BLIND ESTIMATION OF SYMBOL TIMING AND CARRIER FREQUENCY OFFSET IN PULSE SHAPING OFDM SYSTEMS

Helmut Bölcskei

Institute of Communications and Radio-Frequency Engineering
Vienna University of Technology, Gusshausstrasse 25/389, A-1040 Vienna, Austria
phone: +43 1 58801 38927; fax: +43 1 587 05 83; e-mail: hboelcsk@aurora.nt.tuwien.ac.at

Abstract—We introduce a blind algorithm for the joint estimation of symbol timing and carrier frequency offset in pulse shaping OFDM systems. The proposed estimator exploits the cyclostationarity of the received OFDM signal and can be seen as an extension of the Gini-Giannakis estimator [1] for single-carrier systems. An important feature of our method is the capability to perform a carrier frequency acquisition over the entire bandwidth of the OFDM signal. Furthermore, our estimator can be applied even if no cyclic prefix is used. We provide simulation results demonstrating the performance of the new estimator.

1. INTRODUCTION AND OUTLINE

Orthogonal frequency division multiplexing (OFDM) systems [2]-[6] are in general more sensitive to symbol timing errors and carrier frequency offsets than single-carrier systems [7]. In an OFDM system synchronization errors cause both intersymbol-interference (ISI) and intercarrier-interference (ICI). Most OFDM time-frequency offset estimators proposed in the literature require pilot symbols (e.g. [8, 9]). However, the use of pilot symbols lowers the data rate. Therefore, methods that do not need pilot symbols are desirable. Such estimators [10, 11] make use of the redundancy introduced by the cyclic prefix (CP).

In this paper, we present a blind algorithm[1] for the joint estimation of symbol timing and carrier frequency offset in *pulse shaping OFDM systems.* Our estimator exploits the cyclostationarity of the received OFDM signal and can be viewed as an extension of the Gini-Giannakis estimator [1] for single-carrier systems. We shall next summarize important novel features of the proposed method:

- it applies to pulse shaping OFDM systems with arbitrary pulse shapes.
- it can be used in OFDM systems employing arbitrary time-frequency guard regions.
- it is capable of performing a carrier frequency acquisition over the entire bandwidth of the OFDM signal.
- it applies to time-dispersive environments.
- it does not need a CP (in this case the estimators proposed in [10, 11] would break down).
- it is FFT-based and hence computationally efficient.

The paper is organized as follows. In Section 2 we briefly describe OFDM systems employing a time-frequency guard region [12, 13]. Section 3 introduces the new estimator and discusses its properties. Section 4 presents simulation results, and finally Section 5 concludes the paper.

[1]The algorithm is blind because it does not need pilot symbols. In fact, it does not even need a CP.

2. OFDM SYSTEMS EMPLOYING A TIME-FREQUENCY GUARD REGION

Time-frequency guard region. The baseband equivalent of a pulse shaping OFDM system is given by

$$x(t) = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} c_{k,l}\, g(t-lT)\, e^{j2\pi kF(t-lT)},$$

where T is the symbol duration, F denotes the subcarrier spacing, N is the number of carriers, $g(t)$ is the transmitter pulse shaping filter, and $c_{k,l}$ denotes the data symbols. The reconstructed symbols $\hat{c}_{k,l}$ are obtained as $\hat{c}_{k,l} = \langle x, h_{k,l} \rangle$, where $h_{k,l}(t) = h(t-lT)e^{j2\pi kF(t-lT)}$ with the receiver pulse shaping filter $h(t)$. In an OFDM system employing a CP [3] $g(t)$ is a rectangular pulse of duration T, $h(t)$ is a rectangular pulse of duration $T - T_c$ with T_c denoting the length of the CP (typically $T_c = 0.25T$), and $F = \frac{1}{T-T_c}$, so that $TF > 1$. The CP acts as a temporal guard interval and allows for equalization using simple divisions [3].

In this paper we consider pulse shaping OFDM systems with $TF \geq 1$ and $g(t)$ and $h(t)$ satisfying the biorthogonality relation $\langle g, h_{k,l} \rangle = \delta[k]\delta[l]$ (this guarantees perfect demodulation in the absence of a channel). For $TF > 1$, the system is said to employ a time-frequency guard region [12, 13]. In fact, OFDM systems using a CP [3] can be seen as a special case thereof with the time-frequency guard region being a temporal guard region only. Note, however, that $TF > 1$ can not only be achieved by insertion of a CP; for example, one can introduce spectral guard regions by spacing the subcarriers further apart to avoid ICI in frequency-dispersive environments. Although the use of a time-frequency guard region reduces spectral efficiency [6], the resulting advantages such as increased dispersion robustness [6, 13], pulse shaping filters with improved time-frequency localization [6, 13], and the possibility to perform blind equalization [14], generally motivate its use. We finally note that pulse shaping is especially important for reducing out-of-band emission in wireless OFDM.

Discrete-time model. Assuming that the total bandwidth of the OFDM signal $x(t)$ is approximately NF and setting[2] $TF = \frac{M}{N} \geq 1$, a critically sampled version of $x(t)$ is obtained as $x[n] = \sum_{k=0}^{N-1} \sum_{l=-\infty}^{\infty} c_{k,l}\, g[n - lM]e^{j\frac{2\pi}{N}k(n-lM)}$, where $x[n] \triangleq x\left(\frac{n}{NF}\right)$ and $g[n] \triangleq g\left(\frac{n}{NF}\right)$. The reconstructed data symbols are given by $\hat{c}_{k,l} = \langle x, h_{k,l} \rangle$ with $h_{k,l}[n] = h[n-lM]e^{j\frac{2\pi}{N}k(n-lM)}$. The real-valued pulses

[2]Note that usually N is very large, so that this choice for TF does not impose a severe restriction on the possible values of TF.

$g[n]$ and $h[n]$ are biorthogonal if their cross-ambiguity function $A^{(g,h)}[k,\theta] = \sum_{n=-\infty}^{\infty} g[n]h[n-k]e^{-j2\pi n\theta}$ [15] satisfies

$$A^{(g,h)}\left[lM, \frac{k}{N}\right) = \delta[l]\delta[k]. \quad (1)$$

For $h[n] = g[n]$ Eq. (1) reduces to $A^{(g,g)}\left[lM, \frac{k}{N}\right) = \delta[l]\delta[k]$ with the auto-ambiguity function $A^{(g,g)}[k,\theta)$ of $g[n]$ [15].

3. BLIND ESTIMATION OF TIMING ERRORS AND CARRIER FREQUENCY OFFSETS

For the sake of simplicity, we shall first formulate the new estimator in the absence of a channel. The extension to time-dispersive environments is provided later in this section.

Assumptions. The received OFDM signal is given by

$$r[n] = x[n-n_e]e^{j(2\pi\theta_e n+\phi)} + \rho[n], \quad (2)$$

where $n_e \in \mathbb{Z}$ is the timing offset, θ_e denotes the carrier frequency offset, ϕ is the initial phase, and $\rho[n]$ is a wide-sense-stationary noise process, independent of $c_{k,l}$. The data symbols $c_{k,l}$ are drawn from a finite-alphabet complex constellation and satisfy[3] $\mathcal{E}\{c_{k,l}c^*_{k',l'}\} = \sigma_c^2\delta[k-k']\delta[l-l']$.

We furthermore assume that the subcarriers are transmitted with different powers. The corresponding subcarrier weighting function is given by $w[k]$, i.e., the symbols on the k-th subcarrier are multiplied by[4] $w[k]$. This simple modification will be seen later to allow a carrier frequency acquisition over the entire bandwidth of the OFDM signal. The OFDM transmit signal is now given by

$$x[n] = \sum_{k=0}^{N-1}\sum_{l=-\infty}^{\infty} c_{k,l}w[k]g[n-lM]e^{j\frac{2\pi}{N}k(n-lM)}. \quad (3)$$

In the receiver, the data symbols can be retrieved according to $\hat{c}_{k,l} = \left\langle r, \frac{h_{k,l}}{w[k]}\right\rangle$.

Cyclostationarity. We shall next show that under quite general conditions the received signal $r[n]$ in (2) is cyclostationary (CS) [16], which constitutes the basis for our estimation algorithm. The correlation function of a nonstationary stochastic process is defined as $c_r[n,\tau] = \mathcal{E}\{r[n]r^*[n-\tau]\}$ with τ being an integer lag parameter.[5] The signal $r[n]$ is said to be second-order CS with period M if $c_r[n,\tau] = c_r[n+M,\tau]$ [16, 1]. Using (2) and (3), the correlation function of the received OFDM signal $r[n]$ is given by [17]

$$\begin{aligned} c_r[n,\tau] &= \sigma_c^2 e^{j2\pi\theta_e\tau}\Gamma_N[\tau]\sum_{l=-\infty}^{\infty} g[n-n_e-lM] \\ &\quad g[n-\tau-n_e-lM] + c_\rho[\tau], \end{aligned} \quad (4)$$

where $c_\rho[\tau] = \mathcal{E}\{\rho[n]\rho^*[n-\tau]\}$ is a function of τ only because the noise process $\rho[n]$ was assumed to be stationary, and $\Gamma_N[\tau] = \sum_{k=0}^{N-1}|w[k]|^2 e^{j\frac{2\pi}{N}k\tau}$ is the N-point IDFT of $|w[k]|^2$. From (4) it easily follows that $c_r[n,\tau]$ is M-periodic in n, i.e., $c_r[n,\tau] = c_r[n+M,\tau]$ for every τ. Now, if $c_r[n,\tau]$ depends on n, the received OFDM signal $r[n]$ is CS with period M. There are several possibilities to introduce cyclostationarity in an OFDM signal, either by the use of time-frequency guard regions (for example a CP), by employing pulse shaping, or by using different transmit powers on the subcarriers [17]. If no time-frequency guard region is used (i.e. $N = M$) one can still evoke cyclostationarity by using a pulse shaping filter $g[n]$ (i.e. a filter $g[n]$ other than the rectangular function). One can even show that for $N = M$ and no pulse shaping (i.e. $g[n]$ is a rectangular pulse) the OFDM signal $r[n]$ is cyclostationary if the subcarriers are transmitted with different powers. Therefore, the estimators to be presented below can be applied even if no CP is used, in which case methods relying on the redundancy introduced by the CP such as those in [10, 11] would break down[6].

For a fixed lag τ, the M-periodic correlation function $c_r[n,\tau]$ can be expanded into a Fourier series with coefficients given by

$$\mathcal{C}_r[k,\tau] = \frac{1}{M}\sum_{n=0}^{M-1} c_r[n,\tau]e^{-j\frac{2\pi}{M}kn}.$$

Using (4) it follows after some manipulations that [17]

$$\begin{aligned} \mathcal{C}_r[k,\tau] &= \frac{\sigma_c^2}{M}e^{j2\pi\theta_e\tau}e^{-j\frac{2\pi}{M}kn_e}\Gamma_N[\tau]A^{(g,g)}\left[\tau, \frac{k}{M}\right) \\ &+ c_\rho[\tau]\delta[k]. \end{aligned} \quad (5)$$

Since σ_c^2, $g[n]$, $w[k]$ and hence $\Gamma_N[\tau]$ are known at the receiver their influence can be eliminated by defining

$$\mathcal{C}[k,\tau] = \begin{cases} \frac{\mathcal{C}_r[k,\tau]}{\frac{\sigma_c^2}{M}\Gamma_N[\tau]A^{(g,g)}\left[\tau,\frac{k}{M}\right)}, & [k,\tau] \in \mathcal{I}, \\ 0, & \text{else}, \end{cases} \quad (6)$$

where $\mathcal{I} := \{[k,\tau] | \Gamma_N[\tau]A^{(g,g)}\left[\tau, \frac{k}{M}\right) \neq 0\}$. We thus have

$$\mathcal{C}[k,\tau] = e^{j2\pi\theta_e\tau}e^{-j\frac{2\pi}{M}kn_e} + \frac{c_\rho[\tau]}{\frac{\sigma_c^2}{M}\Gamma_N[\tau]A^{(g,g)}\left[\tau,\frac{k}{M}\right)}\delta[k] \quad (7)$$

for $[k,\tau] \in \mathcal{I}$. The effect of the additive noise-term can be eliminated by considering $\mathcal{C}[k,\tau]$ for $k = 1, 2, ..., M-1$ only.

Estimation of symbol timing and carrier frequency offset. Now, following a procedure first suggested by Gini and Giannakis for single-carrier systems [1], the carrier frequency offset can be retrieved as

$$\theta_e = \frac{\arg\{\mathcal{C}[k,\tau]\mathcal{C}[M-k,\tau]\}}{4\pi\tau}, \; k \in [1, L_k], \; |\tau| \in [1, L_\tau], \quad (8)$$

where $[k,\tau] \in \mathcal{I}$, *arg* denotes the unwrapped phase, $L_k = \frac{M}{2}$ for M even and $L_k = \frac{M-1}{2}$ for M odd, and L_τ is the maximum τ in $\mathcal{I}$. Given the carrier frequency offset θ_e, the timing error can be obtained as

$$n_e = -\frac{M}{2\pi k}\arg\{\mathcal{C}[k,\tau]e^{-j2\pi\theta_e\tau}\}, \; k \in [1, M-1], \; |\tau| \in [0, L_\tau], \quad (9)$$

where again $[k,\tau] \in \mathcal{I}$. From (8) it follows that in order to avoid ambiguity due to spectral folding we have to require $|4\pi\tau_{min}\theta_e| < \pi$, where $|\tau_{min}| \geq 1$. Therefore, provided that $|\tau_{min}| = 1$, the maximum allowed frequency offset is $|\theta_e| < \frac{1}{4}$, i.e., half of the bandwidth of the OFDM signal $x[n]$. Similar arguments reveal that the timing offset has to satisfy $|n_e| < L_k$, i.e., roughly speaking the timing offset can be corrected over one symbol interval. Estimation of θ_e and n_e according to (8) and (9), respectively, will in general require phase unwrapping. The cyclic spectrum approach discussed below will be shown to avoid phase unwrapping. Moreover, it allows a full range carrier frequency acquisition (i.e. $|\theta_e| < \frac{1}{2}$).

[3] $\mathcal{E}$ denotes the expectation operator.

[4] The $w[k]$ need not be real-valued; rather it is important that $|w[k]|$ is different for different subcarriers.

[5] For stationary processes the correlation function $c_r[n,\tau]$ depends on τ only.

[6] OFDM systems not making use of a CP have been proposed in [18] for wireless high-data-rate applications. Note that the use of a CP lowers the data rate and results in a loss of SNR.

The cyclic spectrum approach. We shall next present an alternative method for estimating θ_e and n_e from $r[n]$. This approach first suggested in [1] for single-carrier systems uses the cyclic spectrum which is defined as the Fourier transform of $\mathcal{C}[k,\tau]$ with respect to τ, i.e., $\mathcal{S}[k,f] = \sum_{\tau=-\infty}^{\infty} \mathcal{C}[k,\tau] e^{-j2\pi\tau f}$. From (7) we obtain

$$\mathcal{S}[k,f] = e^{-j\frac{2\pi}{M}kn_e} \frac{\sin\left(2\pi(\theta_e - f)\left(L_\tau + \frac{1}{2}\right)\right)}{\sin(\pi(\theta_e - f))},$$

where we assumed that $k \geq 1$ and $[k,\tau] \in \mathcal{I}$ for $|\tau| \leq L_\tau$. The carrier frequency offset can now be obtained as

$$\theta_e = \arg\max_{|f|<\frac{1}{2}} |\mathcal{S}[k,f)|. \qquad (10)$$

From (10) it follows that ambiguity due to spectral folding is avoided if $|\theta_e| < \frac{1}{2}$, which means that the carrier frequency acquisition range is the entire bandwidth of the OFDM signal $x[n]$. For given carrier frequency offset θ_e the timing offset can be retrieved as

$$n_e = -\frac{M}{2\pi k} \arg\{\mathcal{S}[k,\theta_e)\}. \qquad (11)$$

Equal transmit power on all subcarriers. We shall next specialize our results to the case where all subcarriers use the same transmit power[7]. Here, $\Gamma_N[\tau] = N\delta_N[\tau]$ with $\delta_N[\tau]$ denoting an N-periodic pulse train with amplitude 1. It follows from (6) that $\mathcal{C}[k,\tau] = 0$ for $\tau \neq rN$ with $r \in \mathbb{Z}$. Therefore, $|\tau_{min}| \geq N$ which using $|4\pi\tau_{min}\theta_e| < \pi$ implies $|\theta_e| < \frac{1}{4N}$ for $|\tau_{min}| = N$. The carrier frequency acquisition range is therefore restricted to half the subcarrier spacing. Assuming that $k \in [1, M-1]$ is such that $[k,\tau] \in \mathcal{I}$ for $\tau = rN$ with $r \in \mathbb{Z}$, the cyclic spectrum is given by

$$\mathcal{S}[k,f) = e^{-j\frac{2\pi}{M}kn_e} \sum_{l=0}^{N-1} \frac{\sin\left(2\pi\left(\theta_e - \left(f - \frac{l}{N}\right)\right)\left(L_\tau + \frac{1}{2}\right)\right)}{\sin\left(\pi\left(\theta_e - \left(f - \frac{l}{N}\right)\right)\right)},$$

which shows that using the cyclic spectrum approach the maximum allowable carrier frequency offset is one subcarrier spacing, i.e., $|\theta_e| < \frac{1}{N}$. This result demonstrates that the use of different subcarrier transmit powers is crucial for increasing the carrier frequency acquisition range. In particular, $g[n]$ and $w[k]$ should be chosen such that $|\tau_{min}| = 1$.

Specialization to systems employing a CP. Let us assume that the length of the CP is P, i.e., $M = N + P$. The filter $g[n]$ is a rectangular pulse of length M and $h[n]$ is a rectangular pulse of length N. Inserting into (4) it can be shown that $r[n]$ is CS with period M [14, 17]. Symbol timing and carrier frequency offset can now be retrieved using one of the methods described above. The cyclostationarity induced by the CP has been exploited previously for blind equalization [14].

Estimation in presence of a channel. We shall next adapt our estimator to time-dispersive environments. Assume that the channel impulse response $d[n]$ is known at the receiver or has been identified previously. A somewhat lengthy calculation reveals that [17]

$$\mathcal{C}_r[k,\tau] = \frac{\sigma_c^2}{M} e^{j2\pi\theta_e\tau} e^{-j\frac{2\pi}{M}kn_e} \hat{A}^{(g,g,d)}\left[\tau, \frac{k}{M}\right) + c_\rho[\tau]\delta[k],$$

where $\hat{A}^{(g,g,d)}[\tau,\theta) =$

$$\sum_s d[s] \sum_{s'} d[s'] \Gamma_N[s'-s+\tau] \sum_n g[n-s] g[n-s'-\tau] e^{-j2\pi n\theta}.$$

Since the channel impulse response $d[n]$, the transmitter pulse shaping filter $g[n]$, and σ_c^2 are all known at the receiver, we can define $\mathcal{C}[k,\tau] = \frac{\mathcal{C}_r[k,\tau]}{\hat{A}^{(g,g,d)}\left[\tau,\frac{k}{M}\right)\frac{\sigma_c^2}{M}} =$

$$e^{j2\pi\theta_e\tau} e^{-j\frac{2\pi}{M}kn_e} + \frac{M}{\sigma_c^2} c_\rho[\tau] \hat{A}^{(g,g,d)^{-1}}\left[\tau, \frac{k}{M}\right) \delta[k]$$

for $[k,\tau] \in \mathcal{I}$ with $\mathcal{I} := \{[k,\tau] | \hat{A}^{(g,g,d)}\left[\tau, \frac{k}{M}\right) \neq 0\}$ and $\mathcal{C}[k,\tau] = 0$ else. Finally, θ_e and n_e can be retrieved using (8) and (9) or (10) and (11), respectively.

Estimation of cyclic statistics. In practice the cyclic statistics $\mathcal{C}_r[k,\tau]$ can be estimated from a finite data record of length L according to [1]

$$\hat{\mathcal{C}}_r[k,\tau] = \frac{1}{L} \sum_{n=0}^{L-1} r[n] r^*[n-\tau] e^{-j\frac{2\pi}{M}kn}. \qquad (12)$$

For a discussion of the statistical properties of this estimator the interested reader is referred to [1]. Note that (12) is basically an FFT of the signal $r[n]r^*[n-\tau]$.

Estimators. Similar to the single-carrier case, estimators for θ_e and n_e can be obtained from (8) and (9) by averaging over $\mathcal{I}$ according to

$$\hat{\theta}_e = \frac{1}{4\pi|\mathcal{I}'|} \sum_{[k,\tau]\in\mathcal{I}'} \frac{1}{\tau} \arg\{\hat{\mathcal{C}}[k,\tau]\hat{\mathcal{C}}[M-k,\tau]\} \qquad (13)$$

$$\hat{n}_e = -\frac{M}{2\pi|\mathcal{I}''|} \sum_{[k,\tau]\in\mathcal{I}''} \frac{1}{k} \arg\{\hat{\mathcal{C}}[k,\tau] e^{-j2\pi\hat{\theta}_e\tau}\}, \qquad (14)$$

where $\hat{\mathcal{C}}[k,\tau]$ is obtained from (6) by replacing $\mathcal{C}_r[k,\tau]$ with $\hat{\mathcal{C}}_r[k,\tau]$, $\mathcal{I}' = \mathcal{I} \setminus \{k=0, \tau=0\}$ (i.e. the set $\mathcal{I}$ except for the $k=0$ axis and the $\tau=0$ axis), and $\mathcal{I}'' = \mathcal{I} \setminus \{k=0\}$.

Similarly, estimators based on the cyclic spectrum can be obtained from (10) and (11) as

$$\hat{\theta}_e = \frac{1}{|\mathcal{K}|} \sum_{k\in\mathcal{K}} \arg\max_{|f|<\frac{1}{2}} |\hat{\mathcal{S}}[k,f)| \qquad (15)$$

$$\hat{n}_e = -\frac{M}{2\pi|\mathcal{K}|} \sum_{k\in\mathcal{K}} \frac{1}{k} \arg\{\hat{\mathcal{S}}[k,\hat{\theta}_e)\}, \qquad (16)$$

where $\mathcal{K}$ denotes the set of all $k \geq 1$ used in (15) and (16), respectively, and $\hat{\mathcal{S}}[k,f)$ is obtained by windowing $\hat{\mathcal{C}}[k,\tau]$ with $W[\tau]$ and taking the Fourier transform as [1]

$$\hat{\mathcal{S}}[k,f) = \sum_{\tau=-L_\tau}^{L_\tau} W[\tau] \hat{\mathcal{C}}[k,\tau] e^{-j2\pi\tau f}.$$

4. SIMULATION RESULTS

In this section we provide simulation results demonstrating the performance of the proposed estimators. We simulated an OFDM system with $N = 8$ channels, $M = 16$, and pulse shaping filters $h[n] = g[n]$ of length 96. The data symbols were i.i.d. 4-PSK symbols with $\sigma_c^2 = 2$. The signal-to-noise-ratio (SNR) was defined as SNR$=\sigma_c^2/\sigma_\rho^2$, where σ_ρ^2 is the variance of the white noise process $\rho[n]$. All results were obtained by averaging over 200 Monte Carlo trials. Each realization consisted of 1024 data symbols. Furthermore, the estimates of the cyclic statistics were obtained using the entire data record in (12). The subchannel weighting vector was chosen as $\mathbf{w} = [1.1\ 2.0\ 1.4\ 1.33\ 1.0\ 0.6\ 0.8\ 1.2]$.

Simulation Example 1. In the first simulation example we computed the bias and the mean squared error (MSE) of the carrier frequency offset estimator (13) and the timing offset estimator (14) for $n_e = 2$ and $\theta_e = 0.0625$, i.e., half the subcarrier spacing. Figs. 1 (a) and (b) show the bias and the MSE, respectively, of $\hat{\theta}_e$ versus θ_e as a function of the SNR in dB. Figs. 1 (c) and (d) show the bias

[7] For the sake of simplicity we assume that $|w[k]| = 1$ for $k = 0, 1, ..., N-1$.

and the MSE, respectively, of $\frac{\hat{n}_e}{M}$ versus $\frac{n_e}{M}$ as a function of the SNR. We can see that our estimator performs well even for small SNR values.

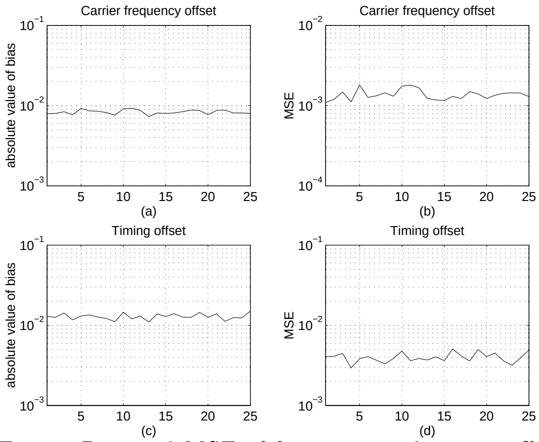

Fig. 1: Bias and MSE of frequency and timing offset estimators (13) and (14), respectively, versus SNR/dB: a) bias and b) MSE of $\hat{\theta}_e$, c) bias and d) MSE of $\hat{n}_e/M$.

Simulation Example 2. In the second simulation example (see Fig. 2) we computed the bias and the MSE of the carrier frequency offset estimator (13) at an SNR of 9dB as a function of the frequency offset.[8]

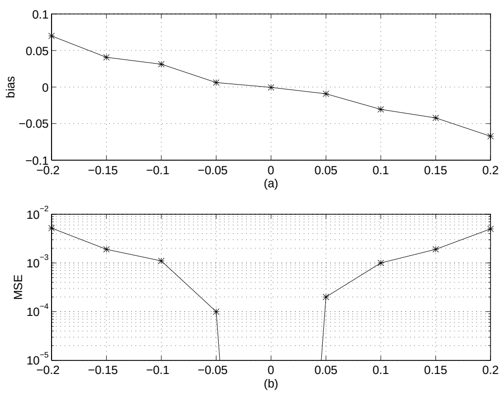

Fig. 2: (a) Bias and (b) MSE[9] of $\hat{\theta}_e$ according to (13) versus θ_e as a function of θ_e.

Simulation Example 3. In the last simulation example, we consider estimation of the carrier frequency offset using the cyclic spectrum approach. Fig. 3 shows $|\hat{\mathcal{S}}[1, f)|$ for $\theta_e = -0.45$.

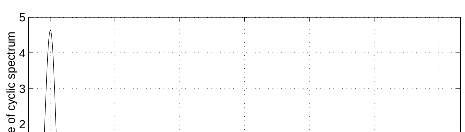

Fig. 3: Magnitude of the cyclic spectrum $\hat{\mathcal{S}}[1, f)$.

[8] Recall that the acquisition range of the estimator (13) is restricted to $|\theta_e| < 0.25$. For $|\theta_e|$ close to 0.25, however, the estimator gets inaccurate. We therefore simulated the range $|\theta_e| \leq 0.2$ only.

[9] For $\theta_e = 0$ the MSE of $\hat{\theta}_e$ was in the order of MATLAB's computational accuracy.

5. CONCLUSION

We introduced a blind time-frequency offset estimator for pulse shaping OFDM systems. The proposed method is computationally efficient and allows to perform a carrier frequency acquisition over the entire bandwidth of the OFDM signal. Furthermore, it needs neither pilot symbols nor a CP. Our estimator exploits the cyclostationarity of the received signal and can be seen as an extension of the Gini-Giannakis estimator [1] to the multi-carrier case. We finally provided simulation examples demonstrating the performance of the new estimator.

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