REALIZABLE MIMO DECISION FEEDBACK EQUALIZERS

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ABSTRACT

A general MMSE decision feedback equalizer (DFE) for multiple input-multiple output channels is presented. It is derived under the constraint of realizability, requiring finite smoothing lag and causal filters both in the forward and feedback link. The proposed DFE, which provides an optimal structure as well as optimal filter degrees, is obtained as an explicit solution in terms of the channel and noise description. A generalization of the scalar zero-forcing DFE is also presented. Conditions for the existence of such an equalizer are discussed. We argue that when no zero-forcing equalizer exists, poor near-far resistance of the corresponding optimal minimum mean square error DFE can be expected. A numerical example indicates the potential advantages of using a decision feedback equalizer with appropriate structure. The main improvements are obtained at moderate to high signal-to-noise ratios.

1. INTRODUCTION

During the last years, communication channels with several inputs and outputs have received increasing interest. For example, such channels occur in cellular systems with antenna arrays at the receiver. A detector employed for this purpose must be able to cancel the effect of the dispersive channel as well as the interfering cross-talk. Multivariable generalizations of equalizers are potential candidates for such a detector.

One equalizer which has good performance at moderate complexity is the *decision feedback equalizer (DFE)*. Traditionally, DFE derivations either results in a DFE with non-causal feedforward filter [1], or a DFE where the structure is fixed prior to the design [2]. In the first case, the DFE cannot be accurately realized, whereas in the second case, the structure of the DFE may be inappropriate for the channel, leading to suboptimal performance.

In this paper we shall use a general *multiple input-multiple output (MIMO)* decision feedback equalizer (GDFE) for the detection. In contrast to a conventional MIMO DFE, the GDFE allows for IIR filters in both the feedforward and the feedback link. We derive closed form expressions for the parameters of a GDFE which minimizes the mean square error under the constraint of realizability, i.e. under the constraint of finite decision delay and causal filters. We also derive a general condition for the existence of a zero-forcing MIMO DFE. It turns out that the existence of a zero-forcing DFE can be used as an indication of near-far resistance of the corresponding minimum mean square DFE.

In a numerical example, we indicate the potential performance improvement of the general DFE as compared to a DFE with fixed structure. For high SNR, the improvement is significant.

2. MULTIVARIABLE CHANNEL MODELS

2.1. Prerequisites

A communication channel having multiple inputs and multiple outputs will be called a *multivariable* channel or a *multiple input-multiple output (MIMO)* channel. Multivariable channels will be described by *rational matrices*, i.e. matrices whose elements are causal rational functions in the unit delay operator q^{-1} ($q^{-1}x(k) = x(k-1)$). In addition, we assume that the channels are time-invariant and stable.

In some cases, the denominators of all the matrix elements will be constants, rather than polynomials. In these cases, the multivariable channel can be described by a *polynomial matrix* $\boldsymbol{P}(q^{-1}) \stackrel{\triangle}{=} \mathbf{P}_0 + \mathbf{P}_1 q^{-1} + \cdots + \mathbf{P}_{\delta P} q^{-\delta P}$. For any polynomial matrix, we also define

$$\boldsymbol{P}_*(q) \stackrel{\triangle}{=} \mathbf{P}_0^H + \mathbf{P}_1^H q + \cdots + \mathbf{P}_{\delta P}^H q^{\delta P} , \tag{1}$$

where q represents the forward shift operator. The degree of a polynomial matrix $\boldsymbol{P}(q^{-1})$ equals the highest degree of any of its elements, and is denoted δP.

2.2. Detection using antenna arrays

In cellular systems, multi-element antennas, also known as antenna arrays, are frequently used to reject interference and to reduce the effect of fading and noise [3]. The presence of an antenna array at the receiver leads to a channel model with multiple outputs, one for each antenna element.

When several users transmit simultaneously, we can explicitly incorporate multiple users into the model. Assuming that $d_j(k)$ is the symbol sequence transmitted from user j, $y_i(k)$ is the signal received at antenna i, and $v_i(k)$ represents the additive noise received at the same antenna, we define

$$d(k) \stackrel{\triangle}{=} \begin{pmatrix} d_1(k) & d_2(k) & \ldots & d_{n_d}(k) \end{pmatrix}^T \tag{2a}$$

$$y(k) \stackrel{\triangle}{=} \begin{pmatrix} y_1(k) & y_2(k) & \ldots & y_{n_y}(k) \end{pmatrix}^T \tag{2b}$$

$$v(k) \stackrel{\triangle}{=} \begin{pmatrix} v_1(k) & v_2(k) & \ldots & v_{n_y}(k) \end{pmatrix}^T . \tag{2c}$$

We also denote the scalar channel from transmitter j to receiver antenna i as $\mathcal{H}_{ij}(q^{-1})$ and define the rational matrix

$$\boldsymbol{\mathcal{H}}(q^{-1}) \stackrel{\triangle}{=} \begin{pmatrix} \mathcal{H}_{11}(q^{-1}) & \ldots & \mathcal{H}_{1n_d}(q^{-1}) \\ \vdots & \ddots & \vdots \\ \mathcal{H}_{n_y1}(q^{-1}) & \ldots & \mathcal{H}_{n_yn_d}(q^{-1}) \end{pmatrix} . \tag{3}$$

Using (2a), (2b), (2c) and (3), we can now express the signal received at the antenna array by the MIMO model

$$y(k) = \boldsymbol{\mathcal{H}}(q^{-1})d(k) + v(k) . \tag{4}$$

Winters [4] used this model to design a detector, which simultaneously detects the signals from all users.

The channel model (4) can also be used to describe fractionally spaced sampling [5] in which a scalar received signal is sampled several times during a symbol period.

2.3. Multiuser detection in DS-CDMA

In direct sequence code division multiple access (DS-CDMA) systems, the different users will interfere with each other, since the spreading sequences used to multiplex the signals cannot be chosen completely orthogonal, The magnitude of this *multiple access interference (MAI)* is determined by the cross-correlation between the spreading sequences of the different users. The presence of MAI implies that we can formulate a multivariable model to describe this scenario [6]:

$$y(k) = \mathbf{R}(1)\mathbf{W}d(k) + \mathbf{R}(0)\mathbf{W}d(k-1) + \mathbf{R}(-1)\mathbf{W}d(k-2) + v(k)\ . \quad (5)$$

In (5), the matrices $\mathbf{R}(m)$ contain partial cross-correlations between the spreading sequences $s_i(t)$:

$$(\mathbf{R}(m))_{i,j} \triangleq \int_{\tau_i}^{T_s+\tau_i} s_i(t-\tau_i)s_j^*(t+mT_s-\tau_j)dt\ ,$$

where $s_i(t) = 0$ outside $t \in [0, T_s[$; $\tau_i \in [0, T_s[$ is the propagation delay of user i with T_s and $(\cdot)^*$ denoting the symbol period and complex conjugate, respectively. The elements of the diagonal matrix $\mathbf{W}$ represent the energies and phase shifts of the respective users. The output $y(k)$ of the channel model is the sampled output from the correlators used to despread the received signal, and $v(k)$ constitutes noise. Obviously this model can be expressed by means of (4).

Design of multiuser detectors for DS-CDMA based on the model (5) has been studied extensively over the last decade, see e.g. [7].

3. PROBLEM STATEMENT

Consider the received sequence of measurement vectors $y(k)$. Assume that each vector can be described as a sum of the output from a dispersive, multivariable channel and a multivariate noise term as depicted in Fig. 1. Both the channel and the noise model

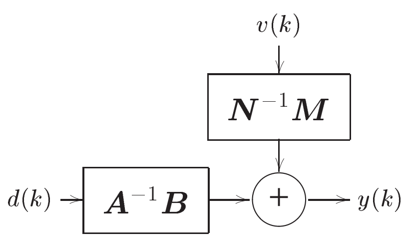

Figure 1: The multivariable system model.

are parameterized by *left matrix fraction descriptions (MFD:s)*:

$$y(k) = \boldsymbol{A}^{-1}(q^{-1})\boldsymbol{B}(q^{-1})d(k) + \boldsymbol{N}^{-1}(q^{-1})\boldsymbol{M}(q^{-1})v(k)\ . \quad (6)$$

The polynomial matrix $\boldsymbol{B}(q^{-1})$ has n_y rows and n_d columns, whereas $\boldsymbol{A}(q^{-1})$, $\boldsymbol{M}(q^{-1})$ and $\boldsymbol{N}(q^{-1})$ are square polynomial matrices of dimension n_y. These three matrices are assumed to be *stably invertible*, i.e. the roots of $\det \boldsymbol{A}(z^{-1}) = 0$, $\det \boldsymbol{M}(z^{-1}) = 0$ and $\det \boldsymbol{N}(z^{-1}) = 0$ all lie inside the unit circle $|z| = 1$.

We assume that the leading matrix coefficient of $\boldsymbol{M}(q^{-1})$ is non-singular and that the denominator matrices $\boldsymbol{A}(q^{-1})$ and $\boldsymbol{N}(q^{-1})$ are assumed monic. To simplify the presentation and the design equations, $\boldsymbol{A}(q^{-1})$ and $\boldsymbol{N}(q^{-1})$ are also assumed to be *diagonal*. The polynomial elements in the matrices may have complex coefficients, and are assumed to be correctly estimated.

Each element in the vector $d(k)$ is taken from a finite set of values, the so-called *symbol alphabet*. Each $d_i(k)$ is considered to be a stochastic variable with zero mean, which is uncorrelated with the disturbance vector $v(k)$. Finally, we assume that the transmitted symbol vectors are white with covariance matrix

$$Ed(k)d^H(k) = \lambda_d \mathbf{I}\ . \quad (7)$$

The noise vector $v(k)$ in (6) has n_y elements. It is a possibly complex-valued, white stochastic process with zero mean and covariance matrix

$$E[v(k)v^H(k)] = \lambda_v \mathbf{I}\ . \quad (8)$$

For future reference, we also define

$$\rho \triangleq \frac{\lambda_v}{\lambda_d}\ . \quad (9)$$

Our primary goal is to reconstruct the sequence of symbol vectors $d(k)$ from the measurements of $y(k)$. For this purpose, we introduce the general MIMO-decision feedback equalizer (GDFE):

$$\begin{aligned} \hat{d}(k-\ell|k) &= \boldsymbol{\mathcal{R}}(q^{-1})y(k) - \boldsymbol{\mathcal{F}}(q^{-1})\tilde{d}(k-\ell-1) \\ \tilde{d}(k-\ell) &= \mathrm{f}(\hat{d}(k-\ell|k))\ . \end{aligned} \quad (10)$$

See Fig. 2. The feedforward filter $\boldsymbol{\mathcal{R}}(q^{-1})$ and the feedback filter $\boldsymbol{\mathcal{F}}(q^{-1})$ are stable and causal rational matrices. The design variable ℓ is the *smoothing lag*, i.e. the number of future measurements used to estimate the current symbol. The function $\mathrm{f}(\cdot)$ constitutes the decision non-linearity, which maps the estimate $\hat{d}_i(k-\ell|k)$ to the closest point in the symbol constellation. The vector $\tilde{d}(k-\ell)$ thus constitutes the decision made on the estimate $\hat{d}(k-\ell|k)$.

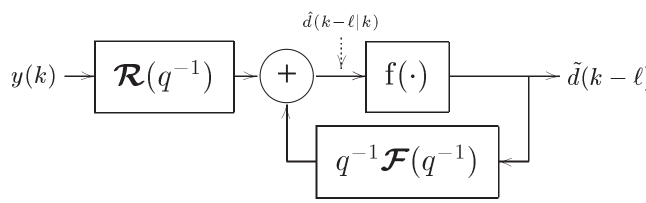

Figure 2: The general IIR decision feedback equalizer (GDFE).

It is important to note that the GDFE (10) is required to be *realizable*. This constraint implies that the smoothing lag ℓ must be finite, and the filters must be causal and stable. In [5], it is illustrated what happens when the constraint of realizability is relaxed.

Given the received sequence of symbol vectors $y(k)$ and the channel model (6), we want to find the stable and causal rational matrices $\{\boldsymbol{\mathcal{R}}(q^{-1}), \boldsymbol{\mathcal{F}}(q^{-1})\}$ which minimize the estimation error covariance matrix [1]

$$\mathbf{P} \triangleq E\varepsilon(k-\ell)\varepsilon^H(k-\ell) \quad (11)$$

[1]The covariance matrix is minimized in the sense that any other admissible choice of $\{\boldsymbol{\mathcal{R}}(q^{-1}), \boldsymbol{\mathcal{F}}(q^{-1})\}$ will result in an estimation error covariance matrix $\tilde{\mathbf{P}}$ such that $\tilde{\mathbf{P}} - \mathbf{P}$ is positive definite.

where the estimation error $\varepsilon(k-\ell)$ is defined as

$$\varepsilon(k-\ell) \stackrel{\triangle}{=} d(k-\ell) - \hat{d}(k-\ell|k) \ . \tag{12}$$

We are also interested in finding the conditions under which a *zero-forcing* solution to the equalization problem exists. While a scalar zero-forcing equalizer removes all intersymbol interference from the symbol estimate, a natural extension to the multivariable case would be to require that both the intersymbol interference and the co-channel interference can be removed [8]. A multivariable zero forcing (ZF) equalizer can then be defined accordingly:

Definition 1 *Consider the channel model* (6) *and a multivariable equalizer which forms the estimate* $\hat{d}(k-\ell|k)$ *of a transmitted symbol vector* $d(k-\ell)$*. If*

$$\hat{d}(k-\ell|k) = d(k-\ell) - \varepsilon(k-\ell) \tag{13}$$

where $\varepsilon(k-\ell)$ *is uncorrelated with all transmitted symbol vectors* $d(m)\ \forall\ m$*, then the equalizer is said to be* zero-forcing*.*

To obtain a closed form expression for the parameters of the optimal GDFE, we adopt the usual assumption that all decisions affecting the current estimate are correct.

4. OPTIMUM GENERAL DECISION FEEDBACK EQUALIZERS

4.1. The optimum MMSE GDFE

Introduce the polynomial matrices

$$\mathbf{\Gamma}(q^{-1}) \stackrel{\triangle}{=} \boldsymbol{A}(q^{-1})\boldsymbol{M}(q^{-1}) \tag{14a}$$
$$\boldsymbol{\tau}(q^{-1}) \stackrel{\triangle}{=} \boldsymbol{N}(q^{-1})\boldsymbol{B}(q^{-1}) \ . \tag{14b}$$

and define $\tilde{\mathbf{\Gamma}}(q^{-1})$ and $\tilde{\boldsymbol{\tau}}(q^{-1})$ by the *coprime factorization*

$$\tilde{\boldsymbol{\tau}}(q^{-1})\tilde{\mathbf{\Gamma}}^{-1}(q^{-1}) = \mathbf{\Gamma}^{-1}(q^{-1})\boldsymbol{\tau}(q^{-1}) \tag{15}$$

where $\tilde{\boldsymbol{\tau}}(q^{-1})\tilde{\mathbf{\Gamma}}^{-1}(q^{-1})$ constitutes an irreducible MFD.

The general MMSE DFE is then given by the following theorem:

Theorem 1 *Assume that a multivariable channel can be described by* (6)*, and that the transmitted data and the noise are described by* (7) *and* (8) *respectively. Assuming correct past decisions, the general multivariable DFE* (10) *minimizes the estimation error covariance matrix* (11) *if and only if*

$$\boldsymbol{\mathcal{R}}(q^{-1}) = \boldsymbol{S}(q^{-1})\boldsymbol{M}^{-1}(q^{-1})\boldsymbol{N}(q^{-1}) \tag{16a}$$
$$\boldsymbol{\mathcal{F}}(q^{-1}) = \boldsymbol{Q}(q^{-1})\tilde{\mathbf{\Gamma}}^{-1}(q^{-1}) \ . \tag{16b}$$

Above, $\boldsymbol{S}$ *and* $\boldsymbol{Q}$*, together with the polynomial matrices* $\boldsymbol{L}_1$ *and* $\boldsymbol{L}_2$ *satisfy the two coupled polynomial matrix equations*

$$\tilde{\mathbf{\Gamma}} - q^{\ell}\boldsymbol{S}\tilde{\boldsymbol{\tau}} + q^{-1}\boldsymbol{Q} = \boldsymbol{L}_{1*}\tilde{\mathbf{\Gamma}} \tag{17a}$$
$$q^{-\ell}\boldsymbol{L}_{1*}\boldsymbol{\tau}_* - \rho\boldsymbol{S}\mathbf{\Gamma}_* = q\boldsymbol{L}_{2*} \tag{17b}$$

where the degrees of the unknown polynomials satisfy

$$\begin{aligned} \delta S &= \ell, \quad \delta Q = \max(\delta\tilde{\Gamma}, \delta\tilde{\tau}) - 1 \\ \delta L_1 &= \ell, \quad \delta L_2 = \max(\delta\tau, \delta\Gamma) - 1 \ . \end{aligned} \tag{18}$$

Proof: See [5]. ■

The presented MMSE solution provides an *optimal DFE structure* with *optimal filter degrees*. On the other hand, the conventional DFE structure, where both the feedforward and the feedback filters have finite impulse responses, is optimal *only* when $\boldsymbol{A}(q^{-1}) = \boldsymbol{M}(q^{-1}) = \mathbf{I}$.

In addition to providing an optimal DFE structure and optimal filter degrees, Theorem 1 gives guidelines on how to choose the filter degrees in a conventional structure when the optimum structure is deemed inappropriate.

4.2. The zero-forcing GDFE

We now turn our attention to the general zero-forcing DFE.

Theorem 2 *Consider the multivariable channel model* (6) *and the general DFE* (10)*. There exists a multivariable DFE satisfying the zero-forcing condition* (13) *if and only if there exist rational matrices* $\boldsymbol{\mathcal{R}}(q^{-1})$ *and* $\boldsymbol{\mathcal{F}}(q^{-1})$ *such that*

$$q^{-\ell}\mathbf{I} = \boldsymbol{\mathcal{R}}\boldsymbol{A}^{-1}\boldsymbol{B} - q^{-\ell-1}\boldsymbol{\mathcal{F}} \ . \tag{19}$$

Proof: See [5]. ■

Equation (19) may have several solutions. However, in some cases no solution to (19) will exist. The precise condition for this is stated in Lemma 1.

Lemma 1 *There exists a solution to* (19) *if and only if every common right factor*[2] *of* $\boldsymbol{A}^{-1}\boldsymbol{B}$ *and* $q^{-\ell-1}\mathbf{I}$ *is also a right factor of* $q^{-\ell}\mathbf{I}$*.*

Proof: The results follows from the general theory of Diophantine equations. See [9]. ■

Lemma 1 can be used to determine if a zero-forcing solution exists for any given channel. However, we can also use Lemma 1 to find cases where trivial channel characteristics make the existence of a zero-forcing equalizer impossible. The following two corollaries deal with this issue.

Corollary 1 *If* $n_d > n_y$*, then* $\boldsymbol{A}^{-1}\boldsymbol{B}$ *and* $q^{-\ell-1}\mathbf{I}$ *will always have a common right factor which is not a right factor of* $q^{-\ell}\mathbf{I}$*.*

Proof: See [5]. ■

If we define

$$\Delta_i \stackrel{\triangle}{=} \text{the minimal delay of user } i \text{ in any channel.} \tag{20}$$

then the second corollary can be stated as follows:

Corollary 2 *If* $\Delta_i > \ell$ *for some* i*,* $\boldsymbol{A}^{-1}\boldsymbol{B}$ *and* $q^{-\ell-1}\mathbf{I}$ *will always have a common right factor which is not a right factor of* $q^{-\ell}\mathbf{I}$*.*

Proof: See [5]. ■

When no ZF equalizer can be found, there will be some residual interference at the output of the corresponding MMSE equalizer, irrespective of the noise level. In the limit as the powers of the interfering signals go to infinity, the MMSE detector will become useless. This phenomenon is called the *near-far* problem in the CDMA literature, and detectors that are capable of handling a situation with very disparate transmitter powers are said to be *near-far resistant*.

The discussion above suggests that the existence of a zero-forcing DFE can be used as an indicator of the near-far resistance of the corresponding MMSE DFE.

[2] The right factors should be members of the ring of stable and causal rational matrices [9].

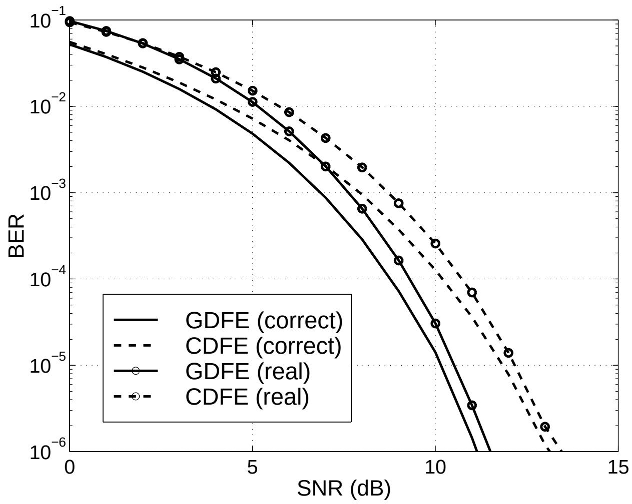

Figure 3: The BER of the GDFE (solid) compared to the BER of the conventional DFE (dashed) for correct decisions and real decisions.

5. A NUMERICAL EXAMPLE

To illustrate the potential performance improvements of the GDFE, we have performed a Monte Carlo simulation. Consider the two input-two output FIR channel

$$\boldsymbol{B}(q^{-1}) = \begin{pmatrix} 0.979 + 0.204q^{-1} & 0.826 + 0.563q^{-1} \\ -0.843 - 0.538q^{-1} & 0.403 + 0.915q^{-1} \end{pmatrix}.$$

Over this channel, we transmit two BPSK modulated signals, i.e. $d_j(k) = \{+1, -1\},\ j = 1, 2$. At the receiver, noise is added. The noise is Gaussian and can be described by the MA model

$$\boldsymbol{M}(q^{-1}) = \begin{pmatrix} -0.481 - 0.148q^{-1} & -0.629 + 0.592q^{-1} \\ -0.53 + 0.265q^{-1} & 0.795 - 0.132q^{-1} \end{pmatrix}.$$

This noise model has zeros in $z_{1,2} = 0.298 \pm 0.321i$, and represents rather weakly colored noise. We compare the performance of two DFE:s with smoothing lag $\ell = 3$:

- The GDFE, designed using Theorem 1.
- The fixed structure, or conventional, DFE described in [2] with feedforward filter degree $\delta S = 4$ and feedback filter degree $\delta Q = 1$.[3]

In Fig. 3, the bit error rate (BER) is shown as a function of the signal-to-noise ratio (SNR) of a single user. The SNR:s of the two users are identical. The simulations are performed using either correct decisions or decisions from the decision device.

From Fig. 3, we conclude that it is clearly advantageous to take the noise model into account. The GDFE does this in an optimal way, whereas the conventional DFE does not. The performance of the two DFE:s are almost identical for low SNR, but for high SNR, the difference is significant.

From Fig. 3, we also note that with real decisions, the performance of both DFE:s worsen, but that the difference in performance between the two DFE:s remains, except at low SNR:s.

[3]These degrees have been chosen so that both DFE:s are described using the same number of parameters.

6. CONCLUSIONS

From a practical point of view, a decision feedback equalizer must be realizable. Also, significant performance improvements can be achieved if the structure and filter degrees of the DFE are appropriate. We have presented a solution which fulfills both these requirements. The solution, which is explicit in terms of the channel and noise description, offer both insight and useful design equations, properties which are vital for detector design.

We have also presented new findings regarding the near-far resistance of the MMSE decision feedback equalizer. By investigating the possible existence of a ZF MIMO DFE, important conclusions can be drawn: If such an equalizer does not exist the corresponding MMSE DFE will not be near-far resistant.

The utility of the general DFE is demonstrated in a numerical example. This example indicates that for colored noise and moderate to high signal-to-noise ratios, the GDFE outperforms the conventional DFE. The performance improvement may however depend on the location of the zeros of the noise description. We anticipate this conclusion to hold also for time-varying channels. Investigation of these aspects is currently underway.

7. REFERENCES

[1] Alexandra Duel–Hallen, "Equalizers for multiple input/multiple output channels and PAM systems with cyclostationary input sequences," *IEEE Journal on Selected Areas in Communications*, vol. 10, no. 3, pp. 630–639, Apr. 1992.

[2] Claes Tidestav, Mikael Sternad, and Anders Ahlén, "Reuse within a cell—multiuser detection or interference rejection?," submitted.

[3] Peter Monsen, "MMSE equalization of interference on fading diversity channels," *IEEE Transactions on Communications*, vol. 32, no. 1, pp. 5–12, Jan. 1984.

[4] Jack Winters, "On the capacity of radio communication systems with diversity in a Rayleigh fading environment," *IEEE Journal on Selected Areas in Communications*, vol. 5, no. 5, pp. 871–878, June 1987.

[5] Claes Tidestav, Anders Ahlén, and Mikael Sternad, "The structure and design of realizable MIMO decision feedback eqaulizers," submitted.

[6] Ruxandra Lupas and Sergio Verdú, "Near-far resistance of multi-user detectors in asynchronous channels," *IEEE Transactions on Communications*, vol. 38, no. 4, pp. 496–508, Apr. 1990.

[7] Alexandra Duel-Hallen, Jack Holtzman, and Zoran Zvonar, "Multiuser detection for CDMA systems," *IEEE Personal Communications Magazine*, vol. 2, no. 2, pp. 46–58, Apr. 1995.

[8] Philip Balaban and Jack Salz, "Optimum diversity combining and equalization in digital data transmission with applications to cellular mobile radio — part I: Theoretical considerations," *IEEE Transactions on Communications*, vol. 40, no. 5, pp. 885–894, 1992.

[9] Vladimír Kučera, *Analysis and Design of Discrete Linear Control Systems*, Prentice Hall, 1991.