ERROR PROBABILITIES AND PERFORMANCE COMPARISONS OF FFH/BFSK RECEIVERS WITH MULTITONE JAMMING AND AWGN

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ABSTRACT

This paper studies the bit-error rate (BER) performance of a fast frequency-hopped (FFH) binary frequency-shift-keying (BFSK) clipper receiver in the presence of multitone jamming (MTJ) and additive white Gaussian noise (AWGN). By using the Taylor-series expansion and the quantization approach, the BER expressions for higher diversity levels can be obtained without much extra computational complexity. The analytical BER results, validated by simulations, show that there is an optimum diversity level for the clipper receiver. Performance comparisons among various receivers demonstrate that the BER performance of the clipper receiver is significantly better than that of the linear-combining receiver. In addition, the clipper receiver also outperforms the product-combining receiver and the self-normalizing receiver provided that the clipping threshold is set at the desired signal power level.

1. INTRODUCTION

The issues of interference rejection in fast frequency-hopped (FFH) spread-spectrum (SS) systems have become increasingly important in both commercial and military applications. Recently, we have analyzed the bit-error rate (BER) performance of three FFH binary frequency-shift-keying (BFSK) receivers; namely, the linear-combining receiver [1], the product-combining receiver [2], and the self-normalizing receiver [3], under the conditions of multitone jamming (MTJ) and additive white Gaussian noise (AWGN) . It has been shown in [1] that there is no diversity improvement for the linear-combining receiver against the $n = 1$ band MTJ (i.e., jammed hop bands contain only one jamming tone at one of the two adjacent frequency slots corresponding to bits 1 and 0, respectively) and AWGN; that is, the BER performance is further degraded as the diversity level is increased. In contrast, it has been shown in [2] and [3] that under the same jamming conditions, there are optimum diversity levels for the product-combining and self-normalizing receivers. The BER results of these two nonlinear diversity-combining receivers are significantly better than that of the linear-combining receiver.

In this paper, we consider the clipper receiver, which is another type of nonlinear diversity-combining receivers. It has been shown in [4] that the FFH/BFSK clipper receiver possesses reasonably good partial-band noise jamming (PBNJ) rejection capability compared to that of the soft-decision linear-combining receiver. When the AWGN is not taken into consideration, the results in [5] also show that the FFH/BFSK clipper receiver has some advantages over other types of receivers in suppressing stronger MTJ. In this paper, we study the BER performance of the FFH/BFSK clipper receiver under the conditions of MTJ and AWGN. In particular, we make use of Taylor-series expansion as proposed in [1]-[3] to obtain the probability density function (pdf) of the clipper output and derive a computationally efficient BER expression for any diversity level. In addition, we also present the BER performance comparisons among the clipper receiver and the other three types of receivers presented in [1]-[3] under the conditions of the worst-case band MTJ and AWGN.

2. SYSTEM MODEL

In [1]-[3], the system models are described in detail. In this paper, only a brief definition of the system parameters is given. The block diagram of the FFH/BFSK clipper receiver is shown in Figure 1. The received signal at the receiver front end consists of the desired signal corrupted by MTJ and AWGN. This received signal is down-converted, dehopped, and band-pass filtered to produce

$$r_l(t) = \sqrt{2}a_S \cos(2\pi f_i t + \theta_l) + w(t) + n_J(t), \quad i = 1 \text{ or } 2, \tag{1}$$

where $\sqrt{2}a_S = \sqrt{2E_b/(LT_h)}$ is the signal amplitude, E_b is the bit energy, L is the diversity level of the system,

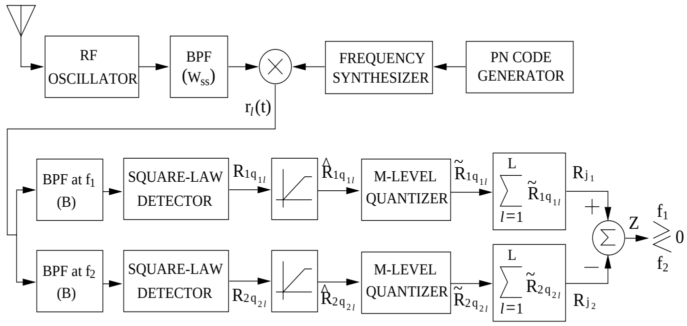

Figure 1: FFH/BFSK clipper receiver block diagram.

T_h is the hop duration, θ_l is the random phase, f_i is the baseband frequency, $w(t)$ is the noise term due to AWGN, and $n_J(t)$ is the jamming component. Note that the random noise $w(t)$ is a band-limited, zero-mean additive Gaussian process with variance over a bandwidth of $B = 1/T_h$ (Hz) given by $\sigma_w^2 = E\{w^2(t)\} = N_0 B$, where N_0 is the one-sided power spectral density of the AWGN. When the MTJ is present at a particular frequency slot, the jamming tone over a hop duration can be expressed as

$$J(t) = \sqrt{2}a_J \cos(2\pi f_J t + \psi_l), \tag{2}$$

where $\sqrt{2}a_J$ is the amplitude, ψ_l is the random phase, and f_J is the frequency of each multiple interfering tone. The total power P_{J_T} (W) of the MTJ is distributed over Q equal-power interfering tones and the power of each jamming tone is $P_J = P_{J_T}/Q$ (W). We define the signal-to-jamming ratio of the system as $SJR \triangleq E_b/N_J$, where $N_J \triangleq P_{J_T}/W_{SS}$ is the equivalent one-sided power spectral density of the MTJ and W_{SS} is the total SS bandwidth. In this paper, only the $n = 1$ band MTJ is considered as it has been shown in [2, 3] that the $n = 1$ band MTJ gives the worst-case BER performance as compared to other types of MTJ models.

The dehopped signal $r_l(t)$ is demodulated by two square-law detectors matched to frequencies f_1 and f_2, respectively. These outputs are denoted as $R_{iq_{il}}$, where $q_{il} = 0$ or 1 is the jamming indicator function representing the jamming state of the lth hop with frequency $f_i,\ i = 1, 2$. The square-law detector outputs pass through a linear clipper to produce

$$\hat{R}_{iq_{il}} = \min[R_{iq_{il}}, C],\ i = 1, 2,\ l = 1, 2, \cdots, L, \tag{3}$$

where the clipping level C is expressed as a fraction c times the expected output power a_S^2 due to the desired signal alone. The parameter c is called the relative clipping level of the system. The clipper outputs are approximated by an M-level uniform quantizer and the resultant outputs are summed for all L hops to give $R_{j_i},\ i = 1, 2$. The decision variable $Z = R_{j_1} - R_{j_2}$ is then compared with a zero threshold to make the final decision.

3. PROBABILITY OF BIT ERROR

Without loss of generality, we assume that f_1 was transmitted. The conditional pdf of R_{11} is [1]-[3]

$$\begin{aligned} p_{R_{11}|\cos\phi}(r_1|\cos\phi) &= \frac{1}{2\sigma_w^2}\exp\left(-\frac{r_1 + 2(a_S^2 + a_J^2 + 2a_S a_J\cos\phi)}{2\sigma_w^2}\right) \\ &\cdot I_0\left(\frac{\sqrt{2r_1(a_S^2 + a_J^2 + 2a_S a_J\cos\phi)}}{\sigma_w^2}\right)U(r_1), \end{aligned} \tag{4}$$

where $U(\cdot)$ is the unit step function, $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and ϕ is the random phase difference of the two tones uniformly distributed over $[-\pi, \pi]$. By using the Taylor-series approximation as proposed in [1]-[3], the pdf of R_{11} can be simplified to yield

$$p_{R_{11}}(r_1) \approx \sum_{k=-1}^{1}\frac{1}{3}p_{R_{11}|\cos\phi}\left(r_1 \middle| k\frac{\sqrt{3}}{2}\right). \tag{5}$$

The pdf's of R_{10}, R_{21}, and R_{20} can be obtained from (5) by setting $a_J = 0$, $a_S = 0$, and $a_S = a_J = 0$, respectively. For the convenience of mathematical computation, we used an M-level uniform quantizer to approximate the operation of the ideal soft-limiter and the outputs of the quantizers are defined as

$$\tilde{R}_{iq_{il}} = \begin{cases} a_m, & \text{if } a_m \le \hat{R}_{iq_{il}} < a_{m+1}, \\ & m = 0, \cdots, M-2, \\ a_{M-1}, & \text{if } \hat{R}_{iq_{il}} \ge a_{M-1} \end{cases} \tag{6}$$

for $i = 1, 2$, $l = 1, 2, \cdots, L$, and $a_m = mC/(M-1)$. The discrete pdf's of $\tilde{R}_{iq_{il}}$ can be expressed as

$$p_{\tilde{R}_{iq_{il}}}(\alpha) = \sum_{m=0}^{M-1} V_{iq_{il}}[m]\delta(\alpha - a_m),\ i = 1, 2, \tag{7}$$

where

$$\begin{aligned} V_{iq_{il}}[m] &\triangleq \Pr\{\tilde{R}_{iq_{il}} = a_m \mid f_1\} \\ &= \begin{cases} \int_{a_m}^{a_{m+1}} p_{R_{iq_{il}}}(r)dr, & m = 0, 1, \cdots, M-2, \\ \int_{C}^{\infty} p_{R_{iq_{il}}}(r)dr, & m = M-1. \end{cases} \end{aligned} \tag{8}$$

In particular, it can be shown that

$$V_{11}[m] \approx \begin{cases} \frac{1}{3}\sum_{k=-1}^{1}\left[Q\left(\frac{\sqrt{2\left(a_S^2+a_J^2+k\sqrt{3}a_S a_J\right)}}{\sigma_w}, \sqrt{\frac{a_m}{\sigma_w^2}}\right) \right. \\ \qquad \left. -Q\left(\frac{\sqrt{2\left(a_S^2+a_J^2+k\sqrt{3}a_S a_J\right)}}{\sigma_w}, \sqrt{\frac{a_{m+1}}{\sigma_w^2}}\right)\right], \\ \qquad m = 0, 1, \cdots, M-2, \\ \frac{1}{3}\sum_{k=-1}^{1} Q\left(\frac{\sqrt{2\left(a_S^2+a_J^2+k\sqrt{3}a_S a_J\right)}}{\sigma_w}, \sqrt{\frac{C}{\sigma_w^2}}\right), \\ \qquad m = M-1, \end{cases} \tag{9}$$

where

$$Q(a,b) \triangleq \int_b^\infty x e^{-(x^2+a^2)/2} I_0(ax)\, dx \qquad (10)$$

is the Marcum's Q-function. Similarly, the values of $V_{10}[m]$, $V_{21}[m]$, and $V_{20}[m]$ can be obtained from (9) by setting $a_J = 0$, $a_S = 0$, and $a_S = a_J = 0$, respectively. The quantizer outputs are summed for L hops to form the decision statistics

$$R_{j_i} = \sum_{l=1}^{L} \tilde{R}_{iq_{il}},\ i = 1, 2, \qquad (11)$$

where the subscript $j_i \triangleq \sum_{l=1}^{L} q_{il}$ denotes the number of hops jammed out of the total L diversity receptions. Finally, the probability of bit error P_b can be expressed as

$$P_b = \sum_{j=0}^{L} \binom{L}{j} \left(\frac{Q}{N_h}\right)^j \left(1 - \frac{Q}{N_h}\right)^{L-j} \cdot \left[\sum_{h=0}^{j} \left(\frac{1}{2}\right)^j \binom{j}{h} P_e(h, j-h)\right], \qquad (12)$$

where

$$P_e(j_1, j_2) = \sum_{n=0}^{L(M-1)-1} \sum_{m=n+1}^{L(M-1)} \Pr\{R_{j_1} = na_1\} \Pr\{R_{j_2} = ma_1\} + \frac{1}{2} \sum_{m=0}^{L(M-1)} \Pr\{R_{j_1} = R_{j_2} = ma_1\} \qquad (13)$$

is the conditional BER given that j_1 and j_2 hops are being jammed by the $n = 1$ band MTJ for frequencies f_1 and f_2, respectively. In (13), the values of $\Pr\{\cdot\}$ can be obtained from the $(L-1)$-fold discrete convolution of the pdf's of $\tilde{R}_{iq_{il}}$ as defined in (7).

4. NUMERICAL RESULTS AND DISCUSSION

The worst-case BER results of the FFH/BFSK clipper receiver are calculated from (12) with the following parameters: E_b/N_0 =13.35dB, $C = a_S^2$, and $M = 64$.

In Figure 2, both the analytical and simulation BER results of the FFH/BFSK clipper receiver with different diversity levels are presented under the conditions of the worst-case $n = 1$ band MTJ and AWGN. The close matches in both analytical and simulation results validate the BER expressions derived in Section 3. In contrast to the linear-combining receiver presented in [1], we observe from Figure 2 that an optimum diversity scheme can be employed in the region of 10dB $< SJR <$ 40dB for the given E_b/N_0.

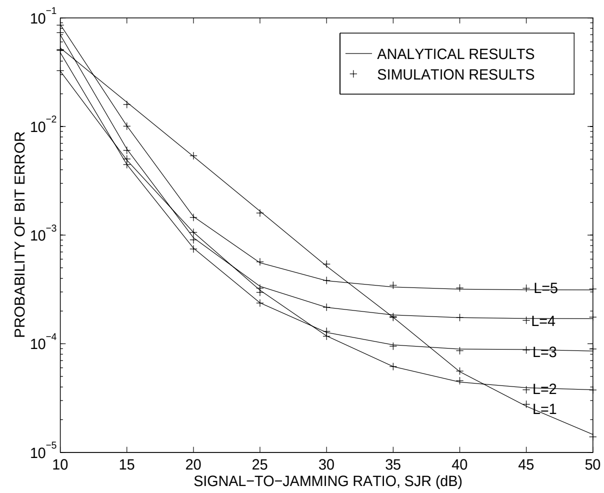

Figure 2: Comparisons of the worst-case analytical and simulation results for the clipper receiver.

This is due to the fact that the jamming power mitigation achieved by the clipper receiver at the optimum diversity level outweighs the corresponding noncoherent combining loss of the system [4].

In Figures 3 and 4, we show the BER results of the linear-combining [1], product-combining [2], self-normalizing [3], and clipper receivers with $L = 3$ and 5, respectively, under the conditions of the worst-case $n = 1$ band MTJ and AWGN. These results show that the BER performance of the clipper receiver is much better than that of the linear-combining receiver. This is because the linear-combining receiver does not provide any jamming power mitigation mechanism to suppress the MTJ while the clipper receiver limits the strong jamming power in each diversity reception before diversity combining. Comparisons among the three nonlinear diversity-combining receivers show that the clipper receiver marginally outperforms the self-normalizing receiver and the product-combining receiver. It should be noted that the clipper receiver requires the desired signal power level whereas both the product-combining receiver and the self-normalizing receiver are nonparametric. Nevertheless, it can be shown that the BER performance of the clipper receiver is not sensitive to minor error in estimating the actual desired signal power. Hence, if the desired signal power level is known *a priori* or can be estimated with reasonably good accuracy, the clipper receiver gives the best BER performance among the four receiver structures presented in Figures 3 and 4. On the other hand, if the side information on the desired signal level is not available, then the self-normalizing receiver will be preferred in rejecting the band MTJ and AWGN.

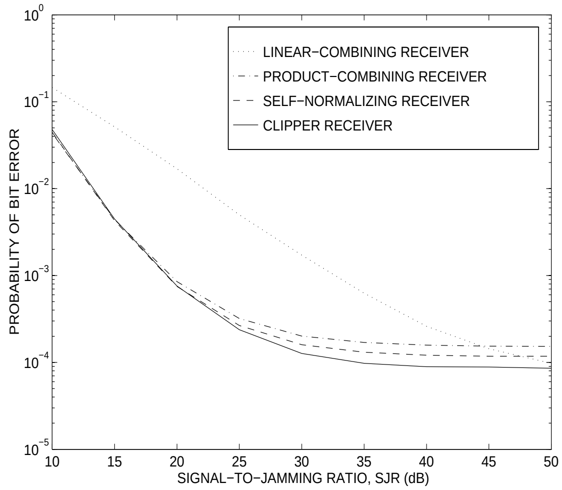

Figure 3: The worst-case BER performance comparisons among various receivers with $L = 3$.

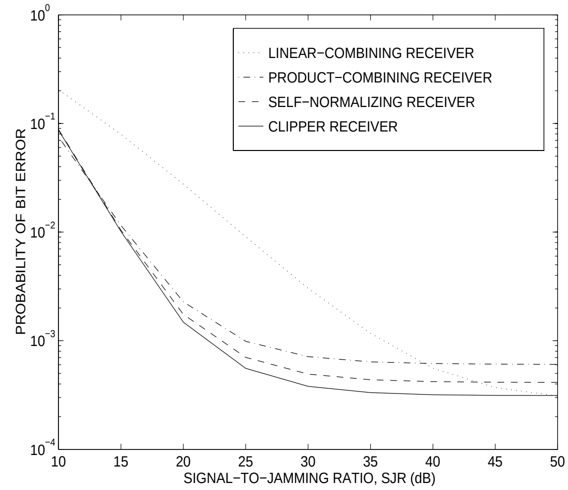

Figure 4: The worst-case BER performance comparisons among various receivers with $L = 5$.

5. CONCLUSION

In this paper, we have presented the performance analysis of the FFH/BFSK clipper receiver against band multitone jamming and AWGN. The BER expressions derived based on Taylor-series expansion are applicable to any arbitrary diversity level without extra computational complexity. Under the fixed bit energy conditions, the analytical BER results show that there is an optimum diversity level for the clipper receiver. Performance comparisons among the linear-combining, product-combining, self-normalizing, and clipper receivers reveal that the clipper receiver gives the best BER performance provided that the clipping threshold is set at the desired signal power level. The linear-combining receiver, which is the simplest to implement but does not provide jamming power rejection capabilities, is significantly outperformed by the other three nonlinear diversity-combining receivers. If the side information on the signal power level is not available, then the self-normalizing receiver gives better BER performance.

6. REFERENCES

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