MULTI-CHANNEL HIGH RESOLUTION BLIND IMAGE RESTORATION

Wirawan, Pierre Duhamel, and Henri Maître

Dépt. Traitement du Signal et des Images, ENST Paris
46, rue Barrault, 75634 Paris Cedex 13 France
e-mail: {wirawan,maitre}@ima.enst.fr, duhamel@sig.enst.fr

ABSTRACT

We address the reconstruction problem of a high resolution image from its undersampled measurements accross multiple FIR channels with unknown response. Our method consists of two stages : blind multi-input multi-output (MIMO) deconvolution using FIR filters and blind separation of mixed polyphase components. The proposed deconvolution method is based on the *mutually referenced equalizers* (MRE) algorithm previously developed for blind equalization in digital communications. For sources separation, a method is proposed for separating mixed polyphase components of a bandlimited signal. The existing blind source separation algorithms assume that the source signals are either independent or uncorrelated, which is not the case when the sources are polyphase components of a bandlimited signal. Simulation results on artificial and photographics images are given.

1. INTRODUCTION

Multi-channel image restoration and reconstruction have gained interest among workers in the last two decades. Multichannel image reconstruction in this paper refers to the methods to obtain higher resolution image from low resolution multiframe images. Since the generalized sampling theorem was proposed by Papoulis [9], for multi-channel reconstruction of a bandlimited signal from its undersampled measurements, a great effort has been put forward employing it in many applications and extending it to related problems. For example, Unser and Zerubia [11] enlarged the class of input signals to L_2 signals. As for restoration, exact deconvolution approach by using multiple image sensors to overcome the ill-posedness associated with the single image restoration problem has been developed into a working theory by Berenstein et. al., see for example [1, 2].

While each of the restoration and reconstruction tasks is most of the time addressed independently in the litterature, combined effort in a particular problem, namely high resolution restoration, has received less attention [3, 10]. The need of high resolution multichannel image restoration arises, for example, when the observed images are not only degraded by blurs and noise but also suffer from undersampling during image acquisition step. More challenging to this problem is when no sufficient information about the blur PSF is available [10], for example in imaging through a stochastically varying medium, such as a turbulent athmosphere.

In this paper we address blind reconstruction of a discrete bandlimited image from its undersampled measurements accross several unknown FIR channels. Due to the undersampling process, each low resolution frame is a linear combination of the polyphase components of the high resolution input image, weighted by the polyphase components of the individual channel impulse response. Accordingly, the problem considered here (blind high resolution image restoration) can be represented as the blind 2-D deconvolution of a MIMO (multi-input-multi-output) system (sometimes called convolutive mixture) driven by polyphase components of a bandlimited signal. It is known that blind MIMO deconvolution based on second order statistiques contains some inherent indeterminations. That is, in general, we cannot identify the order, the power and the time delay for each source. This means that after deconvolution, the polyphase components still need to be separated. Usually this instantaneous separation is done by using some independence assumption between the sources, which cannot be done here, since the corresponding sources are highly correlated (polyphase components of the same signal).

Our algorithm consists of two stages : 2-D MIMO partial deconvolution and separation of polyphase components. For the deconvolution stage we have extended the generalized MRE method, previously proposed by Gesbert et. al [4], from 1-D to 2-D signals. For the separation stage we propose a source separation algorithm that minimizes out-of-band spectral energy resulting from instantaneous mixture of polyphase components. The deconvolution method using multiple FIR filters we propose belongs to the class of algebraic methods recently studied as an extension from 1-D signals [6, 7, 8], although these works concentrate mainly on blind identification/equalization. Here, we extend these works to high resolution imaging.

2. PROBLEM STATEMENT

The signal measurement model relating a high-resolution (HR) continuous image $x(t_1, t_2)$, $t_i \in \mathbb{R}, i = 1, 2$ and a set of K low-resolution (LR) discrete images $y_k(n_1, n_2), n_i \in \mathbb{Z}, i = 1, 2, k = 1, \ldots, K$, consists of two parts. The first is multiple, linear and space invariant filters representing the degradations or blurs due to the diffraction limited optical system, finite sensor dimension, and medium impairments. The second part is a sampling stage followed by a discretization process. Due to undersampling, aliasing would be present in the observed LR images $y_k(n_1, n_2)$.

We assume that the image at the focal plane of the imaging system, where the sensor is placed, is bandlimited. This is a reasonable assumption since the optical systems are bandlimited in nature. With this assumption there exists an ideal HR detector the individual sensor element of which (giving rise to a single HR image pixel) has uniform response $u(t_1, t_2)$ over the entire Nyquist sampling period T_1 and T_2 in horizontal and vertical directions, respectively.

Given K measured images $y_k(n_1, n_2), k = 1, \ldots, K$, where each image is of size $m_y \times n_y$, we wish to find FIR restoration filters $g_{k,p}^{i,j}(n_1, n_2)$, each of dimension $m_g \times n_g$, so that in noiseless condition, a high resolution image could be reconstructed perfectly from its low resolution measurements:

$$x(n_1 - i, n_2 - j) = \sum_{p=1}^{P} \sum_{k=1}^{K} g_{k,p}^{i,j}(n_1, n_2) * y_k(n_1, n_2) \quad (1)$$

where * denotes 2-D convolution, (i, j) is a restoration delay, and P the number of polyphase components. In this work we consider that the blur functions $h_k(n_1, n_2)$ are unknown, although their support or order is finite and known. Another assumption is that the support of the spectrum of $x(n_1, n_2)$ is known. Our discrete image formation model is exact, in the sense that no zero-padding or circular convolution is used.

3. MULTI-CHANNEL HIGH RESOLUTION IMAGE RESTORATION

The multi-channel high resolution image restoration method that we propose consists of two parts, multichannel partial deconvolution and separation of instantaneous mixture, which we will described in following sections.

3.1. 2-D SIMO Deconvolution

We consider first the case where the detector array is sufficiently dense to sample continuous images without aliasing, which means that $P = 1$. Thus, the necessary task is to restore multiframe images degraded only by multiple FIR blurs [6, 7, 8] and we refer to this problem as SIMO deconvolution.

Consider the images as a set of K separate windows of size $(m_g \times n_g)$, and consider the images as the concatenation of these windows, written as vectors as follows:

$$Y(n_1, n_2) = [\mathbf{y}_1^t \ldots \mathbf{y}_K^t]^t \quad (2)$$

where:

$$\begin{aligned} \mathbf{y}_k \;\; = \;\; & [y_k(n_1, n_2), \ldots, y_k(n_1 - m_g + 1, n_2), \\ & y_k(n_1, n_2 - 1), \ldots, y_k(n_1 - m_g + 1, n_2 - n_g + 1)]^t, \\ & k = 1, \ldots, K \end{aligned}$$

each vector has size $(m_g \times n_g)$, the full image has size $K m_g n_g$, m_g and n_g are the horizontal and vertical dimensions of the restoration filters.

$$Y(n_1, n_2) = \mathcal{H} X(n_1, n_2) + B(n_1, n_2) \quad (3)$$

$$\begin{aligned} X(n_1, n_2) \;\; = \;\; & [x(n_1, n_2), \ldots, x(n_1 - m_h - m_g + 2, n_2), \\ & x(n_1, n_2 - 1), \ldots, \\ & x(n_1 - m_h - m_g + 2, n_2 - n_h - n_g + 2)]^t \end{aligned}$$

where $\mathcal{H} = [\mathbb{H}_1^t \ldots \mathbb{H}_K^t]^t$ denotes the channel matrix of dimension $(K m_g n_g) \times (m_h + m_g - 1)(n_h + n_g - 1)$,

$$\mathbb{H}_k = \begin{bmatrix} \mathbf{H}_k(0) & \ldots & \mathbf{H}_k(m_h - 1) & & \mathbf{0} \\ \mathbf{0} & \ddots & & \ddots & \\ \vdots & \ddots & & \ddots & \\ \mathbf{0} & \mathbf{H}_k(0) & & \ldots & \mathbf{H}_k(m_h - 1) \end{bmatrix} \quad (4)$$

$$\mathbf{H}_k(m) = \begin{bmatrix} h_k(m, 0) \;\ldots\; h_k(m, n_h - 1) & & 0 \\ 0 \;\; \ddots & & \vdots \\ \vdots \;\; \ddots & & \ddots \\ 0 \;\; h_k(m, 0) \;\ldots & & h_k(m, n_h - 1) \end{bmatrix} \quad (5)$$

and B is noise. The sufficient condition for the existence of deconvolver filters is that the channel matrix $\mathcal{H}$ is of full column rank. Although this is not easily characterized in terms of the blurring filter [7, 8], assume this holds in the sequel.

The 1-D MRE algorithm, from which we developed its 2-D version, was proposed for the first time to address 1-D blind multichannel equalization problem with only one input signal [4], and subsequently extended to multi input case [5]. The basic principle of the MRE method is the exploitation of simple relationships that reveal the information redundancy at the outputs of different deconvolver filters (equalizers) with different restoration delays. We wish to compute different sets of restoration filters corresponding to all possible 2-D delays, for example:

$$G^t(i, j) Y(n_1, n_2) = G^t(0, 0) Y(n_1 - i, n_2 - j) \quad (6)$$

where $G(i, j)$ is $(K m_g n_g \times 1)$ restoration filter vector with delay $(i, j), i = 0, \ldots, m_h + m_g - 2, j = 0, \ldots, n_h + n_g - 2$. Using eq. (6), we look for restoration filter matrix $\mathbf{G}$ consisting of restoration filters for all possible delays (i, j) by minimizing:

$$\begin{aligned} J_1 \;\; = \;\; & E \| [I_{N_1} \, 0_{N_1 \times N_2}] \mathbf{G}^t Y(n_1, n_2) - \\ & [0_{N_1 \times N_2} \, I_{N_1}] \mathbf{G}^t Y(n_1, n_2 + 1) \|^2 \end{aligned} \quad (7)$$

where $N_2 = m_h + m_g - 1$, $N_1 = N_2(n_h + n_g - 1)$, I_{N_1} is $(N_1 \times N_1)$ identity matrix, and $0_{N1 \times N2}$ the zero matrix of appropriate size. This arrangement corresponds to comparison between two blocks of windowed input image differing only by a single horizontal shift. Other possibilities are comparison between up-down neighboring windowed image, diagonals neighboring windowed images etc. J_1 being quadratic, its minimization is very simple [4]. It can be shown that under noiseless condition the minimum corresponds to actual restoration filters.

3.2. 2-D MIMO Deconvolution

When undersampling occurs, aliasing contributes to the loss of resolution, beside blurs and noise. The system model now becomes:

$$Y(n_1, n_2) = \mathsf{H} \mathbf{X}(n_1, n_2) + B(n_1, n_2) \quad (8)$$

where $\mathsf{H} = [\mathcal{H}_1 \ldots \mathcal{H}_P]$, $\mathbf{X}(n_1, n_2) = [X_1^t(n_1, n_2), \ldots, X_P^t(n_1, n_2)]^t$, and $X_p(n_1, n_2)$, $p = 1, \ldots, P$, are the polyphase components of the original image. The channel matrix H has dimension $(Km_gn_g) \times P(m_h + m_g - 1)(n_h + n_g - 1)$ and the restoration condition requires, as usual, that H be of full column rank with necessary condition:

$$(Km_gn_g) \geq P(m_h + m_g - 1)(n_h + n_g - 1) \tag{9}$$

The assumptions concerning the size of H are:

1. all HR channel filters have the same support, (m_H, n_H),
2. m_H and n_H are integer multiples of m_h and n_h, i.e. support of LR channel filters or polyphase components of HR filters.

This is somewhat restrictive conditions. Nevertheless, $\mathcal{H}$ could still be reconfigured to account for zero coefficients, since we assume that (m_H, n_H) is known.

After the deconvolution step, the resulting P polyphase component images are not the original ones, namely that obtained from the HR discrete image, but a mixture of them. This difficulty is inherent to any MIMO deconvolution problem. The mixture in our case could be parametrized as an unknown $P \times P$ non-singular matrix. In what follows we describe the proposed algorithm to separate mixture of polyphase components which does not use the assumption about the independency between sources.

3.3. Separation of polyphase components of a bandlimited signal

The separation step of mixed polyphase components that we address in this work amounts to finding separating matrix giving an image satisfying *a priori* information about the support of its spectrum, i.e. bandlimited. An $l_2(\mathbb{Z}^N)$ signal is said to be *bandlimited* to $\Omega \in [-\pi, \pi]^N$ if there exists $X(\omega) \in L_2([-\pi, \pi]^N)$ such that:

$$x(\mathbf{n}) = \frac{1}{(2\pi)^N} \int_\Omega X(\omega) e^{j<\omega,n>} d\omega \tag{10}$$

or alternately, if $X(\omega) = H_{lpf}(\omega)X(\omega)$, where $H_{lpf}(\omega)$ is the ideal lowpass filter frequency response:

$$H_{lpf}(\omega) = \begin{cases} 1, & \omega \in \Omega \\ 0, & \omega \in \Omega^c \end{cases} \tag{11}$$

Proposition 1

A 1-D discrete time signal $s(n)$, whose polyphase components are mixture of polyphase components of a bandlimited signal $x(n)$, in $\Omega = [-\omega_m, \omega_m]$, $0 < \omega_m < \pi$, with mixture characterized by a nonsingular matrix $\mathbf{B} \in \mathbb{R}^{P \times P}$:

$$\mathbf{s}_{pol}(n) = \mathbf{B}\mathbf{x}_{pol}(n) \tag{12}$$

where $\mathbf{s}_{pol}(n) = [s_1(n) \ \ldots \ s_P(n)]^T$ and $\mathbf{x}_{pol}(n) = [x_1(n) \ \ldots,$ $x_P(n)]^T$, $P > 1$, is also bandlimited to Ω if and only if:

$$\mathbf{B} = \alpha.\mathbf{I}, \ \alpha \in \mathbb{R} \tag{13}$$

Corollary 1

A non bandlimited signal whose polyphase components are mixture of polyphase components of a bandlimited signal defined in Proposition 1 can be transformed into a bandlimited signal $z(\mathbf{n})$ in Ω by:

$$\mathbf{z}_{pol}(\mathbf{n}) = \mathbf{A}\mathbf{s}_{pol}(\mathbf{n}) \tag{14}$$

if and only if:

$$\mathbf{AB} = \alpha.\mathbf{I} \tag{15}$$

Applying to our problem, the original image and its reconstruction both represented also in the form of polyphase vectors:

$$\hat{\mathbf{x}}_{pol}(n_1, n_2) = \mathbf{AB}\mathbf{x}_{pol}(n_1, n_2) \tag{16}$$

where $\mathbf{x}_{pol}(n_1, n_2) = [x_1(n_1, n_2) \ldots x_P(n_1, n_2)]^t$ and, as before, $\mathbf{A}$ is a $P \times P$ separating matrix.

The separating matrix is the one minimizing:

$$J_2(\mathbf{A}) = \sum_{(\omega_1,\omega_2) \neq \Omega} |\widehat{X}(\omega_1, \omega_2)|^2 \tag{17}$$

under the condition that $\mathbf{A}$ be a full rank matrix, where Ω denotes the support of $X(\omega_1, \omega_2)$, DFT of $x(n_1, n_2)$ and $\widehat{X}(\omega_1, \omega_2)$ is DFT of $\widehat{x}(n)$, the reconstructed image.

The assumption that the support of $X(\omega_1, \omega_2)$ is known seems to be overrealistics. However, simulation results show that over determination of support area does not affect very much the optimal solution, whereas if the support area is underdetermined we have erroneous result.

4. SIMULATION

For simulation, first we simulate bandlimited images using a simulated image and *Lena* image which are low-pass filtered so that its spectral energy beyond a given support is zero. The individual bandlimited image is then convolved with 12 blur filters, with impulse response coefficients chosen from random distribution, of size (6×6) and followed by decimation with decimation factor 2 in each direction, so that the number polyphase components is $P = 4$. Using the relation in (9) we choose the size of 12 restoration and reconstruction filters to be (6×6). The following are the steps describing the algorithm :

– First, a partial deconvolution step is carried out directly, without identifying the PSFs, to K low resolution and degraded images, resulting in P images of mixed polyphase components.

– Source separation step is performed using only *a priori* information about the spectral support of the original image, and after that each recovered polyphase components is arranged in its corresponding grid to reconstruct a high resolution image.

The results are shown in Fig. 1 for simulated TP image and Fig. 2 for *Lena* image. It should be noted that our algoritm works as well for blind SIMO deconvolution [6, 7, 8], which is more studied than blind MIMO deconvolution, although the results are not shown here. Compared to other methods the computation complexity of our algorithm does not strongly depend on the image size, but on the size of blur and restoration filters, which certainly gives an advantage in computation complexity for images with large size.

5. CONCLUSIONS

In this paper, we proposed a new blind multi-channel high resolution image restoration by using multiple FIR filters. Multichannel image restoration using FIR filters have received a great

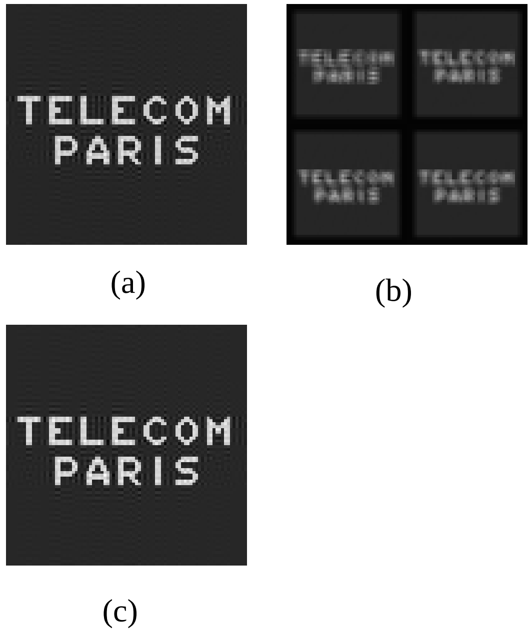

FIG. 1 – *Simulated image. (a) original image, (b) 4 of 12 blurred and undersampled images, (c) restored and reconstructed image.*

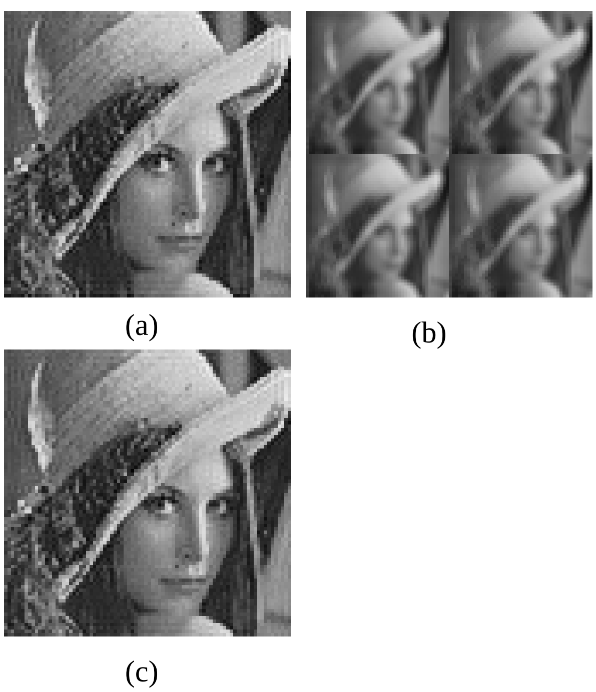

FIG. 2 – *Lena image (a) original image, (b) 4 of 12 blurred and undersampled images, (c) restored and reconstructed image.*

deal of attention recently, either in blind or non-blind setting. However, most of the blind methods developed in the litterature do not consider undersampling process in image acquisition system. Consequently, to obtain high resolution image, restoration task to combat degradation due to blurs is not sufficient when aliasing is present in the observed image. Hence, this paper presents a method able to perform blind reconstruction and restoration of images altogether. This has been undertaken by generalizing algorithm proposed in a communication framework, under various aspects : extension to 2-D and correlated source separation. The method performs very well under “clean” situations (very low noise) and is under study for more adverse conditions.

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