{uv}}(\tau) = \mathbf{B}(\Theta)\mathbf{r_s}(\tau) + \mathbf{e}(\tau) \quad (32)$$

But, now the elements of $\mathbf{r_s}(\tau)$ will contain auto-correlation terms as well as cross-correlation terms due to the correlation between the multi-path signals:

$$\mathbf{r_s}(\tau) = \begin{pmatrix} \sum_{q=1}^{Q_1} < \mathbf{s_{1,1}(t)s_{1,q}(t+\tau)e^{-j2\pi\alpha t}} > \\ .. \\ .. \\ \sum_{q=1}^{Q_L} < \mathbf{s_{L,Q_L}(t)s_{L,q}(t+\tau)e^{-j2\pi\alpha t}} > \end{pmatrix} \quad (33)$$

The DOAs of the multi-paths of all the signals of interest can be estimated using an approach similar to that presented in the previous section if the following condition is satisfied:

$$\sum_{l=1}^{L} Q_l < 2M-1 \quad (34)$$

i.e., the total number of multi-paths should be less than $(2M-1)$.

6. SIMULATIONS

The simulations in this section were carried out for multiple BPSK signals with raised-cosine pulse shaping arriving at a

3-element antenna array with half wavelength spacing of the sensors. We used an oversampling factor of 4 and 30 lags.

In the first experiment, we considered 4 SOIs of symbol rate $1/T$ with the same frequency offset $f_o = 0.3/T$, and an interfering BPSK signal with the same symbol rate but different frequency offset, in an additive Gaussian noise environment with noise variance 0.1. We processed 500 symbols and obtained the DOAs of the four signals. The results of 20 trials are presented in the table below.

True (deg)	Mean (deg)	std (deg)
20	20.2840	0.7735
40	39.4394	0.7577
55	55.7169	0.7782
60	60.8746	0.8080

In the second experiment, we considered a multi-path scenario with 2 SOIs of symbol rate $1/T$ and frequency offset $f_o = 0.3/T$ arriving at the array via two multi-paths, in the presence of an interfering BPSK with the same symbol rate but different frequency offset. The multi-paths were modelled as Rayleigh fading and the additive noise had a variance of 1. The results of 20 trials, with a processing block of 500 symbols, are presented in the table below. The first two DOAs correspond to one signal and the other two to the second signal.

True (deg)	Mean (deg)	std (deg)
20	19.7697	0.8345
40	40.4104	0.7599
55	55.4800	1.0013
60	60.7242	0.9680

In the third experiment, we considered 2 SOIs of symbol rate $1/T$ and frequency offets $f_o = 1/T$, and 2 interfering BPSKs with the same symbol rate but different frequency offsets, and analysed the effect of the length of the processing block and the noise variance on the performance of the algorithm. The results are presented in Figure 1 (for the two DOAs). As expected, due to the additive noise suppression property the performance is less affected by the noise variance; however, as more symbols become available, the estimation error is reduced leading to better performance.

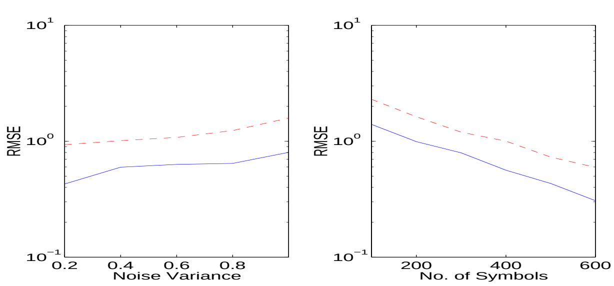

Figure 1: Effect of Noise and Number of Symbols ('-' : SOI 1, '- -' : SOI 2)

7. CONCLUSIONS

We have presented a direction estimation algorithm for multiple signals arriving at an antenna array. The algorithm is based on conjugate cyclic cross-correlations between the sensor outputs and the DOAs are estimated using a Bayesian framework. We have shown that it is possible to estimate the DOAs using only second order statistics and an uniform linear array, even when there are more signals than sensors. The algorithm is insensitive to any additive noise as long as it is wide-sense stationary, and does not require any training sequence. It is also possible to estimate the DOAs in a multi-path fading environment under some conditions about the total number of multi-paths. The drawback is the need for a multi-dimensional optimisation. Although, in this paper we used an iterative approach with a series of one-dimensional optimisations, it is sub-optimal. Recently, Markov chain Monte Carlo (MCMC) approaches are being used for parameter estimation in a Bayesian framework and MCMC techniques could be used to improve the performance of the algorithms presented here.

8. REFERENCES

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