A NEW MULTIFILTER DESIGN PROPERTY FOR MULTIWAVELET IMAGE COMPRESSION*

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ABSTRACT

Approximation order, linear phase symmetry, time-frequency localization, regularity, and stopband attenuation are some criteria that are widely used in wavelet filter design. In this paper, we propose a new criterion called *good multifilter properties* (GMPs) for the design and construction of multiwavelet filters targeting image compression applications. We first provide the definition of GMPs, followed by a necessary and sufficient condition for an orthonormal multiwavelet system to have a GMP order of at least 1. We then present an algorithm to construct orthogonal multiwavelets possessing GMPs, starting from any length-$2N$ scalar CQFs. Image compression experiments are performed to evaluate the importance of GMPs for image compression, as compared to other common filter design criteria. Our results confirmed that multiwavelets that possess GMPs not only yield superior PSNR performances, but also require much lower computations in their transforms.

1. INTRODUCTION

Over the past few years, there has been an increasing number of research activities on multiwavelets, both in pure mathematics (*e.g.* [1], [3], [4]) as well as engineering applications (*e.g.* [7], [8], [10]). Such growing interests in multiwavelets mainly stem from the following facts: (i) multiwavelets can simultaneously possess orthogonality, symmetry, and a high order of approximation for a given support of the scaling functions (this is not possible for any real-valued scalar wavelets [2]); and (ii) multiwavelets have produced promising results in the areas of image compression and denoising.

One of the great challenges to successful application of multiwavelets of multiplicity r is the problem of *initialization / prefiltering*, which requires the generation of r input (vectorized) data streams from a single source stream. Different proposals that address the above problem can be found in [7], [10] and [11]. Another important issue that contributes to the success of multiwavelets lies in the good design and construction of multifilters for a particular application. Several filter design criteria such as regularity, approximation order, and optimum time-frequency resolution have been suggested [1], [5]. In [10], we introduced a new criterion called the *good multifilter properties* (GMPs).

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The focus of this paper is twofold: to introduce a new class of orthonormal multiwavelets possessing a GMP order of at least 1, and to study different multifilter design criteria useful for image compression. For simplicity of exposition, but without loss of generality, we will only consider multiwavelets of multiplicity $r = 2$. Sec. 2 briefly reviews the concepts of an equivalent scalar filter bank system, and the definition of GMPs. Sec. 3 presents a procedure to construct a class of multiwavelets possessing GMPs. A comparison of various multifilter design criteria is carried out in Sec. 4, followed by discussions and the conclusion in Sec. 5.

2. EQUIVALENT SCALAR FILTER BANK SYSTEM AND GOOD MULTIFILTER PROPERTIES

An orthonormal multiwavelet system consists of compactly supported *orthonormal multiscaling function vector* $\mathbf{\Phi} = (\phi_1, \phi_2)^T$ and *orthonormal multiwavelet vector* $\mathbf{\Psi} = (\psi_1, \psi_2)^T$ satisfying

$$\mathbf{\Phi}(x) = \sum_{k\in\mathbb{Z}} \boldsymbol{P}_k \mathbf{\Phi}(2x-k), \quad \mathbf{\Psi}(x) = \sum_{k\in\mathbb{Z}} \boldsymbol{Q}_k \mathbf{\Phi}(2x-k),$$

where $\{\boldsymbol{P}_k\}$ and $\{\boldsymbol{Q}_k\}$ are some finitely supported sequences of 2×2 matrices, which are known as the *matrix lowpass and highpass filters*, respectively. Equivalently, their Fourier transforms, $\boldsymbol{P}(\omega) := \frac{1}{2}\sum_{k\in\mathbb{Z}} \boldsymbol{P}_k e^{-jk\omega}$ and $\boldsymbol{Q}(\omega) := \frac{1}{2}\sum_{k\in\mathbb{Z}} \boldsymbol{Q}_k e^{-jk\omega}$, are referred to as the *refinement mask* and *wavelet mask*, respectively. The orthonormality of $\mathbf{\Phi}$ and $\mathbf{\Psi}$ can also be expressed by the following *perfect reconstruction* (PR) conditions:

$$\boldsymbol{P}(\omega)\boldsymbol{P}^*(\omega) + \boldsymbol{P}(\omega+\pi)\boldsymbol{P}^*(\omega+\pi) = \mathbf{I}_{2\times2}, \quad (1)$$
$$\boldsymbol{P}(\omega)\boldsymbol{Q}^*(\omega) + \boldsymbol{P}(\omega+\pi)\boldsymbol{Q}^*(\omega+\pi) = \mathbf{0}_{2\times2}, \quad (2)$$
$$\boldsymbol{Q}(\omega)\boldsymbol{Q}^*(\omega) + \boldsymbol{Q}(\omega+\pi)\boldsymbol{Q}^*(\omega+\pi) = \mathbf{I}_{2\times2}, \quad (3)$$

where the superscript * denotes conjugate transpose. Specifically, the sequence $\{\boldsymbol{P}_k\}$ that satisfies (1) is called a *conjugate quadrature filter* (CQF). As proved in [5], we know that $\widehat{\mathbf{\Phi}}(0)$, the zero moment of $\mathbf{\Phi}$, also satisfies the following equations:

$$\widehat{\mathbf{\Phi}}^T(0)\boldsymbol{P}(0) = \widehat{\mathbf{\Phi}}^T(0) \quad \text{and} \quad \widehat{\mathbf{\Phi}}^T(0)\boldsymbol{P}(\pi) = \mathbf{0}. \quad (4)$$

In order to analyze the frequency response characteristics of a given multifilter, we introduced in [10] the concept of an *equivalent scalar filter bank system*, which provides a sufficient representation of a given multifilter system. For any orthonormal multiwavelet system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$, we showed that there exists a set of

$r = 2$ equivalent scalar (wavelet) filters, $\mathcal{P}_\nu(\omega), \nu = 1, 2$, with the frequency responses satisfying

$$[\mathcal{P}_1(\omega), \mathcal{P}_2(\omega)]^T = \boldsymbol{P}(2\omega)[1, e^{-j\omega}]^T, \tag{5}$$

such that the two systems produce identical input-output relationships. The corresponding definition of $\mathcal{Q}_\nu(\omega)$ associated with $\{\boldsymbol{Q}_k\}$ is similar to (5). It was further shown that, in fact, $\boldsymbol{P}(\omega)$ and $\boldsymbol{Q}(\omega)$ are the polyphase matrices of $\mathcal{P}_\nu(\omega)$ and $\mathcal{Q}_\nu(\omega)$, $\nu = 1, 2$, respectively.

In [10], we also presented an approach to transform $\boldsymbol{P}(\omega)$ and $\boldsymbol{Q}(\omega)$ into

$$\boldsymbol{P}^\sharp(\omega) = \boldsymbol{U}\boldsymbol{P}(\omega)\boldsymbol{U}^{-1} \quad \text{and} \quad \boldsymbol{Q}^\sharp(\omega) = \boldsymbol{U}\boldsymbol{Q}(\omega)\boldsymbol{U}^{-1}, \tag{6}$$

using an orthogonal matrix $\boldsymbol{U}$ such that the vector $\boldsymbol{U}\widehat{\boldsymbol{\Phi}}(0)$ is parallel to the vector $[1, 1]^T$. Clearly, $\boldsymbol{\Phi}^\sharp(x) = \boldsymbol{U}\boldsymbol{\Phi}(x)$ and $\boldsymbol{\Psi}^\sharp(x) = \boldsymbol{U}\boldsymbol{\Psi}(x)$ also constitute an orthonormal multiwavelet system with $\boldsymbol{P}^\sharp_k = \boldsymbol{U}\boldsymbol{P}_k\boldsymbol{U}^{-1}$ and $\boldsymbol{Q}^\sharp_k = \boldsymbol{U}\boldsymbol{Q}_k\boldsymbol{U}^{-1}$, $k = 0, \ldots, N$, for some fixed positive integer N. We later exploited the simplicity of matrix $\boldsymbol{U}$ to introduce a generalized, low complexity, and compact representation paradigm for multiwavelet initialization and discrete multiwavelet transforms.

Having established the relationship between a multifilter system and its equivalent scalar filter system, we can now characterize a given multiwavelet as follows:

Definition 1. A given orthonormal multiwavelet system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$ is said to possess a *GMP order* α if the $\{\boldsymbol{P}^\sharp_k\}$'s equivalent scalar lowpass filters, $\{\mathcal{P}^\sharp_\nu(\omega)\}$, possess the following properties:

$$\frac{d^\ell}{d\omega^\ell}\mathcal{P}^\sharp_\nu(\pi) = 0, \quad \ell = 0, 1, \ldots, \alpha - 1, \tag{7}$$

for all $\nu = 1, 2$, and $\alpha \geq 1$.

Condition (7) ensures that $\{\mathcal{P}^\sharp_\nu(\omega)\}, \nu = 1, 2$, have at least one vanishing moment, which also implies that there is no DC leakage. This is very important for image coding as it helps to prevent checkerboard artifacts in the reconstructed images. In general, the frequency properties of the highpass filters can be completely determined from those of the lowpass filters [10].

3. CONSTRUCTION OF MULTIWAVELETS WITH GMP'S

In this section we first give a necessary and sufficient condition by means of its refinement mask to check whether a given multifilter possesses GMPs. Starting from any scalar filter bank system, we then present a 4-step procedure for the construction of a class of orthogonal multiwavelets possessing GMPs.

Proposition 1. *A given orthonormal multiwavelet system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$ will have a GMP order of at least 1 iff the matrix $\boldsymbol{P}(0)$ is singular; else, it is considered to possess no GMPs.*

Proof. Sufficient: It is clear form (5) and the definition of GMPs in (7) that $\boldsymbol{P}^\sharp(0)[1, -1]^T = \boldsymbol{0}$, i.e., $\boldsymbol{P}(0)$ is singular.
Necessary: Suppose that $\boldsymbol{P}^\sharp(0) = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. From the similarity transformation in (6), we always have $\boldsymbol{P}^\sharp(0)[1, 1]^T = [1, 1]^T$. In addition to the fact that $\boldsymbol{P}^\sharp(0)$ also satisfies (4), we have $a = d$ and $c = b$. If $\boldsymbol{P}^\sharp(0)$ is singular, we have $a = b = c = d = \frac{1}{2}$, which implies that the multiwavelet system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$ must possess a GMP order of at least 1. □

Now we will present the algorithm to construct an orthonormal multiwavelet system with GMPs, given any length-$2N$ scalar CQF, $\{h_k\}_{k=0}^{2N-1}$.

Step 1: Construct a $2N \times 2N$ matrix $\boldsymbol{M} = [m_{i,j}]_{i,j=1,\ldots,2N}$ that has only the following non-zero elements:

$$m_{2i-1,2i-1} = 1, \qquad m_{2i-1,2i} = -\tau,$$
$$m_{2i,2N-2i+1} = 1, \qquad m_{2i,2N-2i+2} = \tau.$$

where $\tau = 1$ or -1.

Step 2: Construct a length-$4N$ scalar CQF $\{{}_Nh_k\}_{k=0}^{4N-1}$ such that the even and odd filter taps are given by

$$\begin{aligned}[\cdots {}_Nh_{2k} \cdots]^T &= \frac{1}{2}\boldsymbol{M}[\cdots h_k \cdots]^T, \\ {}_Nh_{2k+1} &= \tau(-1)^{k+1}{}_Nh_{2k}.\end{aligned}$$

Step 3: Construct the matrix lowpass filter $\{\boldsymbol{P}_k\}_{k=0}^{2N-1}$ using one of the following two possibilities:
(i) $\tau = 1$

$$\begin{aligned}\boldsymbol{P}_{2k} &= \begin{bmatrix} {}_Nh_{4N-4k-2} & -{}_Nh_{4k} \\ {}_Nh_{4N-4k-2} & {}_Nh_{4k} \end{bmatrix}, \\ \boldsymbol{P}_{2k+1} &= \begin{bmatrix} {}_Nh_{4k+2} & {}_Nh_{4N-4k-4} \\ -{}_Nh_{4k+2} & {}_Nh_{4N-4k-4} \end{bmatrix},\end{aligned}$$

(ii) $\tau = -1$

$$\begin{aligned}\boldsymbol{P}_{2k} &= \begin{bmatrix} {}_Nh_{4k} & {}_Nh_{4N-4k-2} \\ -{}_Nh_{4k} & {}_Nh_{4N-4k-2} \end{bmatrix}, \\ \boldsymbol{P}_{2k+1} &= \begin{bmatrix} {}_Nh_{4N-4k-4} & -{}_Nh_{4k+2} \\ {}_Nh_{4N-4k-4} & {}_Nh_{4k+2} \end{bmatrix}.\end{aligned}$$

Step 4: Construct the associated matrix highpass filter $\{\boldsymbol{Q}_k\}_{k=0}^{2N-1}$

$$\boldsymbol{Q}_k = (-1)^k\boldsymbol{P}_{2N-1-k}\boldsymbol{A}, \quad \text{where} \quad \boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

regardless of whether $\tau = 1$ or -1.

It can easily be checked that the multifilter system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$ satisfies PR conditions (1) – (3). We also have the following relations:

$$\boldsymbol{P}_k = \boldsymbol{S}\boldsymbol{P}_{2N-1-k}\boldsymbol{S}, \quad \boldsymbol{Q}_k = \boldsymbol{S}\boldsymbol{Q}_{2N-1-k}\boldsymbol{S},$$

where $\boldsymbol{S} = \mathrm{diag}(1, -1)$, which imply that the multiscaling functions and multiwavelets are symmetric-antisymmetric pairs centered at the point $N - \frac{1}{2}$, respectively. By Proposition 1, we can further prove that the multifilter system $\{\boldsymbol{P}_k, \boldsymbol{Q}_k\}$ possesses a GMP order of at least 1.

As a constructive example, starting from the following parameterized length-4 scalar CQF

$$\frac{\alpha(\alpha-1)}{\alpha^2+1}, \quad \frac{1-\alpha}{\alpha^2+1}, \quad \frac{1+\alpha}{\alpha^2+1}, \quad \frac{\alpha(\alpha+1)}{\alpha^2+1},$$

we can construct, for the case of $\tau = 1$, a family of orthonormal multiwavelets, $\{\boldsymbol{P}_k\}_{k=0}^3$, with a GMP order of at least 1, as

$$\boldsymbol{P}_0 = \frac{1}{2}\begin{bmatrix} \frac{(\alpha-1)^2}{\alpha^2+1} & \frac{1-\alpha^2}{\alpha^2+1} \\ \frac{(\alpha-1)^2}{\alpha^2+1} & \frac{\alpha^2-1}{\alpha^2+1} \end{bmatrix}, \boldsymbol{P}_1 = \frac{1}{2}\begin{bmatrix} \frac{(\alpha+1)^2}{\alpha^2+1} & \frac{1-\alpha^2}{\alpha^2+1} \\ \frac{-(\alpha+1)^2}{\alpha^2+1} & \frac{1-\alpha^2}{\alpha^2+1} \end{bmatrix},$$

$\boldsymbol{P}_2 = \boldsymbol{S}\boldsymbol{P}_1\boldsymbol{S}$ and $\boldsymbol{P}_3 = \boldsymbol{S}\boldsymbol{P}_0\boldsymbol{S}$. The associated matrix highpass filter $\{\boldsymbol{Q}_k\}_{k=0}^3$ can be obtained from **Step 4**. The parameter α provides one degree of freedom for selecting another useful multifilter design criterion, which we will show later.

	Multiwavelets							Scalar Wavelets	
	GHM	REG	AP	T-F	SA4(1)	SA4(2)	SA4(3)	D4	D8
Reference	[3]	[5]	[1]	[5]	[8] & [10]			[2]	[2]
Filter Taps	4	4	4	4	4	4	4	4	8
Orthogonal	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Symmetric	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Approx. Order	2	2	3	2	1	2	1	2	4
Regularity	1.0	1.2668	0.9408	1.2303	*	1.0270	*	0.55	1.275
Stopband Error	0.6012	0.3695	0.3565	0.3812	0.3459	0.3571	0.3236	0.4478	0.3165
GMP Order	None	None	None	None	2	1	1	N.A.	N.A.

Table 1: Properties of various multiwavelet and scalar wavelet filters. *Note that the regularity of **SA4(1)** and **SA4(3)** could not be computed accurately, as limited by the fact that their approximation orders are less than 2.

4. PERFORMANCE ANALYSIS OF VARIOUS MULTIFILTER DESIGN CRITERIA

In this section, we will compare and contrast the relative importance of various multifilter design criteria used in the construction of multiwavelets targeting image compression applications. The following *six* multifilters design criteria are investigated:

• **Approximation Order.** A higher approximation order of the multiscaling functions corresponds to higher vanishing moments of the multiwavelets. As signals are projected onto the space spanned by the multiscaling functions, multifilters with a higher approximation order usually leads to better energy compaction (or higher coding gain).

• **Regularity/Smoothness.** Regularity provides a measure of the smoothness of the functions. Smoother functions (particularly for the synthesis multifilters) contribute to the reduction of checkerboard artifacts in the reconstructed images.

• **Time-Frequency Localization.** Finite-length multiwavelet filters provide a flexible trade-off between time and frequency (scale) localizations. Higher localizations may contribute to more efficient coding of high-frequency wavelet coefficients.

• **Linear Phase Symmetry.** Phase linearity of a transform is determined by the symmetry of the multifilters. Symmetric multifilters help reduce phase distortions around edges and borders of the reconstructed images.

• **Stopband Attenuation.** Stopband attenuation measures the passband and stopband deviations from the ideal brick-wall filter. A sharper cutoff frequency at the transition band is useful but it usually results in longer multifilters.

• **Good Multifilter Properties.** GMPs characterize the magnitude responses of the equivalent scalar filter bank associated with a multifilter. Multifilters possessing GMPs help prevent both DC and high-frequency leakages across bands, which can contribute to reduced smearing, blocking and ringing artifacts.

To perform a consistent and thorough comparative study of multifilter design criteria for image compression, we tested the following seven symmetric-antisymmetric (except for GHM which both scaling functions are symmetric), orthonormal 4-tap multiwavelets: (i) **GHM** is one of the earliest multiwavelets constructed using fractal interpolation [3]; (ii) **REG** has the highest regularity [5]; (iii) **AP** has the highest approximation order [1]; (iv) **T-F** has optimal time-frequency localization [5]; (v) **SA4(1)** has the highest GMP order of 2; (vi) **SA4(2)** has the highest approximation order while possessing GMPs; and (vii) **SA4(3)** has optimal stopband attenuation while possessing GMPs. The Daubechies' scalar wavelets D4 and D8 are used for benchmarking purposes. It is noted that only the SA4 family [8], [10] possesses a GMP order of at least 1. For example, we obtain **SA4(1)** when $\alpha = \sqrt{15}/5$, **SA4(2)** when $\alpha = (\sqrt{19} - 2)/3$, and **SA4(3)** when $\alpha = 0.749423$. Some specific properties of each multiwavelet are summarized in Table 1.

For the purpose of fair comparisons, the same still image codec [6] is used for all the multifilters. Similar comparative results were also obtained for codec [9]. Table 2 compares the compression performances of the multifilters, each having a different combination of useful multifilter design criteria. Bold values represent the filters that perform best for particular image / compression ratio pairs. It can be concluded that **SA4(3)** has emerged as the winner for all the tested images over a wide range of bit rates. We also showed [8], [10] that the SA4 family of multiwavelets that possess GMPs have much lower computational complexity; for example, the scalar D8 has 2.67 times higher complexity than SA4.

5. DISCUSSIONS AND CONCLUSIONS

The simulation results verified the usefulness of GMPs for constructing multiwavelets targeting image compression applications. Fig. 1 provides some insights by analyzing the magnitude responses of the respective equivalent scalar filters corresponding to different multiwavelets. It is clear from the plots that the **GHM**, **REG**, **AP**, and **T-F** multiwavelets do not vanish at $\omega = \pi$, which also imply that they do not possess GMPs. The SA4 family of multiwavelets, on the other hand, have a GMP order of at least 1. In particular, we also noted that the plot of **AP** is nearly zero ($\approx$ 0.036) at $\omega = \pi$, which likely explains its better performance as compared to other multiwavelets without GMPs.

In conclusion, the following observations can be made from our study:

(i) A GMP order of at least 1 is critical for ensuring no frequency leakages across bands; hence, improving compression performance;

(ii) A sharper cutoff frequency at the transition band can be more useful than having a higher approximation order (comparing **SA4(3)** against **SA4(2)**);

(iii) Having the highest GMP order alone may not yield the best multifilter for image compression; and

(iv) Other design criteria such as regularity, approximation order, time-frequency localization, symmetry, etc. are also useful for image compression, provided that the multifilters have a GMP order of *at least* 1.

	CR	GHM	REG	AP	T-F	SA4(1)	SA4(2)	SA4(3)	D4	D8
Lena	32:1	33.58	34.14	34.37	34.01	34.42	34.33	**34.47**	33.23	34.00
	64:1	30.52	31.23	31.40	31.14	31.46	31.38	**31.50**	30.24	30.97
	128:1	27.75	28.59	28.73	28.54	28.75	28.71	**28.78**	27.72	28.33
Barbara	16:1	31.14	31.79	32.06	31.64	32.13	32.02	**32.27**	30.52	31.67
	32:1	27.44	28.15	28.23	28.07	28.30	28.23	**28.39**	27.12	27.86
	64:1	24.85	25.49	25.53	25.47	25.30	25.53	**25.59**	24.71	25.12
Boat	16:1	33.65	34.20	34.46	34.09	34.49	34.42	**34.55**	33.42	33.95
	32:1	30.04	30.68	30.85	30.59	30.87	30.82	**30.93**	29.98	30.44
	64:1	27.37	27.94	28.08	27.85	28.10	28.06	**28.12**	27.34	27.74
Baboon	16:1	25.23	25.54	25.59	25.50	25.61	25.58	**25.67**	25.11	25.43
	32:1	23.06	23.33	23.36	23.30	23.38	23.36	**23.41**	22.97	23.18
	64:1	21.49	21.68	21.70	21.67	21.72	21.70	**21.74**	21.49	21.64
Goldhill	16:1	32.51	33.02	33.12	32.96	33.14	33.10	**33.18**	32.41	32.66
	32:1	29.87	30.49	30.58	30.45	30.60	30.57	**30.62**	29.84	30.09
	64:1	27.83	28.45	28.50	28.41	28.52	28.50	**28.55**	27.80	28.00

Table 2: Comparisons of PSNR values (in dB) of various wavelet filters using different images and compression ratios (CR). Bold entries indicate the best filters for particular image/CR combinations.

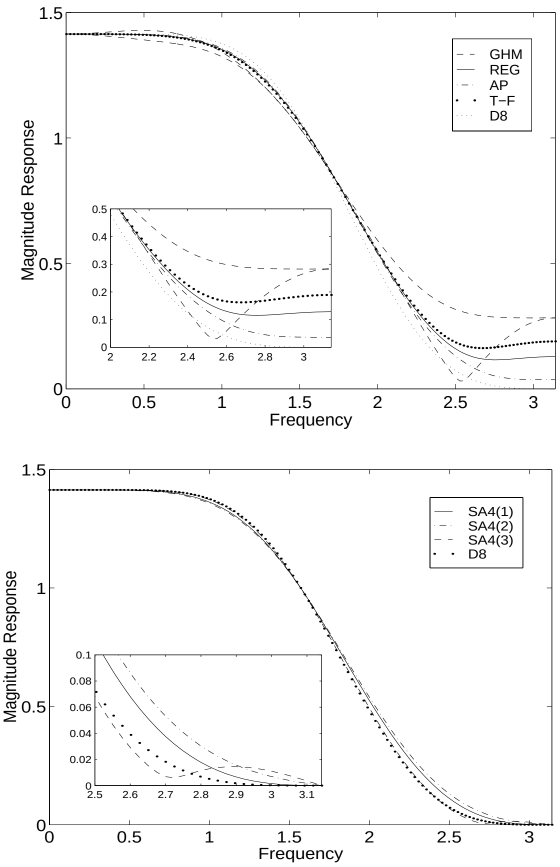

Figure 1: Magnitude responses of the equivalent scalar filters associated with different multifilters. Scalar wavelet D8 (maximum flatness) serves as a reference. It is noted that the two magnitude responses of GHM are different, whereas the magnitude responses of each of the other multifilters coincide perfectly [8].

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