A COMPARATIVE STUDY BETWEEN PARAMETRIC BLUR ESTIMATION METHODS

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ABSTRACT

In pattern recognition problems, the effectiveness of the analysis depends heavily on the quality of the image to be processed. This image may be blurred and/or noisy and the goal of digital image restoration is to find an estimate of the original image. A fundamental issue in this process is the blur estimation. When the blur is not readily avalaible, it has to be estimated from the observed image. Two main approaches can be found in the literature. The first one identify the blur parameters before any restoration whereas the second one realizes these two steps jointly. We present a comparative study of several parametric blur estimation methods, based on a parametric ARMA modeling of the image, belonging to the first approach. Our purpose is to evaluate the acuracy of the various methods, on which the restoration procedure relies, and their robustness to modeling assumptions, noise, and size of support.

1. INTRODUCTION

It is well known that in all problem of pattern recognition by vision, the quality of the analysis depends heavily on that of the image to process (possibly blurred and/or noisy version of the original image) and the filtering or restoration operation has to eliminate at best the degradation that allocates the image while preserving the relevant information. Thus, the goal of digital image restoration is to estimate the original image from the degraded observed image at the output of the imaging system. To solve this problem, there exists in the literature two great approaches: the first consists in identify parameters of the blur beforehand to all processing of restoration, the second realizes jointly the two stages. As a result, when the point spread function (PSF) is not readily available, a fundamental issue in methods of both approaches is the blur estimation. This general problem can be divided into several levels of difficulty. These levels relate mainly to the extent of the blur, to the availability of an analytic PSF, to the space-variance or space-invariance of the PSF and to the noise properties of the whole image formation process. So, it is very difficult to *a priori* choose a precised method to solve a particular restoration problem. The purpose of this paper is to present the results of a comparative study of methods belonging to the fisrt appoach; more precisely of most common blur estimation methods which consist in identify blur from the observed image, without any other *prior* knowledge. This comparative study will allow us to evaluate on the one hand, the efficiency of the retained methods, from which the quality of the restoration procedure highly depends, and on the other hand, to specify their robustness to hypotheses they take into account. It is obvious that the success of any estimation procedure is closely linked to the accuracy of the observation model that describes the relationship between the input (original image) and the output (observed image) of the imaging system. In spite of the great number of imaging systems, in many practical situations, a standard observation model is used: the image degradation process is approximated by a linear space-invariant blur with an additive gaussian noise [1]. Although this may seem quite restrictive, many common blurs are adequately described by a parametric function with few parameters. The noise is assumed to be an independant identically distributed zero mean random process and covers measurement errors and quantization errors in the sampling process.

We begin by introducing the image and observation models that will be used in this work. The original sampled image and the noise process are represented by matrix F and N of size $N \times N$. By using the $vec()$ operator [6] we obtain an algebraic expression of the image restoration problem. The standard observation model is then expressed as:

$$g = H\ f + n \tag{1}$$

where $f = vec(F), n = vec(N), g$ represents the observed image and H is the $N^2 \times N^2$ matrix of the linear transformation built from the discrete PSF [1].

When the PSF is space-invariant, the matrix H is block Toeplitz. By using the circulant-to-Toeplitz approximation [1], which amounts to approximating linear convolution by circular convolution, computations can be simplified in the Fourier domain. In order to avoid Fourier induced artefacts, the boundaries of g may have to be preprocessed.

Now, assuming an AR representation of the original image, which is widely used [7] because it is useful for approximating the second order statistics of an image, the original image f is considered as the output of a 2D AR filter whose input is a zero-mean white noise process u with variance σ_u^2. The spatial coordinates regression is expressed as:

$$f(x,y) = \sum_{S_a} \sum a(k,l) f(x-k, y-l) + u(x,y) \tag{2}$$

with $f(x,y)$ the value of the original image at coordinates (x,y), $u()$ the noise process and $a()$ the coefficients of the filter of support S_a.

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A more compact expression is derived by using the $vec()$ operator [6] in order to stack the matrix F and U representing the original image and the input noise columnwise:

$$f = A\ f + u \tag{3}$$

where $f = vec(F)$, $u = vec(U)$ and the matrix A is built with the AR coefficients [2].

The image autocorrelation is then expressed as:

$$\Gamma_f = E[\ f\ f^T] = \sigma_u^2\ (I - A)^{-1}{(I - A)^{-1}}^T \tag{4}$$

Then, grouping the previous formation equation (3) and the observation equation (1) yields the following state-space representation:

$$f = A\ f + u \tag{5}$$
$$g = H\ f + n \tag{6}$$

It can be reduced to a single equation:

$$g = H\ f + n \tag{7}$$
$$= H\ (A\ f + u) + n \tag{8}$$
$$= H\ A\ f + Hu + n + A\ n - A\ n \tag{9}$$
$$= A\ H\ f + An + H\ u + (I - A)\ n \tag{10}$$
$$= A\ (H\ f + n) + H\ u + (I - A)\ n \tag{11}$$
$$= A\ g + H\ u + n \tag{12}$$

In equation (10), matrix H and A permute because of their structure. In equation (12), $(I - A)\ n$ is approximated by n. Thus, equation (12) represents an ARMA model which approximates the observed image g.

2. PSF ESTIMATION

Now, we present a brief review of the retained blur estimation methods. The earlier techniques of blur identification were focused on PSF with regular patterns of zeros in the frequency domain. The periodicity (in the case of a motion blur) or pseudo-periodicity (in the case of a defocusing blur) of these zeros can be identified by power spectrum or cepstral averaging [10]. These techniques do not take into account the observation noise and cannot be used with other PSF.

More recently, other blur identification techniques were developed using the ARMA modeling (12) of the observed image [7]. As a result, the PSF coefficients are the MA coefficients of the model whereas the AR coefficients represent the image autocorrelation and translates a certain degree of knowledge on the original image.

This way, the PSF estimation becomes a problem of estimating the MA parameters of a stochastic parametric ARMA model.

As precised before, different hypotheses on the statistical characteristics of the distribution of n (12) can be considered and lead to different methods. That is why we have selected firstly the method of identification by decomposition in parallel ARMA1D models proposed by [2], and secondly a method by resolution of a linear system built from second order statistics of the observed image and proposed by [5].

Now, let us consider the restoration problem. It consists in inversing the direct model (1). It belongs to the class of ill-posed inverse problems for which a unique and stable solution is not available [1]. Small variations of the degraded image can cause large variations of the restored image. Regularization is a technique that uses *prior* information in order to obtain a satisfactory solution.

The Tikhonov-Miller regularization has been developed in a deterministic framework [1] and results in a stabilizing functionnal being added to the least-squares solution of (1):

$$\hat{f}_{TM} = \arg\min_f J(f) = \|g - H\ f\|^2 + \lambda\ \|C\ f\|^2 \tag{13}$$

where C incorporates prior knowledge about f and λ is the regularization parameter.

Usually, C is a low-order differenciation operator, such as a first order difference operator. The regularization parameter λ controls the trade-off between fidelity to the available data g and smoothness of the estimate $\hat{f}$. By introducing a bias on the estimate, the mean square error of the solution is reduced [4].

Several methods exist in order to choose the value of the regularization parameter λ [4]. The generalized cross validation (GCV) criterion is very popular because it provides good estimates of the optimal value of λ without *prior* knowledge of the observation noise variance.

The GCV criterion can be expressed in the spatial domain as:

$$GCV(\lambda) = \frac{\|\ (I - H\ H_r(\lambda))\ g\ \|^2}{[\ trace(\ I - H\ H_r(\lambda)\ )\ ]^2} \tag{14}$$

where I is the identity matrix and H_r is the restoration matrix obtained from (13):

$$H_r = \left(\ H^T\ H + \lambda\ C^T\ C\ \right)^{-1} H^T \tag{15}$$

The optimal value $\hat{\lambda}$ of the regularization parameter is chosen as the one that minimises the GCV criterion.

$$\hat{\lambda} = \arg\min_\lambda\ GCV(\lambda) \tag{16}$$

However, the GCV criterion may not have a unique minimum [4] but this non-unicity is quite uncommon and happens mainly with low levels of noise.

Assuming an ARMA modeling (12) of the observed image, the GCV criterion can be expressed in the spatial domain as [8]:

$$GCV(\theta) = \frac{\|\ (I - [H\ H_r](\theta))\ g\ \|^2}{[\ trace(\ I - [H\ H_r](\theta)\ )\ ]^2} \tag{17}$$

where, using (15), the expression of the Wiener or Minimum Mean Square Estimate of the original image:

$$\hat{f}_{MMSE} = (H^T \Gamma_n^{-1} H + \Gamma_f^{-1})^{-1} H^T \Gamma_n^{-1} g \tag{18}$$

and finally (4):

$$[H\ H_r](\theta) = H\ \left(H^T\ H + \lambda\ (I - A)^T\ (I - A)\right)^{-1}\ H^T \tag{19}$$

and the vector of unknown parameters is:

$$\theta = [\ h(k,l), a(p,q), \lambda\ ]^T \qquad (k,l) \in S_h \quad (p,q) \in S_a \tag{20}$$

Reeves and Mersereau [9] show that the derivative of the expected value of the GCV criterion is equal to zero when the estimated parameters are the actual parameters. For blurs described by a single parameter θ_h, they used an iterative two-steps search

strategy. In a first step, starting with an initial guess for the AR coefficients of the image, the optimal value of the regularization parameter λ is evaluated for each value of the parameter θ_h in a predetermined set. The range of the parameter θ_h in which the GCV criterion atains a minimum value is thus bracketed. In a second step, the AR coefficients are estimated . These two steps are iterated until the estimated parameters reach a sufficient accuracy. This third method is retained in our comparative study.

3. THE PROTOCOL OF COMPARISON

To evaluate the efficiency and the robustness of the retained parametric blur identification methods, we propose a comparison of their performances [11], on firstly a synthetic image constituted by a random field, and secondly on a real image of the french data bank of the CNRS GDR-PRC-ISIS (http://www-isis.enst.fr). The synthetic image has been obtained by filtering of an independant, identically and exponentially distributed noise of unit variance. The filter used is a symmetrical causal AR filter of support 3 whose coefficients are:

$$\begin{bmatrix} 0.1101 & -0.4000 & 0.1101 \\ -0.2752 & 1.0000 & -0.2752 \\ 0.1101 & -0.4000 & 0.1107 \end{bmatrix} \quad (21)$$

Next, we have considered PSF with simple characteristics: symmetry and shift-invariance. These assumptions are common to all the tested methods. The retained PSF are those of a uniform motion blur (parallel linear movement to the plan of the image); a defocusing blur without aberration; a truncated gaussian blur and a pillbox blur.

As a result, for the synthetic blurred image, the previous assumed image modelisation is well suited to the data and second order statistics exist. For the real image, the stationarity of the stochastic process modeling the original image and the ARMA assumption are stronger assumptions, that need to be evaluated. These two considered images are shown in figure 1.

Now, to evaluate the robustness of each selected method to variations of the PSF from the previous assumptions or the value of the signal to noise ratio,we have defined 6 scenarios which can be summarized as follows:

Scenario 1: blurred original image;

Scenario 2: blurred original image with a gaussian zero mean additive white noise of variance 2;

Scenario 3: blurred original image with a gaussian zero mean additive white noise of variance 6;

Scenario 4: blurred original image with a gaussian zero mean additive white noise of variance 14;

Scenario 5: blurred original image, while the blur was subject to a stochastic deformation (gaussian zero mean additive white noise of variance 10^{-3});

Scenario 6: blurred original image, while the blur was subject to a deterministic deformation (similarity with report 0.7 of its Fourier transform).

The scenarios 2, 3 and 4 allow us to evaluate the robustness of each estimation method to various levels of observation noise. The scenario 5 gives insights on their respective robustness to the symetry hypothesis.

The scenario 6 highlights their respective robustness to good *a priori* information on the actual value of the blur support.

Finally, the last parameter that we have made vary is the length of the support of the PSF. Three different values of the blur support have been taken into account: 3x3, 5x5 and 15x15 pixels (3x1, 5x1 and 15x1 for the uniform motion blur).

The presentation of the results of this comparative study is done using the Mean Square Error Criterion in percent between the estimated PSF and the actual PSF. The calculation of the MSE is given by:

$$MSE(\hat{h}) = 100\, \frac{\sum_{(k,l)} \left(a\, \hat{h}(k,l) - h(k,l) \right)^2}{\sum_{(k,l)} h^2(k,l)} \quad (22)$$

where $\hat{h}(k,l)$ is the current estimated value of $h(k,l)$ and the parameter a is a scaling factor $a = \frac{\sum_{(k,l)} h(k,l)\, \hat{h}(k,l)}{\sum_{(k,l)} \hat{h}^2(k,l)}$

4. SIMULATIONS AND RESULTS

For both images, results are tabulated for a value of the blur support equal to 5x5 pixels and then 15x15 pixels. We do not present the results for a value of the blur support equal to 3x3 pixels since they are quite similar to those obtained with a value of the blur support equal to 5x5 pixels. Each table (table 1 and then 2) displays the three retained methods (columnwise) against the six scenarios and records in each square, blurs belonging to the four previously precised, for which the obtained value of the MSE is leather than 10%. For each refered method, the first line of results has been obtained with the synthetic image 1-a) and the second line of results with the real image 1-b). A different symbol is used for each type of blur:

- m: uniform motion blur;
- d: defocusing blur without aberration;
- g: truncated gaussian blur;
- p: pillbox blur.

		Scenario					
		1	2	3	4	5	6
[2]	a)	m g p				m g p	m
	b)	m d g p	m	m		m g p	m d g
[5]	a)	m g p	p			m g p	g
	b)	m p	m p	m p	m p	m p	
[9]	a)	g p	m d g p	m d g p	m d g p	m g p	m p
	b)	m d g p	m d g p	m d g p	m d g p	m d g p	m d

Table 1: Blur Support equal to 5x5 pixels

The results given by the tables bring to light the behavior of the retained methods of blur estimation. They allow us to give some precised comments about the cases where the methods are especially efficient or deficient. Indeed, for each method, we can easily see for which type of blurs, one gets best results.

5. CONCLUSION

A first conclusion can be drawn. Results given by the method by decomposition in parallel ARMA1D models validate the use of the ARMA modeling to identify synthetic blur on a real image. The

		Scenario					
		1	2	3	4	5	6
[2]	a)	m g p					m
	b)		m				m
[5]	a)						
	b)						
[9]	a)	m g p	m d g p	m d g p	m d g p	m g	m g p
	b)	m d g p	m d g p	m d g p	m d g p	m d g	m d g p

Table 2: Blur Support equal to 15x15 pixels

most precise results are obtained with the synthetic image without any observation noise. The accuracy of these methods decreases when the size of the support of the PSF increases and become unacceptable in the presence of an observation noise, even with a very small variance. The results obtained with the real image are less precise without any observation noise but they are far less sensitive to the presence of an observation noise. However, when the size of the support of the PSF increases, the acuracy of the estimations decreases more sharply than in the case of a synthetic image. Results obtained with scenarios 5 and 6 lead us to underline the sensitivity of the algorithm to first, the no symmetry property of the coefficients of the PSF and, second, to the *a priori* surevaluation of the support. The more important the size of the support, the more marked this sensitivity. Moreover, we have been able to observe the importance of the characteristics of the PSF on the accuracy of the estimation. This accuracy decreases when the PSF have null coefficients on the outskirts as is the case with a defocalisation blur and zeros in the frequency domain. The pillbox blur is an example of such a PSF. These observations explain that most precise results are obtained when the PSF is a truncated gaussian one. So, we can conclude that this algorithm is interesting for real images and symetrical blur with limited support.

However, the *a priori* estimation of the size of the support in the general case remains to specify.

The method by resolution of a linear system built up from second order statistics seems not to be very robust to modelisation errors: results obtained with the synthetic image are not found with the real image. Moreover, when the size of the support increases, one gets a "limit solution" independently of the observation noise.

The method of the monoparametric generalized crossed validation is the one which gives the best results, especially on real images. It presents the advantage to directly estimate the support and to be extremely robust to the noise. It needs on the other hand a parametric form of the blur. This may limit its use when the PSF to identify does not possess an analytic form. The use of a first order differentiation matrix in the computation of the criterion makes no longer necessary to iteratively estimate both the parameter of interest of the PSF and the vector of AR parameters of the original image. Results obtained with the real image and the image realizing an AR model show this approximation is entirely justified [3].

Figure 1: a) Synthetic image b) Real image

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