MEAN FIELD DECOMPOSITION OF A POSTERIORI PROBABILITY FOR MRF-BASED UNSUPERVISED TEXTURED IMAGE SEGMENTATION

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ABSTRACT

This paper proposes a Markov random field (MRF) model-based method for unsupervised segmentation of multispectral images consisting of multiple textures. To model such textured images, a hierarchical MRF is used with two layers, the first layer representing an unobservable region image and the second layer representing multiple textures which cover each region. This method uses the Expectation and Maximization (EM) method for model parameter estimation, where in order to overcome the well-noticed computational problem in the expectation step, we approximate the Baum function using mean-field-based decomposition of a posteriori probability. Given provisionally estimated parameters at each iteration in the EM method, a provisional segmentation is carried out using local a posteriori probability of each pixel's region label, which is derived by mean-field-based decomposition of a posteriori probability of the whole region image.

1. INTRODUCTION

Segmentation of images consisting of multiple textures is the most difficult problem among others since in textured images correlations of gray level at each pixel with those of adjacent pixels should be considered. For the textured image segmentation, hierarchical Markov random field (MRF) models [1],[3]-[4] are commonly used with two layers. One layer called the hidden layer is an unobservable stochastic process (region label process) which represents the geometry of regions covered with different textures. The other layer consists of a number of observable stochastic processes (observation process) which represent the covering textures. In the hierarchical MRF models, all processes are described by MRF models.

It is desirable that image segmentation can be carried out in an unsupervised way; estimation of model parameters and image segmentation can be done simultaneously by using only a given image. For parameter estimation of the hierarchical MRF models with an unobservable region process, the Expectation and Maximization (EM) method [5] is naturally used since the EM method is exactly for Maximum Likelihood (ML) parameter estimation with incompletely observed data. However the Baum function in the expectation step, the summation over all possible configurations of the hidden region process is difficult (practically impossible) to be computed (See **3.1**). To overcome this problem, a promising approach using the mean field theory has been proposed to approximate the Baum function [2]. However in [2] any method applicable to textured images is not given. Later an approach applicable to textured images has been proposed [3] but its derivation seems to be rather ad hoc.

This paper presents a better refined approach to approximate the Baum function, which is derived based on mean-field-based decomposition[1] of a posteriori probability of the hidden region process. The a posteriori probability of the whole region image is decomposed into the product of local a posteriori probabilities (LAPs) for all pixels and the LAPs are computed consistently taking them as the mean fields. At each iteration in the EM method, a provisional segmentation is carried out using the LAPs with provisionally estimated parameters. In this paper we treat multispectral images for general use.

2. MULTISPECTRAL TEXTURED IMAGE MODEL

A multispectral image comprising of different textures is considered as a realization of a collection of interacting random variables $(\mathbf{X}_{\mathcal{L}}, \mathbf{Y}_{\mathcal{L}})$[2] defined on a finite two dimensional lattice $\mathcal{L}$, $\mathcal{L} = \{(i,j)\}, 1 \le i \le N_1, 1 \le j \le N_2$. The observation process $\mathbf{Y}_{\mathcal{L}} = \{\mathbf{Y}_{ij}\}, (i,j) \in \mathcal{L}$, where $\mathbf{Y}_{ij}$ is a vector representing multispectral gray levels at (i,j)-pixel, is assumed to be a function of the region label process, $\mathbf{X}_{\mathcal{L}} = \{\mathbf{X}_{ij}\}, (i,j) \in \mathcal{L}$. $\mathbf{X}_{ij}$ is also a vector as described soon. The interacting processes $(\mathbf{X}_{\mathcal{L}}, \mathbf{Y}_{\mathcal{L}})$ can be characterized completely by a joint probability $p(\mathbf{x}_{\mathcal{L}}, \mathbf{y}_{\mathcal{L}})$ or equivalently by $p(\mathbf{x}_{\mathcal{L}})$ and $p(\mathbf{y}_{\mathcal{L}} \mid \mathbf{x}_{\mathcal{L}})$, where $\mathbf{x}_{\mathcal{L}}$ and $\mathbf{y}_{\mathcal{L}}$ represent a realization of $\mathbf{X}_{\mathcal{L}}$ and that of $\mathbf{Y}_{\mathcal{L}}$.

We consider images consisting of M distinct textural regions. Then suppose that $\mathbf{x}_{ij}$ is an indicator vector taking one from the vector set $Q_X = \{\mathbf{e}_1, \ldots, \mathbf{e}_M\}$, where $\mathbf{e}_m, 1 \le m \le M$ is the M dimensional unit vector whose mth component is 1 and all other components are 0. When the region label of (i,j)-pixel is m, $\mathbf{x}_{ij}$ takes $\mathbf{e}_m$. To model the hidden region label process, we adopt a multi-level logistic MRF (LMRF) with the second-order neighborhood[3] system. In this model all clique[3] energies are assumed to be zero except the doubleton clique energies which are given by

$$U(\mathbf{x}_{ij}, \mathbf{x}_{ij+\tau}) = \begin{cases} -\beta & \text{if } \mathbf{x}_{ij} = \mathbf{x}_{ij+\tau} \\ \beta & \text{otherwise.} \end{cases} \quad (1)$$

In (1), $\tau \in \mathcal{N} = \{(0,1),(0,-1),(1,0),(-1,0),(1,1),(-1,-1),(-1,1),(1,-1)\}$. For example when $\tau = (0,1)$, $\mathbf{x}_{ij+\tau} = \mathbf{x}_{i,j+1}$.

[1] The idea of decomposition using the mean field approximation is applied to the Maximum A Posteriori (MAP) estimation for textured image segmentation by us [6], [4].

[2] In this paper, x_A and $f(x_A)$ denote the set $\{x_{a_1}, \ldots, x_{a_l}\}$ and the multivariable function $f(x_{a_1}, \ldots, x_{a_l})$ respectively, where $A = \{a_1, \ldots, a_l\}$.

[3] For details on MRFs and related concepts such as the neighborhoods and cliques, see [7].

The local conditional probability for the hidden region label process is given by

$$p(\mathbf{x}_{ij} \mid \mathbf{x}_{\eta_{ij}^X}) = \frac{\exp\{-\sum_{\tau\in\mathcal{N}} U(\mathbf{x}_{ij}, \mathbf{x}_{ij+\tau})\}}{\sum_{\mathbf{x}_{ij}\in Q_X} \exp\{-\sum_{\tau\in\mathcal{N}} U(\mathbf{x}_{ij}, \mathbf{x}_{ij+\tau})\}}, \tag{2}$$

where η_{ij}^X denotes the (i,j)-pixel's neighborhood on $\mathbf{X}_{\mathcal{L}}$.

The observed multispectral textures are modeled by multidimensional Gaussian MRFs (GMRFs) with the second-order neighborhood systems characterized by the following local conditional pdfs

$$p(\mathbf{y}_{ij} \mid \mathbf{y}_{\eta_{ij}}, \mathbf{x}_{ij}=\mathbf{e}_m) = \frac{1}{(2\pi)^{K/2}|\Sigma_m|^{1/2}} \cdot \exp\{-\frac{1}{2}(\mathbf{y}_{ij} - \hat{\mathbf{y}}_{ij}^m)^T \Sigma_m^{-1} (\mathbf{y}_{ij} - \hat{\mathbf{y}}_{ij}^m)\} \tag{3}$$

$$\hat{\mathbf{y}}_{ij}^m = \mu_m + \sum_{\tau\in\mathcal{N}} \mathbf{B}_{m,\tau}(\mathbf{y}_{ij+\tau} - \mu_m). \tag{4}$$

Here K denotes the dimension of $\mathbf{y}_{ij}$, i.e., the number of spectral bands, $\hat{\mathbf{y}}_{ij}^m$ denotes the predicted vector using neighboring vectors $\mathbf{y}_{\eta_{ij}^Y}$, and μ_m, $\mathbf{\Sigma}_m$, and $\mathbf{B}_{m,\tau}$ stand for mean vector, covariance matrix of the prediction error vectors $(\mathbf{y}_{ij} - \hat{\mathbf{y}}_{ij}^m)$ and spatial interaction parameter matrix for pairwise cliques, all depending on the class label m. The spatial interaction parameter matrix is reminiscence of the prediction coefficient matrix in the linear vector prediction and therefore can be simply referred as the prediction matrix. The prediction matrices are assumed to be symmetric, i.e., $\mathbf{B}_{m,\tau} = \mathbf{B}_{m,-\tau}$ (if $\tau = (i,j)$, then $-\tau = (-i,-j)$).

3. UNSUPERVISED SEGMENTATION ALGORITHM

3.1. Parameter Estimation

The EM method [5] is an iterative method to perform the ML estimation with incompletely observed data. It is considered that an observed image $\mathbf{y}_{\mathcal{L}}$ is imperfect data and a set of an observed image $\mathbf{y}_{\mathcal{L}}$ and a region label image (in fact a set of region indicator vectors) $\mathbf{x}_{\mathcal{L}}$ is complete data. However only an observed image is available. The EM method consists of the expectation step (E-step to obtain the Baum function) and the maximization step (M-step).

- E-step:

$$Q(\lambda \mid \lambda^{(p)}) = \sum_{\mathbf{x}_{\mathcal{L}}\in\Omega_X} p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)}) \log p(\mathbf{x}_{\mathcal{L}}, \mathbf{y}_{\mathcal{L}}; \lambda) \tag{5}$$

- M-step:

$$\lambda^{(p+1)} = \arg\max_{\lambda} Q(\lambda \mid \lambda^{(p)}), \tag{6}$$

where λ represents a set of all parameters to be estimated (MRF parameters of the region process and the observation process) and $\lambda^{(p)}$ is a provisionally estimated set at the p-th iteration.

The Baum function in (5) represents the sum over all possible configurations of $\mathbf{x}_{\mathcal{L}}$, $\Omega_X = Q_X^{N_1\times N_2}$, and it is difficult (practically impossible) to calculate this. To overcome this problem, mainly two approaches to approximate the Baum function have been proposed. The first approach of approximation is to replace the sum over all possible configurations of $\mathbf{x}_{\mathcal{L}}$ by only its current estimate $\mathbf{x}_{\mathcal{L}}^{(p)}$[4]. The second approach uses the mean field approximation to calculate the Baum function[2]. We present a better established algorithm for the latter approach applicable to multispectral textured images based on the mean-field-based decomposition of a posteriori probability.

The a posteriori probability $p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)})$ in (5) is described as

$$p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)}) = \frac{p(\mathbf{y}_{\mathcal{L}} \mid \mathbf{x}_{\mathcal{L}}; \lambda_Y^{(p)}) p(\mathbf{x}_{\mathcal{L}}; \lambda_X^{(p)})}{\sum_{\mathbf{x}_{\mathcal{L}}\in\Omega_X} p(\mathbf{y}_{\mathcal{L}} \mid \mathbf{x}_{\mathcal{L}}; \lambda_Y^{(p)}) p(\mathbf{x}_{\mathcal{L}}; \lambda_X^{(p)})}, \tag{7}$$

where λ_X is a parameter set for the region process and λ_Y is that for the observation process, and $\lambda = \{\lambda_X, \lambda_Y\}$. By using the mean field approximation, $p(\mathbf{y}_{\mathcal{L}} \mid \mathbf{x}_{\mathcal{L}}; \lambda_Y^{(p)})$ and $p(\mathbf{x}_{\mathcal{L}}; \lambda_X^{(p)})$ are decomposed as [6], [4]

$$p(\mathbf{y}_{\mathcal{L}} \mid \mathbf{x}_{\mathcal{L}}; \lambda_Y^{(p)}) \simeq \prod_{(i,j)\in\mathcal{L}} p(\mathbf{y}_{ij} \mid \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y^{(p)}) \tag{8}$$

$$p(\mathbf{x}_{\mathcal{L}}; \lambda_X^{(p)}) \simeq \prod_{(i,j)\in\mathcal{L}} p(\mathbf{x}_{ij} \mid \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda_X^{(p)}), \tag{9}$$

where $\langle \bullet \rangle$ is the mean field for $\bullet$. Substituting (8) and (9) into (7), and replacing $\sum_{\mathbf{x}_{\mathcal{L}}\in\Omega_X} \prod_{(i,j)\in\mathcal{L}}$ by $\prod_{(i,j)\in\mathcal{L}} \sum_{\mathbf{x}_{ij}\in Q_X}$, $p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)})$ is also decomposed as

$$p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)}) \simeq \prod_{(i,j)\in\mathcal{L}} p(\mathbf{x}_{ij} \mid \mathbf{y}_{ij}, \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda^{(p)}), \tag{10}$$

where

$$p(\mathbf{x}_{ij} \mid \mathbf{y}_{ij}, \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda^{(p)}) = \frac{p(\mathbf{y}_{ij} \mid \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y^{(p)}) p(\mathbf{x}_{ij} \mid \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda_X^{(p)})}{\sum_{\mathbf{x}_{ij}\in Q_X} p(\mathbf{y}_{ij} \mid \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y^{(p)}) p(\mathbf{x}_{ij} \mid \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda_X^{(p)})}. \tag{11}$$

$p(\mathbf{x}_{ij} \mid \mathbf{y}_{ij}, \langle \mathbf{y} \rangle_{\eta_{ij}^Y}, \langle \mathbf{x} \rangle_{\eta_{ij}^X}; \lambda^{(p)})$ is considered as a local a posteriori probability (LAP) and hereafter we write it as $z_{ij}^{(p)}(\mathbf{x}_{ij})$ for short. Then the LAPs for all region indicators form a vector (LAP vector), $\mathbf{z}_{ij}^{(p)} = (z_{ij}^{(p)}(\mathbf{x}_{ij} = \mathbf{e}_1), \cdots, z_{ij}^{(p)}(\mathbf{x}_{ij} = \mathbf{e}_M))^T$. It is reasonable to use the LAP vector $\mathbf{z}_{ij}^{(p)}$ as the mean field of $\mathbf{x}_{ij}$, $\langle \mathbf{x} \rangle_{ij}$[4] . Then, from (11) $z_{ij}^{(p)}(\mathbf{x}_{ij})$ is computed by

$$z_{ij}^{(p)}(\mathbf{x}_{ij}) = \frac{p(\mathbf{y}_{ij} \mid \mathbf{y}_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y^{(p)}) p(\mathbf{x}_{ij} \mid \mathbf{z}_{\eta_{ij}^X}^{(p)}; \lambda_X^{(p)})}{\sum_{\mathbf{x}_{ij}\in Q_X} p(\mathbf{y}_{ij} | \mathbf{y}_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y^{(p)}) p(\mathbf{x}_{ij} | \mathbf{z}_{\eta_{ij}^X}^{(p)}; \lambda_X^{(p)})}, \tag{12}$$

here $\mathbf{y}_{\eta_{ij}^Y}$ is simply used for $\langle \mathbf{y} \rangle_{\eta_{ij}^Y}$ and $\mathbf{z}_{\eta_{ij}^X}^{(p)} \triangleq \{\mathbf{z}_{kl}^{(p)}, (k,l) \in \eta_{ij}^X\}$. Note that in order to calculate $\mathbf{z}_{ij}^{(p)}$, the LAP vector for $\mathbf{x}_{ij}$, we need $\mathbf{z}_{\eta_{ij}^X}^{(p)}$, those for $\mathbf{x}_{\eta_{ij}^X}$. Therefore the LAP vectors are calculated by iterative procedures. In $p(\mathbf{x}_{ij} \mid \mathbf{z}_{\eta_{ij}^X}^{(p)}; \lambda_X^{(p)})$ the doubleton

[4] Given $\mathbf{y}_{\mathcal{L}}$, the mean field $\langle \mathbf{x} \rangle_{ij}$ can be defined as $\langle \mathbf{x} \rangle_{ij} = \sum_{\mathbf{x}_{\mathcal{L}}\in\Omega_X} \mathbf{x}_{ij} p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)})$. Using the decomposition (10), it is easily shown that $\langle \mathbf{x} \rangle_{ij} = \mathbf{z}_{ij}^{(p)}$.

clique energy in (1) should be changed as

$$U(\mathbf{x}_{ij}, \mathbf{z}_{ij+\tau}) = -\beta \mathbf{x}_{ij}^T \mathbf{z}_{ij+\tau} + \beta(1 - \mathbf{x}_{ij}^T \mathbf{z}_{ij+\tau}) = \beta(1 - 2\mathbf{x}_{ij}^T \mathbf{z}_{ij+\tau}). \quad (13)$$

Now we can approximately calculate the Baum function with the mean-field-based decomposition of $p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)})$, $\prod_{(k,l)\in\mathcal{L}} z_{kl}^{(p)}(\mathbf{x}_{kl})$, and with the same decomposition for $\log p(\mathbf{x}_{\mathcal{L}}, \mathbf{y}_{\mathcal{L}}; \lambda)$.

$$\begin{aligned} Q(\lambda \mid \lambda^{(p)}) \simeq & \sum_{\mathbf{x}_{\mathcal{L}} \in \Omega_X} \prod_{(k,l)\in\mathcal{L}} z_{kl}^{(p)}(\mathbf{x}_{kl}) \cdot \\ & \{ \sum_{(i,j)\in\mathcal{L}} \log p(\mathbf{y}_{ij} \mid \mathbf{y}_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y) + \sum_{(i,j)\in\mathcal{L}} \log p(\mathbf{x}_{ij} \mid \mathbf{z}_{\eta_{ij}^X}^{(p)}; \lambda_X) \} \\ = & \sum_{(i,j)\in\mathcal{L}} \sum_{\mathbf{x}_{ij}\in Q_X} z_{ij}^{(p)}(\mathbf{x}_{ij}) \log p(\mathbf{y}_{ij} \mid \mathbf{y}_{\eta_{ij}^Y}, \mathbf{x}_{ij}; \lambda_Y) \\ & + \sum_{(i,j)\in\mathcal{L}} \sum_{\mathbf{x}_{ij}\in Q_X} z_{ij}^{(p)}(\mathbf{x}_{ij}) \log p(\mathbf{x}_{ij} \mid \mathbf{z}_{\eta_{ij}^X}^{(p)}; \lambda_X) \quad (14) \end{aligned}$$

Once the Baum function is obtained, the M-step is carried out straightforwardly ; $\lambda_Y^{(p+1)}$ and $\lambda_X^{(p+1)}$ are derived by maximization of the first and second term of (14), respectively. An estimate of λ_X, β in (13), cannot be given in a mathematically closed form, therefore it needs to be calculated using an appropriate numerical optimization method. In the following experiments we used the Newton method to estimate β. Whereas, the GMRFs' parameters λ_Y can be estimated in a mathematically closed form as follows. The reestimate of the mean vector, $\mu_m^{(p+1)}$ is given as

$$\mu_m^{(p+1)} = \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) \mathbf{y}_{ij} \} / \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) \}, \quad (15)$$

where $z_{ij}^{(p)}(m)$ represents $z_{ij}^{(p)}(\mathbf{x}_{ij} = \mathbf{e}_m)$. Assuming $\mathbf{B}_{m,\tau_s}^{(p+1)} = \mathbf{B}_{m,-\tau_s}^{(p+1)}$, the reestimates of the prediction matrices, $\mathbf{B}_{m,\tau_s}^{(p+1)}$ can be obtained by solving the following matrix equation

$$\begin{aligned} & (\mathbf{B}_{m,\tau_1}^{(p+1)}, \mathbf{B}_{m,\tau_2}^{(p+1)}, \mathbf{B}_{m,\tau_3}^{(p+1)}, \mathbf{B}_{m,\tau_4}^{(p+1)}) \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} & \mathbf{A}_{13} & \mathbf{A}_{14} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{A}_{23} & \mathbf{A}_{24} \\ \mathbf{A}_{31} & \mathbf{A}_{32} & \mathbf{A}_{33} & \mathbf{A}_{34} \\ \mathbf{A}_{41} & \mathbf{A}_{42} & \mathbf{A}_{43} & \mathbf{A}_{44} \end{pmatrix} \\ & = (\mathbf{A}_{01}, \mathbf{A}_{02}, \mathbf{A}_{03}, \mathbf{A}_{04}) \quad (16) \end{aligned}$$

$$\begin{aligned} \mathbf{A}_{st} = & \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) (\mathbf{y}_{ij+\tau_s} + \mathbf{y}_{ij-\tau_s} - 2\mu_m^{(p+1)}) \cdot \\ & (\mathbf{y}_{ij+\tau_t} + \mathbf{y}_{ij-\tau_t} - 2\mu_m^{(p+1)})^T \} / \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) \} \quad (17) \\ \mathbf{A}_{0t} = & \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) (\mathbf{y}_{ij} - \mu_m^{(p+1)}) \cdot \\ & (\mathbf{y}_{ij+\tau_t} + \mathbf{y}_{ij-\tau_t} - 2\mu_m^{(p+1)})^T \} / \{ \sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) \}, \quad (18) \end{aligned}$$

where $\tau_s, \tau_t \in \mathcal{N}' = \{(0,1), (1,0), (1,1), (-1,1)\}$, if $s \neq t$ then $\tau_s \neq \tau_t$. For example $\tau_1 = (0,1)$, $\tau_2 = (1,0)$, $\tau_3 = (1,1)$, $\tau_4 = (-1,1)$. Finally the reestimate of the covariance matrix, $\Sigma_m^{(p+1)}$ is given as

$$\Sigma_m^{(p+1)} = \frac{\sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m) (\mathbf{y}_{ij} - \hat{\mathbf{y}}_{ij}^m)(\mathbf{y}_{ij} - \hat{\mathbf{y}}_{ij}^m)^T}{\sum_{(i,j)\in\mathcal{L}} z_{ij}^{(p)}(m)} \quad (19)$$

$$\hat{\mathbf{y}}_{ij}^m = \mu_m^{(p+1)} + \sum_{s=1}^{4} \mathbf{B}_{m,\tau_s}^{(p+1)} (\mathbf{y}_{ij+\tau_s} + \mathbf{y}_{ij-\tau_s} - 2\mu_m^{(p+1)}). \quad (20)$$

3.2. Image Segmentation

Segmentation of an image into regions of different textures means estimation of the hidden region process $\mathbf{x}_{\mathcal{L}}$. In principle it is carried out with the finally estimated parameters. However a provisional segmentation is also possible with provisionally estimated parameters at each iteration in the EM method. In the following we describe the segmentation in this situation.

Given an observed image $\mathbf{y}_{\mathcal{L}}$ and an estimated parameter set $\lambda^{(p)}$, provisional estimation of $\mathbf{x}_{\mathcal{L}}$ is carried out by maximizing the a posteriori probability $p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)})$ (MAP estimation)[5] .

$$\mathbf{x}_{\mathcal{L}}^{(p)} \triangleq \arg \max_{\mathbf{x}_{\mathcal{L}} \in \Omega_X} p(\mathbf{x}_{\mathcal{L}} \mid \mathbf{y}_{\mathcal{L}}; \lambda^{(p)}) \quad (21)$$

As seen from (10), (11) and (12), this global optimization problem is approximately decomposed into the local optimization problems using the LAPs for $\mathbf{x}_{ij}$s, $z_{ij}^{(p)}(\mathbf{x}_{ij})$s.

$$\mathbf{x}_{ij}^{(p)} = \arg \max_{\mathbf{x}_{ij} \in Q_X} z_{ij}^{(p)}(\mathbf{x}_{ij}) \quad (22)$$

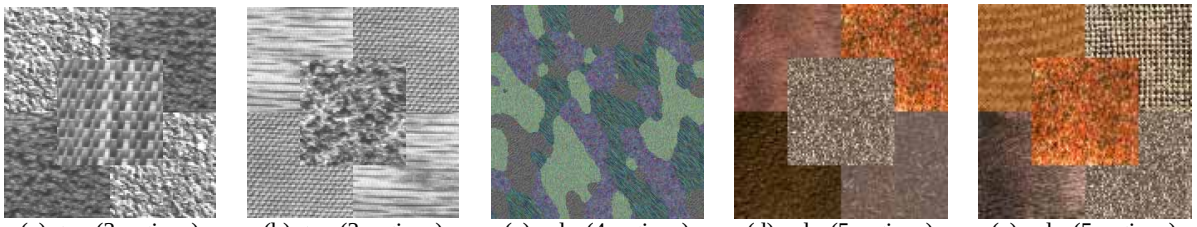
3.3. Initial Parameter Estimation

To start iterative procedures in the EM method, initial values of MRF parameters should be given in advance. These initial values are derived as follows. A given image is divided into small subimages, for example, consisting of 8×8 pixels. Assuming a single texture for each subimage, a set of texture parameters (composing a vector) is estimated. Assuming that the number of regions is known, these vectors derived from all subimages are classified into the known number of regions by using a clustering method. Then the texture parameters for each different region are again estimated using all subimages classified to the same region and are used as initial texture parameters $\lambda_Y^{(0)}$. An appropriate positive value can be used as an initial region parameter $\lambda_X^{(0)}$, $\beta^{(0)}$ in (13). In the following experiments we use $\beta^{(0)} = 0.5$.

4. SIMULATION RESULTS

To evaluate the performance of the proposed unsupervised segmentation method, we applied the method to five textured images shown in Fig. 1. Here (a) and (b) are gray images consisting of three natural textures from the Brodatz album, (c) is a synthesized color multi-textured image generated by the Gibbs sampler [7], and (d) and (e) are color images consisting of five natural textures from the MIT Vision Texture Database. For performance comparison three methods were applied: the proposed method 1 described

[5] To solve this optimization problem, the stochastic relaxation algorithm known as the Simulated Annealing (SA) [7] can be used. However we do not prefer using the SA since it demands formidable computation.



(a) gray(3 regions) (b) gray(3 regions) (c) color(4 regions) (d) color(5 regions) (e) color(5 regions)

Figure 1: Textured images used for segmentation experiments.

Table 1 Segmentation errors(%) by three methods

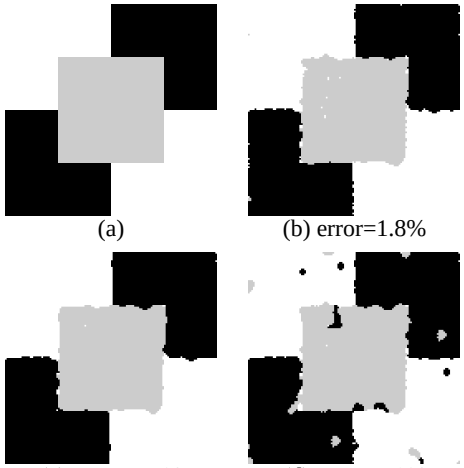
image	proposed 1	proposed 2	conventional
(a)	1.8	1.7	4.0
(b)	2.5	2.7	10.5
(c)	2.0	4.3	8.1
(d)	1.1	1.6	6.5
(e)	2.0	1.3	12.3

in **3**, the method 2 with the first type of approximation of the Baum function where the sum in (5) is replaced by only the current estimate $\mathbf{x}_{\mathcal{L}}^{(p)}$ derived by (22), and a conventional method[4] where the same approximation of the Baum function as in the method 2 is made but with the current estimate derived by the Generalized ICM (GICM) method [6],[4]. The GICM method, which is a generalization of the ICM method [8], is described as

$$\mathbf{x}_{ij}^{(p)}=\arg\max_{\mathbf{x}_{ij}\in Q_X}\{p(\mathbf{y}_{ij}|\mathbf{x}_{ij},\mathbf{y}_{\eta_{ij}^Y};\lambda_Y^{(p)})p(\mathbf{x}_{ij}|\mathbf{x}_{\eta_{ij}^X}^{(p-1)};\lambda_X^{(p)})\}.$$

In the proposed method 2 and the conventional method, the parameter estimation is reduced to maximization of the pseudo-likelihood [8] given the current estimate of the region process.

Segmentation results for Fig. 1(a) as an example are shown in Fig. 2 and segmentation errors for five images are summarized in Table 1. The results by the proposed method 1 and 2 suggest that it does not affect very harmfully to replace the sum in the Baum function by only the current estimate $\mathbf{x}_{\mathcal{L}}^{(p)}$ derived by (22), and comparing the results by the proposed method 2 with those by the conventional method, the MAP estimation by (22) is essential to obtain a good segmentation.



(a) (b) error=1.8% (c) error=1.7% (d) error=4.0%

Figure 2: Segmentation results applied to Fig. 1(a): (a) true region image, (b) segmented image by proposed method 1, (c) by proposed method 2, and (d) by conventional method.

5. CONCLUSION

We have proposed an improved method for unsupervised segmentation of multispectral images consisting of multiple textures. The proposed method is an iterative method based on the EM method where a decomposed a posteriori probability is used. Using the mean field approximation, a posteriori probability of the whole region image is decomposed into the product of local a posteriori probabilities (LAPs) for all pixels. Experiments show that the use of LAPs is essential to perform a good image segmentation.

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