Blind Channel Identification Using RLS Method Based on Second-Order Statistics

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In this paper, we show a new blind identification algorithm which is based on second order statistics and exploits a Single-Input Double-Output(SIDO) model. It is suitable for a real-time processing system because of lower operation and high-speed convergence. The proposed blind identification algorithm is superior to conventional algorithms in view of simple structure and the uniqueness of solution. We also verify its efficiency by computer simulation.

1 INTRODUCTION

Intersymbol interference (ISI) occurs in high speed digital communication due to multipath effects and/or bandwidth constraints. Suppression of ISI or channel equalization, requires either direct or indirect knowledge of the channel impulse response.

There are famous equalization techniques such as LMS or DFE; however, both of them require training sequence. Such methods, though attractive in handling time-variant channels, have to waste a fraction of the transmission time for a training sequence. Particular circumstances, such as time-varying channels, may force frequent re-transmission of the training sequence, leading to inefficient use of available bandwidth.

On the other hand, blind identification and equalization as they do not require training sequences can solve the such problems; therefore, many techniques have been developed. CMA(Constant Modulus Algorithm), such as Sato[1] and Godard[2], suffers from the potential likelihood of being trapped in a local minima due to a nonconvex cost function. The trispectrum-based algorithm[3] can avoid the local minima problem; however, it requires a high computational complexity. In addition, a limitation common to both of these methods is a slow rate of convergence. Recently, many blind channel identification and equalization algorithms using second-order statistics[4]-[7] have been proposed. However, these methods face several difficulties: 1)Nonconvex optimization, 2)Channel order determination, and 3)High computational load numerical arithmetic such as a singular value decomposition.

We have proposed blind channel identification and equalization algorithm which exploits a SIDO(Single-Input Double-Output) model[8]. However, this algorithm needs to compute the correlation matrix and the inverse matrix; therefore, it is difficult to use it in a real-time processing system.

In this paper, we propose a new blind channel identification algorithm that can solve the problems in conventional algorithms. This algorithm is based on second order statistics and exploits a SIDO model. The proposed algorithm is derived from by solving a set of linear simultaneous equation recursively so that we can obtain the unique impulse response. It is suitable for a real-time processing system because of lower operation and high-speed convergence.

The organization of this paper is as follows. The characteristics which is derived from SIDO model is shown in Section II. In Section III, we propose a new cost function in order to blindly identify the channel. Then the blind identification equation is formulated and the existence of its solution is shown. In Section IV, we develop an algorithm to solve the equation above recursively using RLS method. In Section VI, computer simulation results have been shown in order to verify the effectiveness of the proposed algorithm.

2 CHARACTERISTIC OF SIDO MODEL

The concept of SIDO model should not be limited to two physical receivers or sensors. As we has shown, temporally up-sampled with factor2 digital communication signals can also be modeld as a SIDO system as shown in Fig.1[8].

As shown in Fig.1, $y_i(\cdot)$ denotes the output signal from the ith channel with the FIR channel impulse response $h_i(\cdot)$ with L_i taps, which are driven by the same input $x(\cdot)$. We assume in here that $x(\cdot)$ is wide sense stationary(WSS) signal. As a result, we can describe $y_i(\cdot)$ which are WSS as follows;

$$y_i(n) \quad = \quad h_i(n) * x(n) \tag{1}$$

where $*$ is convolution operation.

In Fig.1, the cross-correlation function(CCF) between $y_1(\cdot)$ and $y_2(\cdot)$ can be given as follows.

$$\begin{aligned} r_{y_1y_2}(\tau) &= E[y_1(n)y_2(n+\tau)] \\ &= \sum_{k=0}^{L_1-1}\sum_{l=0}^{L_2-1} h_1(k)h_2(l)R_{xx}(\tau-l+k) \\ &= h_1(-\tau)*h_2(\tau)*R_{xx}(\tau) \end{aligned} \tag{2}$$

Likewise the auto-correlation function(ACF) in terms of $y_1(\cdot)$ is given by

$$\begin{aligned} r_{y_1y_1}(\tau) &= E[y_1(n)y_1(n+\tau)] \\ &= h_1(-\tau)*h_1(\tau)*R_{xx}(\tau) \end{aligned} \tag{3}$$

By cancelling out $R_{xx}(\cdot)$ from both Eq.(2) and Eq.(3), we get

$$h_1(\tau)*r_{y_1y_2}(\tau) = h_2(\tau)*r_{y_1y_1}(\tau) \tag{4}$$

3 BLIND CHANNEL IDENTIFICATION

3.1 Cost Function

We define the estimated value of $h_i(\cdot)$ as $\hat{h_i}(\cdot)$. Substitution $\hat{h_i}(\cdot)$ for $h_i(\cdot)$ of Eq.(4) and expressing the ACF and the CCF as instantaneous value, gives

$$e_1(n) = \hat{h_1}(\tau) * (y_1(n)y_2(n-\tau)) - \hat{h_2}(\tau) * (y_1(n)y_1(n-\tau)) \tag{5}$$

We assume that $\hat{h_1}(0) = 1$. By expressing Eq.(5) in the matrix form, we get

$$e_2(n) = y_1(n)y_2(n) - \hat{\boldsymbol{H}}^t \boldsymbol{Y}(n) \tag{6}$$

where

$$\hat{\boldsymbol{H}} = \left[\hat{h_1}(1) \cdots \hat{h_1}(L_1 - 1) \;\; \hat{h_2}(0) \cdots \hat{h_2}(L_2 - 1)\right]^t \tag{7}$$

$$\begin{aligned}\boldsymbol{Y}(n) = y_1(n)\,[\quad & -y_2(n-1) \quad \cdots \quad -y_2(n - L_1 + 1) \\ & y_1(n) \quad \cdots \quad y_1(n - L_2 + 1)\,]^t\end{aligned} \tag{8}$$

Hence, we define a new cost function, which is mean squared value of Eq.(6), as follows;

$$J(\hat{\boldsymbol{H}}) = E\left[e_2^2(n)\right] = E\left[\left\{y_1(n)y_2(n) - \hat{\boldsymbol{H}}^t \boldsymbol{Y}(n)\right\}^2\right] \tag{9}$$

where $E[\cdot]$ denotes expectation operation.

3.2 Optimum Solution

First, we rewrite Eq.(9) as

$$J(\hat{\boldsymbol{H}}) = r - 2\hat{\boldsymbol{H}}^t \boldsymbol{P} + \hat{\boldsymbol{H}}^t \boldsymbol{R}\hat{\boldsymbol{H}} \tag{10}$$

where

$$\boldsymbol{R} = E\left[\boldsymbol{Y}(n)\boldsymbol{Y}^t(n)\right] \tag{11}$$

$$\boldsymbol{P} = E\left[y_1(n)y_2(n)\boldsymbol{Y}(n)\right] \tag{12}$$

$$r = E\left[\{y_1(n)y_2(n)\}^2\right] \tag{13}$$

The error surface and the contour of cost function J are illustrated in Fig.2 and Fig.3. Impulse response of channels are $\hat{h_1} = [1\ 1.5]$ and $\hat{h_2} = [2]$. From the contour map we can see the existence of the global minimum.

In order to derive the optimum solution of Eq.(10), we take partial differential as

$$\frac{\partial J}{\partial \hat{\boldsymbol{H}}} = 2\boldsymbol{R}\hat{\boldsymbol{H}} - 2\boldsymbol{P} \tag{14}$$

Furthermore, by setting Eq.(14) equals 0, we get

$$\boldsymbol{R}\hat{\boldsymbol{H}} = \boldsymbol{P} \tag{15}$$

We then finally obtain $\hat{\boldsymbol{H}}$ by taking a matrix inversion in Eq.(15) as

$$\hat{\boldsymbol{H}} = \boldsymbol{R}^{-1}\boldsymbol{P} \tag{16}$$

As a result, if $\boldsymbol{R}$ is nonsingular matrix we can estimate channel by only using received signal $y_i(\cdot)$. In other words, we can blindly identify the channel.

3.3 Condition For Channel Identifiability

[*Theorem*]

Since Eq.(11) is nonsingular matrix, Eq.(10) has a unique solution.

[*Proof of theorem*]

We first define

$$\alpha(n - \tau_\alpha) = y_1(n)y_2(n - \tau_\alpha) \tag{17}$$

$$\beta(n - \tau_\beta) = y_1(n)y_1(n - \tau_\beta) \tag{18}$$

$$\tau_\alpha, \tau_\beta \;:\; ragtime$$

where $y_i(\cdot)$ is WSS. Then we obtain

$$r_{\alpha\alpha}(\tau_\alpha) = E[\alpha(n)\alpha(n - \tau_\alpha)] \tag{19}$$

$$r_{\alpha\beta}(\tau_\beta - \tau_\alpha) = E[\alpha(n - \tau_\alpha)\beta(n - \tau_\beta)] \tag{20}$$

$$r_{\beta\beta}(\tau_\beta) = E[\beta(n)\beta(n - \tau_\beta)] \tag{21}$$

The above formulas indicate that the ACF and the CCF between $\alpha(n - \tau_\alpha)$ and $\beta(n - \tau_\beta)$ depend on its ragtime. In this case, the Eq.(11) may be rewritten as

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{r}_{\alpha\alpha} & \boldsymbol{r}_{\alpha\beta} \\ \boldsymbol{r}_{\beta\alpha} & \boldsymbol{r}_{\beta\beta} \end{bmatrix} \tag{22}$$

where

$$\boldsymbol{r}_{\alpha\alpha} = \begin{bmatrix} r_{\alpha\alpha}(0) & r_{\alpha\alpha}(-1) & \cdots & r_{\alpha\alpha}(-L_1+3) & r_{\alpha\alpha}(-L_1+2) \\ r_{\alpha\alpha}(-1) & r_{\alpha\alpha}(0) & \cdots & r_{\alpha\alpha}(-L_1+4) & r_{\alpha\alpha}(-L_1+3) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{\alpha\alpha}(-L_1+3) & r_{\alpha\alpha}(-L_1+2) & \cdots & r_{\alpha\alpha}(0) & r_{\alpha\alpha}(-1) \\ r_{\alpha\alpha}(-L_1+2) & r_{\alpha\alpha}(-L_1+1) & \cdots & r_{\alpha\alpha}(-1) & r_{\alpha\alpha}(0) \end{bmatrix} \tag{23}$$

$$\boldsymbol{r}_{\beta\beta} = \begin{bmatrix} r_{\beta\beta}(0) & r_{\beta\beta}(-1) & \cdots & r_{\beta\beta}(-L_2+2) & r_{\beta\beta}(-L_2+1) \\ r_{\beta\beta}(-1) & r_{\beta\beta}(0) & \cdots & r_{\beta\beta}(-L_2+3) & r_{\beta\beta}(-L_2+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{\beta\beta}(-L_2+2) & r_{\beta\beta}(-L_2+3) & \cdots & r_{\beta\beta}(0) & r_{\beta\beta}(-1) \\ r_{\beta\beta}(-L_2+1) & r_{\beta\beta}(-L_2+2) & \cdots & r_{\beta\beta}(-1) & r_{\beta\beta}(0) \end{bmatrix} \tag{24}$$

$$\boldsymbol{r}_{\alpha\beta} = \begin{bmatrix} -r_{\alpha\beta}(1) & -r_{\alpha\beta}(0) & \cdots & -r_{\alpha\beta}(-L_2+3) & -r_{\alpha\beta}(-L_2+2) \\ -r_{\alpha\beta}(2) & -r_{\alpha\beta}(1) & \cdots & -r_{\alpha\beta}(-L_2+4) & -r_{\alpha\beta}(-L_2+3) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -r_{\alpha\beta}(L_1-2) & -r_{\alpha\beta}(L_1-3) & \cdots & -r_{\alpha\beta}(L_1-L_2) & -r_{\alpha\beta}(L_1-L_2-1) \\ -r_{\alpha\beta}(L_1-1) & -r_{\alpha\beta}(L_1-2) & \cdots & -r_{\alpha\beta}(L_1-L_2+1) & -r_{\alpha\beta}(L_1-L_2) \end{bmatrix} \tag{25}$$

$$r_{\beta\alpha} = r^t_{\alpha\beta} \tag{26}$$

We define a vector $\boldsymbol{v}$ of dimension $(L_1 + L_2 - 1) \times 1$ in order to get the scalar parameter a as follows

$$\begin{aligned} a &= \boldsymbol{v}^t \boldsymbol{Y}(n) \\ &= \boldsymbol{Y}^t(n)\boldsymbol{v} \end{aligned} \tag{27}$$

Then we get

$$\begin{aligned} E[a^2] &= E[\boldsymbol{v}^t \boldsymbol{Y}(n)\boldsymbol{Y}^t(n)\boldsymbol{v}] \\ &= \boldsymbol{v}^t \boldsymbol{R}\boldsymbol{v} \end{aligned} \tag{28}$$

where

$$E[a^2] > 0 \tag{29}$$

Hence,

$$\boldsymbol{v}^t \boldsymbol{R}\boldsymbol{v} > 0 \tag{30}$$

It shows that $\boldsymbol{R}$ is a positive definite matrix,i.e.nonsingular matrix. Consequently, we can conclude that $\boldsymbol{R}$ has an inverse matrix[9].

□

4 RECURSIVE ALGORITHM USING RLS

In order to implement Eq.(16) in real-time processing system, we show a recursive algorithm using RLS method. First, we assume that each matrix of Eq.(16) has time index n and then rewite Eq.(16) as follows

$$\begin{aligned} &\boldsymbol{R}^{-1}(n)\boldsymbol{P}(n) = \\ &\quad \{\boldsymbol{R}(n-1) + \boldsymbol{Y}(n)\boldsymbol{Y}^t(n)\}^{-1} \\ &\quad\quad \{\boldsymbol{P}(n-1) + y_1(n)y_2(n)\boldsymbol{Y}(n)\} \end{aligned} \tag{31}$$

According to the matrix inversion lemma,

$$(\boldsymbol{A} + \boldsymbol{B}\boldsymbol{C})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I} + \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1}$$

we can rewrite the first term of the right side of Eq.(31) as follows

$$\begin{aligned} &\boldsymbol{R}^{-1}(n-1) - \boldsymbol{R}^{-1}(n-1)\boldsymbol{Y}(n) \\ &\quad \{\boldsymbol{I} + \boldsymbol{Y}^t(n)\boldsymbol{R}^{-1}(n-1)\boldsymbol{Y}(n)\}^{-1}\boldsymbol{Y}^t(n)\boldsymbol{R}^{-1}(n-1) \end{aligned} \tag{32}$$

Here we define

$$\boldsymbol{Q}(n-1) = \boldsymbol{R}^{-1}(n-1) \tag{33}$$

then

$$\boldsymbol{K}(n) = \boldsymbol{Q}(n-1)\boldsymbol{Y}(n)\{\boldsymbol{I} + \boldsymbol{Y}^t(n)\boldsymbol{Q}(n-1)\boldsymbol{Y}(n)\}^{-1} \tag{34}$$

Also we define

$$\hat{\boldsymbol{H}}(n-1) = \boldsymbol{Q}(n-1)\boldsymbol{P}(n-1) \tag{35}$$

By substituting Eq.(34)-Eq.(35) into the right side of Eq.(31), we can rewrite

$$\begin{aligned} &\{\boldsymbol{R}(n-1) + \boldsymbol{Y}(n)\boldsymbol{Y}^t(n)\}^{-1}\{\boldsymbol{P}(n-1) + y_1(n)y_2(n)\boldsymbol{Y}(n)\} \\ &= \hat{\boldsymbol{H}}(n-1) + \boldsymbol{K}(n)\{y_1(n)y_2(n) - \boldsymbol{Y}^t(n)\hat{\boldsymbol{H}}(n-1)\} \end{aligned} \tag{36}$$

As a result, a recursive algorithm using RLS method is summarized as follows;

Step*1* $\quad \boldsymbol{K}(n) = \dfrac{\boldsymbol{Q}(n-1)\boldsymbol{Y}(n)}{\{1 + \boldsymbol{Y}^t(n)\boldsymbol{Q}(n-1)\boldsymbol{Y}(n)\}} \quad (37)$

Step*2* $\quad \hat{\boldsymbol{H}}(n) = \hat{\boldsymbol{H}}(n-1) + \boldsymbol{K}(n) \{y_1(n)y_2(n) - \boldsymbol{Y}^t(n)\hat{\boldsymbol{H}}(n-1)\} \quad (38)$

Step*3* $\quad \boldsymbol{Q}(n) = \boldsymbol{Q}(n-1) - \boldsymbol{K}(n)\boldsymbol{Y}^t(n)\boldsymbol{Q}(n-1) \quad (39)$

where initial values are given by

$$\hat{\boldsymbol{H}}(0) = \mathbf{o} \tag{40}$$

$$\boldsymbol{Q}(0) = c\boldsymbol{I} \qquad (c \text{ is constant value}) \tag{41}$$

5 SIMULATION

We show a computer simulation under the following conditions.

Unknown system	24-tap FIR LPF
Input signal $x(n)$	16QAM signal
Evaluation	$\mathrm{IRER} = 10\log_{10}\frac{\|\boldsymbol{h}-\hat{\boldsymbol{h}}\|^t\|\boldsymbol{h}-\hat{\boldsymbol{h}}\|}{\boldsymbol{h}^t\boldsymbol{h}}$ [dB]

We use the channel recommended by ITU-T in our simulation. Fig.4 shows its characteristics in the frequency domain. The convergence property of the proposed scheme evaluated by IRER(Impulse Response Estimation Ratio) is shown in Fig.5, which shows that the channel is well estimated and the convergence speed is fast. As we can see, the proposed algorithm is effective.

6 CONCLUSION

In this paper, we have proposed a new blind channel identification algorithm which is based on second-order statistics and exploits SIDO model. We have developed a recursive algorithm using RLS method and have verified the effectiveness of the proposed algorithm by simulation. As a result, we have shown that we are able to identify channel blindly with lower operation and high-speed convergence.

The future work is to develop adaptive algorithms using stochastic gradient method.

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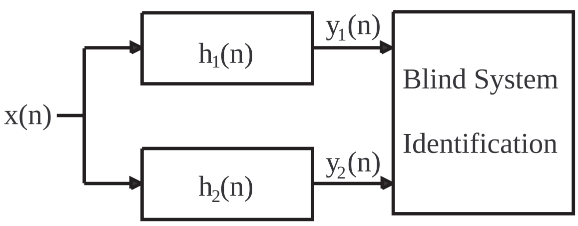

Figure 1: SIDO model

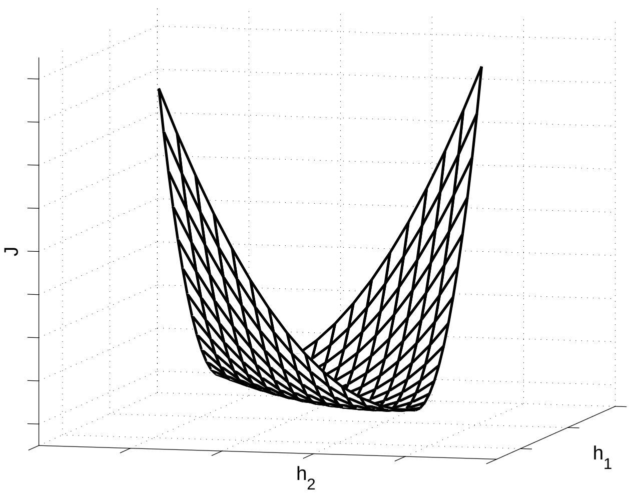

Figure 2: error suface of the cost function

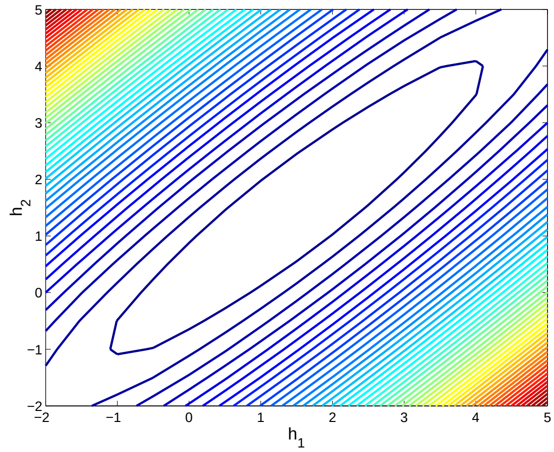

Figure 3: contour of the cost function

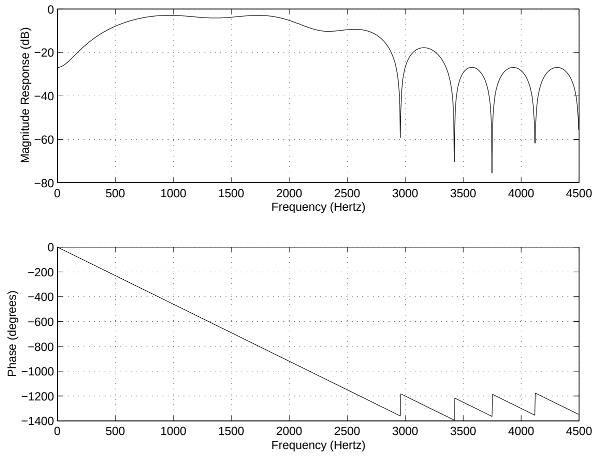

Figure 4: frequency response of the channel

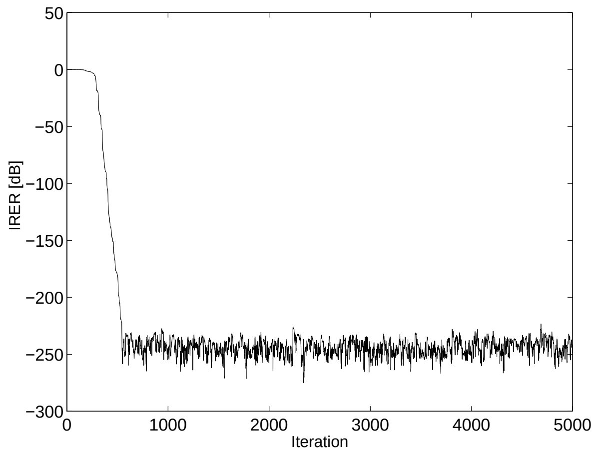

Figure 5: convergence property of IRER(h_2)