NON-UNIFORM FILTERBANK BANDWIDTH ALLOCATION FOR SYSTEM MODELING SUBBAND ADAPTIVE FILTERS

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ABSTRACT

Subband adaptive filters have been used extensively in system modeling configurations to model unknown systems with large impulse responses. This paper will illustrate the advantages of a non-uniform subband adaptive filter over a uniform subband adaptive filter while giving insight to subband bandwidth allocations for system modeling configurations. By implementing small subbands which isolate the transition regions in the unknown system, while using larger subbands for other, more spectrally flat regions, one can minimize convergence time and lower misadjustment.

1. INTRODUCTION

Subband adaptive filters were developed to combat the convergence rate and computational complexity problems associated with large full-band adaptive filters. High order adaptive filters are commonly needed in system modeling applications such as acoustic echo cancellation [6]. A subband adaptive filter in a system modeling configuration is illustrated in Figure 1. In the figure, the unknown system is denoted by $\mathbf{h}$, the analysis and synthesis filters for the n^{th} subband are $\mathbf{f}_n$ and $\mathbf{g}_n$ respectively, and $\uparrow D$ and $\downarrow D$ denote upsampling and downsampling by a factor of D.

Traditionally, uniform filterbanks have been designed for use with subband adaptive filters. To avoid computationally expensive adaptive cross-filters, oversampled filterbanks are usually employed to reduce the amount of aliasing between subbands [9]. However, oversampled filterbanks decrease the spectral efficiency of the subbands thereby preempting a slow asymptotic convergence rate [3, 12] in both the subband mean-squared error (MSE) and the full-band MSE. Some research has been done to improve the overall convergence rate for the oversampled subband adaptive filtering system by implementing increased bandwidth filterbank analysis filters [4, 6].

Non-uniform filterbanks open a new dimension of freedom for subband adaptive filtering, since the filterbank bandwidths can now be adapted. Previous experimental results have shown that non-uniform adaptive filterbanks may have reduced MSE over uniform adaptive filterbanks [10]. This paper presents a method for allocating bandwidths to the individual subband filters, such that the convergence time is minimized. Experimental results also will show the advantages of the non-uniform filterbank over the uniform.

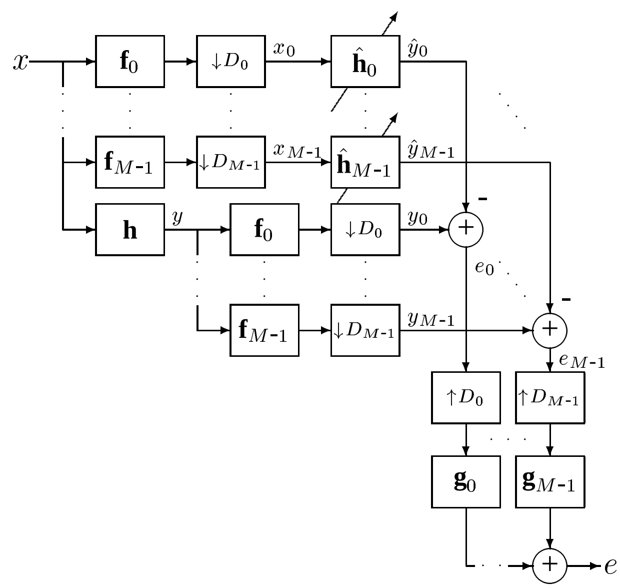

Figure 1: Non-uniform Subband Adaptive Filter System Modeling Configuration

2. NON-UNIFORM BANDWIDTH ALLOCATION

Non-uniform subbands could be allocated such that they fit into two distinct categories. The first classification is for spectrally flat subbands, while the second category distinguishes subbands with a transition. In this way, the unknown spectrum is split into non-uniform subbands, such that the spectrally flat subbands contain as much bandwidth as possible, while the transition subbands contain as little bandwidth as possible while still containing the entire transition band of the unknown filter.

In general, the spectral properties of the unknown filter are not known apriori. However, a uniform (or another initial estimate) filterbank can be used to initially identify properties of the unknown system and then be changed or adapted afterwards. One can maintain estimates of the autocorrelation sequences for the desired signals in each subband (signals $y_0 \dots y_{M\text{-}1}$ in Figure 1). This is given by

$$\mathbf{r}_{y_k}(t) = \lambda \mathbf{r}_{y_k}(t-1) + (1-\lambda) + \tilde{\mathbf{Y}}_k \mathbf{y}_k \tag{1}$$

where the autocorrelation vector of y_k at time t is denoted by $\mathbf{r}_{y_k}(t) = [r_{y_k}(-L+1) \dots r_{y_k}(L-1)]^T$ ($r_{y_k}(n)$ is the n^{th} lag), $\mathbf{y}_k = [y_k(t) \dots y_k(t-L+1)]^T$ is the sequentially time-delayed desired signal vector, and $\tilde{\mathbf{Y}}_k$ is the convolution matrix such that $\tilde{\mathbf{Y}}_k \mathbf{y}_k$ computes the autocorrelation vector for $\mathbf{y}_k$. The autocorrelation forgetting factor is given by λ where $0 \leq \lambda < 1$. Updating the autocorrelation vector in this manner is similar to a rank-1 update for an autocorrelation matrix [7, 14]. This process is effectively $O(L)$ multiplies, since only the multiplies concerning the most recent sample, $y_k(t)$, need be computed. A zero-phase DFT matrix could then be applied[1] to the autocorrelation vector, $\mathbf{r}_{y_k}(t)$, to obtain the power spectral density for the k^{th} subband [13, 14]. The bandwidth of the subband can then be estimated by applying a simple threshold test to the power spectral coefficients.

The resolution of the power spectral density is effectively given by L. L should be chosen such that it is large enough to provide meaningful spectral information, but small enough such that it is computationally possible to compute the DFT. Also, the oversampling must be taken into account. The bandwidth estimations referred to in this paper, refer to the bandwidths that would be measured if the subbands were critically decimated.

Subbands that are bandlimited must contain a spectral transition. These subbands need to be reduced in bandwidth by giving bandwidth to neighboring spectrally flat subbands to effectively isolate the transition. One can determine how to reduce the bandwidth, by examining the subband's power spectral data. $f_{k,1}$ should be increased for spectrally flat regions before the transition and $f_{k,2}$ should be decreased to move spectrally flat regions after the transition to a neighboring spectrally flat subband, where $f_{k,1}$ and $f_{k,2}$ are the lower and higher -3dB cutoff frequencies for the k^{th} analysis filter respectively. Ideally, correctly fitted transition subbands should only be as wide as the transition.

Full bandwidth subbands are spectrally flat. The bandwidth for these subbands may be increased unless a significant spectral transition occurs from the increased bandwidth. Correctly fitted spectrally flat subbands should therefore be as large as possible without containing any spectral transitions.

For example, a non-uniform filterbank can be constructed from uniform constituent filters simply by building a uniform filterbank with a large number of filters and adding subsets of neighboring filters to form the desired subband bandwidths as in [2]. By saving the constituent filterbank, it is possible to modify the subband bandwidths easily, since constituent filters need only to be added and subtracted from the subband filters to exchange bandwidth between subbands. It is sufficient to have L equal to the number of constituent filters incorporated into each subband for critical subsampling. If the non-uniform filterbank is degenerately uniform, then L would be equal for each subband. Since subbands are typically oversampled to avoid cross-filters, one may choose to add more resolution to space the DFT points better within the subbands' passbands. Using significantly more resolution does not make sense, since the filterbank bandwidths are inherently limited by the constituent filters.

If the filterbank can be adapted easily over time, as with non-uniform banks constructed from constituent filters, then the filterbank could be adapted repeatedly as needed. If the unknown system is time-varying, then this would be a necessity for fast convergence. These ideas present an algorithm to adapt the filterbank which is similar to RLS adaptive filtering, since it involves maintaining an estimate of a current autocorrelation sequence. The adaptive algorithm outlined in [10] presents a more LMS-like algorithm.

2.1. Transition Subband Convergence

For the subbands classified as transitional, these subbands will converge quickly simply because the subbands are assumed to contain small amounts of bandwidth. Since the subbands are small, the subbands can be highly decimated, as can the adaptive filter length. Small adaptive filters adapt fast and are computationally efficient, since there are fewer eigenvalue restrictions on step-size and fewer coefficients require adaptation [7].

Compared to a uniform filterbank, most of the bandwidth for the transition subbands has been given to neighboring spectrally flat subbands. Uniform subbands which contain a spectral transition typically are wider than the non-uniform transition bands. The wider bands will typically have similar input eigenvalue spreads, but may have more subdominant eigenvalues than the tighter, non-uniform bands [11]. Subdominant eigenvalues cause the wider, uniform

[1] The DFT is a $O(L^2)$ operation. For more computational efficiency, one could apply the FFT to the latter half of the autocorrelation sequence, $[r_{y_k}(0) \dots r_{y_k}(L-1)]^T$, to give a $O(L \log L)$ operation assuming that L is a power of 2. However, applying the operation to only the latter half of the sequence, does produce some spectral smoothing which could possibly confuse bandwidth estimates made from this sequence. [13, 14]

bans to converge slower than the tighter, non-uniform bands, since coefficient convergence is primarily dependent on the smallest input eigenvalue or slowest mode [7].

2.2. Spectrally Flat Subband Convergence

The spectrally flat subbands converge at the same rate as a comparable uniform spectrally flat subband. The larger non-uniform spectrally flat subband adaptive filters can be viewed as a higher order version of the uniform subband adaptive filter. This implies more input eigenvalues exist that can limit the convergence rate. However, the eigenvalues are distributed in the same way as the uniform case, since the input is essentially the same. Furthermore, with similar input eigenvalue distributions, coefficient convergence will be similar. Also, oversampling will generally induce at least one small eigenvalue regardless of bandwidth [11] which will effectively limit the convergence rate.

Since spectrally flat subbands will essentially adapt to an impulse response, the coefficient vector is very sparse. Sparse adaptive filters, or multi-tap adaptive delay filters, may be used in these subbands to obtain faster convergence with less misadjustment and with less computation than traditional adaptive filters, since only the most significant impulse response coefficients are estimated [1, 8]. Recent work has proposed an even faster algorithm for adaptive delay filtering [15].

3. EXPERIMENTAL RESULTS

Figure 2 illustrates an “unknown” spectrum to model as well as the non-uniform and uniform filterbank analysis filters. 2^{14} samples of white, Gaussian noise were used to form the input signal, x. The subbands were decimated by $\lfloor \mathrm{BW}_k \cdot 1.8 \rfloor$, where BW_k represents the normalized bandwidth of the k^{th} analysis filter ($\frac{f_{k,2}-f_{k,1}}{f_N}$, where f_N is the Nyquist frequency). The “unknown” filter was a 512^{th} order FIR filter, and the subband adaptive filter was also 512^{th} order. NLMS [7] adaptive filters were used in each subband with $\mu = 1$.

The filterbank filters were constructed from a uniform constituent filterbank [2] containing 40 constituent filters. The constituent filters were formed by a complex exponentially modulated prototype filter, p_0, of 160 coefficients as in [5]. The analysis and synthesis modulation equations are given by equations (2) and (3) respectively, where $\theta_k = \frac{\pi}{4}(-1)^k$ and N is the order of the prototype filter. The modulation equations simply complex-modulate the low-pass prototype filter to each subband’s center frequency. The phase adjustment of θ_k is needed to cancel neighboring subband aliasing. For real input signals, n filters will form a sufficient subband decomposition. However, for complex input signals, $2n$ filters are necessary to cover the entire spectrum. This is essentially only a generalization from the traditional pseudo-QMF cosine-modulation equations [16].

$$f_k(n) = p_0(n)\exp\left\{\frac{j\pi}{M}(k+0.5)(n-\frac{N}{2}) + j\theta_k\right\} \quad (2)$$

$$g_k(n) = p_0(n)\exp\left\{\frac{j\pi}{M}(k+0.5)(n-\frac{N}{2}) - j\theta_k\right\} \quad (3)$$

The MSE for the uniform and non-uniform filterbanks is illustrated in Figure 3. The MSE data was averaged over 25 simulations (1 block = 2^{14} input samples). Figure 4 shows the final adapted spectrums of the non-uniform and uniform subband adaptive filters along with the “unknown” spectrum for 2^{14} input samples. Table 1 shows MSE calculations averaged over the last fifth of the MSE data for 2^{14} and 2^{17} input samples.

	2^{14} samples	2^{17} samples
Uniform	6.86×10^{-3}	6.98×10^{-3}
Non-Uniform	3.17×10^{-4}	2.12×10^{-4}

Table 1: Final MSE Comparison

Figure 4 illustrates misadjustments in the second and fourth uniform subband which produce small peaks in the spectrum at $.4$ and $.6$. The non-uniform subband adaptive filter, which has tighter transition subbands converge with less misadjustment and ultimately produces a lower MSE.

4. CONCLUSIONS AND FUTURE WORK

This paper has outlined a technique for allocating bandwidth to non-uniform filterbank filters. By implementing small transition subbands around the transitions in the unknown system and larger spectrally flat subbands elsewhere, MSE and computational complexity is minimized. An RLS-like algorithm has been suggested to determine or adapt subband bandwidths. Finally, experimental results were presented which show that a properly allocated non-uniform subband adaptive filter can converge faster and with less misadjustment than a uniform subband adaptive filter.

Future work will entail further studies of applications with less ideal spectra. Analysis applied to systems with no true spectrally flat regions must be performed to validate this work towards general use. Also, further comparisons will investigate an optimal number of subbands along with an adaptive number of subbands. Work will also be done to relate order-update adaptive algorithms to minimize subband misadjustment.

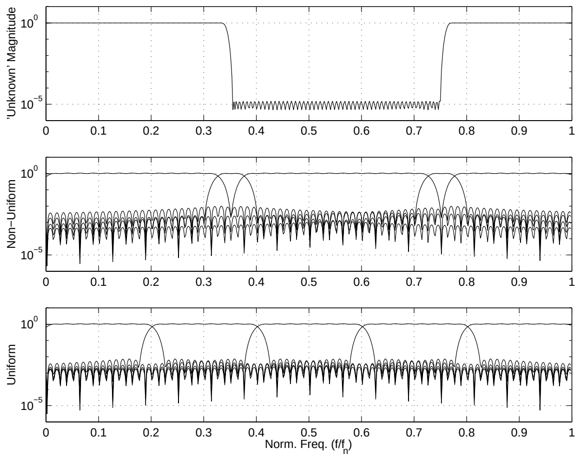

Figure 2: “Unknown” System and Associated Non-Uniform and Uniform Filterbank Analysis Filters ($M = 5$)

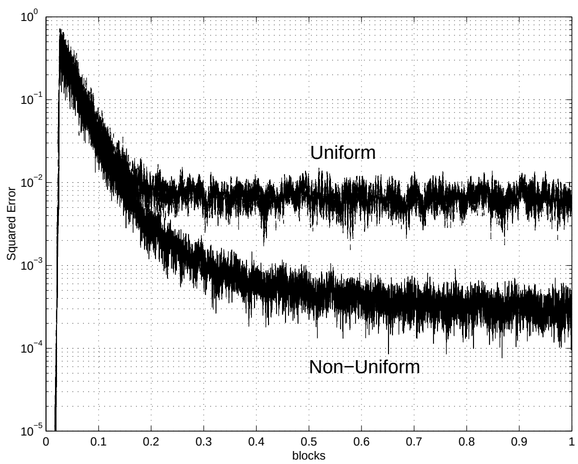

Figure 3: MSE for Uniform and Non-Uniform Subband Adaptive Filters

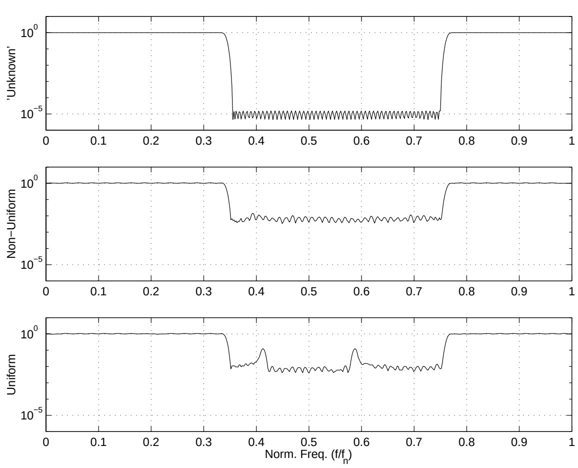

Figure 4: Adapted Spectra

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