A DIRECT METHOD TO COMPUTATIONAL ACOUSTICS

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ABSTRACT

The exact knowledge of the sound field within an enclosure is essential for a number of applications in electro-acoustics. Conventional methods for the assessment of room acoustics model the sound propagation in analogy to the propagation of light. More advanced computational methods rely on the numerical solution of the wave equation. A recently presented method is based on multidimensional wave digital principles. It allows a physically exact numerical modelling of the relevant acoustical effects and yields robust algorithms.

This contribution presents a new foundation of the multidimensional wave digital principle as applied to room acoustics. It starts from the first principles of physics. From there, the derivation of the algorithm only involves basic knowledge of numerical mathematics, linear algebra, and multidimensional system theory. An example for the simulation of dynamic three-dimensional sound propagation demonstrates the capability of the method.

1. INTRODUCTION

Progress in digital audio processing requires a deeper understanding of the acoustical interface. This applies not only to sound transducers, digital hearing aids, or the adaptation of speech communication and recognition systems to adverse acoustical environments. It is also of vital importance for new applications like spatial audio rendering and advanced coding schemes based on 3D scene analysis and synthesis.

The design of advanced audio systems involves careful consideration of the sound field, transducer locations, and digital signal processing. A detailed knowledge of the dynamics of sound propagation in the environment at hand is very helpful but hard to obtain. Measurements are slow, expensive, and only feasible in non-virtual environments. An alternative is the determination of the sound field structure by computer simulations. There are two basic approaches: geometrical and computational methods.

Geometrical methods are based on the assumption of plane waves. This simplifying assumption allows to adapt various methods from computer graphics. They require a modest numerical expense, but neglect some acoustical effects, like diffraction. Computational methods use techniques from numerical mathematics to solve the acoustic wave equation directly. They are numerically expensive, but they model the acoustical effects correctly.

A computational method based on the multidimensional wave digital principle [1] has been presented in [2, 3]. It has been shown that it models free space propagation, diffraction at openings, and reflection at boundaries correctly. The numerical expense of the algorithm increases linearly with the volume of the enclosure, but it does not depend on its shape or structure.

Unfortunately, the derivation of this method relies on a sound knowledge of several classical engineering and mathematical disciplines such as mathematical physics, numerical mathematics, network analysis and synthesis, complex analysis, and multidimensional system theory. Some of these disciplines are currently vanishing from the engineering curricula, as the emphasis in education shifts from classical electrical engineering to information technology. Although a powerful tool, the multidimensional wave digital principle is therefore hardly comprehensible to young engineers with an acoustics or digital signal processing background.

To make this method more accessible, a new foundation of the multidimensional wave digital principle is presented here. Although the approach is more general, it is restricted to acoustic wave propagation in this context. Only a basic knowledge of numerical mathematics, linear algebra, and multidimensional system theory is required to follow our derivation. Classical network theory and complex analysis are avoided, although they would provide intuitive insight to the initiated.

Section 2 sets up the PDE description of our problem. The direct method for the simulation algorithm is derived in section 3. Boundary conditions are treated in section 4. Section 5 presents an example.

2. PROBLEM DEFINITION

The propagation of sound waves in air is governed by the equation of motion and the equation of continuity (see [2, 4]) for the acoustic pressure $p(\mathbf{x}, t)$ and the acoustic fluid velocity vector $\mathbf{v}(\mathbf{x}, t)$. Under reasonable simplifications for sound propagation in air, they are given by

$$\rho_0 \frac{\partial}{\partial t} \mathbf{v}(\mathbf{x}, t) + \operatorname{grad} p(\mathbf{x}, t) \quad = \quad 0 \tag{1}$$

$$\frac{\partial}{\partial t} p(\mathbf{x}, t) + \rho_0 \, c^2 \operatorname{div} \mathbf{v}(\mathbf{x}, t) \quad = \quad 0 \tag{2}$$

where t denotes time and $\mathbf{x}$ the vector of space coordinates x, y, z. ρ_0 is the static density of the air and c is the speed of sound.

For our purposes, a symmetric form of these equations is advantageous. This is achieved by introduction of the normalizing constant (n_d is the number of spatial dimensions)

$$r_0 = \sqrt{n_d} \, \rho_0 c \; . \tag{3}$$

For $n_d = 3$, (1,2) take the form

$$\rho_0 \frac{\partial}{\partial t} \mathbf{v} + r_0 \operatorname{grad} \frac{p}{r_0} \quad = \quad 0 \tag{4}$$

$$r_0 \operatorname{div} \mathbf{v} + 3\rho_0 \frac{\partial}{\partial t} \frac{p}{r_0} \quad = \quad 0 \; . \tag{5}$$

The symmetric form is obvious, when (4,5) are combined into one matrix equation. The operators D_t and D_x denote derivation with respect to time, space component x and similar for y and z.

$$\begin{bmatrix} \rho_0 D_t & 0 & 0 & r_0 D_x \\ 0 & \rho_0 D_t & 0 & r_0 D_y \\ 0 & 0 & \rho_0 D_t & r_0 D_z \\ r_0 D_x & r_0 D_y & r_0 D_z & 3\rho_0 D_t \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_z \\ \dfrac{p}{r_0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

The solution of this set of partial differential equations (PDE) is largely determined by the boundary conditions. These are posed by the shape and the acoustic properties of the enclosure. The latter are given in terms of the acoustic or wall impedance R_w

$$p = R_w v_N \tag{7}$$

where v_N is the component of $\mathbf{v}$ normal to the surface.

To keep the physical interpretation of the problem short, only a simplified model of acoustic wave propagation is given in (6). Neither sound sources nor loss terms, which may occur in media other than air, are considered. However, the derivation of the simulation algorithm in the following section takes into account also these effects.

3. SIMULATION ALGORITHM

The essential elements of the proposed algorithm are best explained for only one spatial variable x. This simplifies the notation even when we consider source and loss terms. The derivation proceeds in the following steps: The PDE in matrix form is separated into a reactive and a resistive part. The reactive part contains the time and space derivatives. These are decoupled by transformation of the dependend and the independend coordinates. In decoupled form, discretization is carried out by numerical integration and a transformation to so called wave quantities. The resulting discrete algorithm is formulated as a state space model.

3.1. one space dimension

Restricting (6) to one dimension in space ($n_d = 1$), including the source terms $e_1(x,t)$ and $e_2(x,t)$ on the right hand side and the loss terms r_1 and r_2 in the operator matrix gives

$$\begin{bmatrix} \rho_0 D_t + r_1 & r_0 D_x \\ r_0 D_x & \rho_0 D_t + r_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}. \tag{8}$$

v_x and p/r_0 have been replaced by i_1 and i_2.

3.1.1. separation

Eq. (8) is now separated into two terms denoted by

$$\boldsymbol{Z}_l = \begin{bmatrix} \rho_0 D_t & r_0 D_x \\ r_0 D_x & \rho_0 D_t \end{bmatrix} \quad \text{and} \quad \boldsymbol{Z}_r = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \tag{9}$$

We call $\boldsymbol{Z}_l$ the reactive and $\boldsymbol{Z}_r$ the resisitive part. To treat both parts separately, introduce the abbreviation $\boldsymbol{u}$ for the reactive part and write with obvious meaning of $\boldsymbol{i}$ and $\boldsymbol{e}$

$$\boldsymbol{Z}_l \boldsymbol{i} = \boldsymbol{u}\,, \tag{10}$$
$$\boldsymbol{u} + \boldsymbol{Z}_r \boldsymbol{i} = \boldsymbol{e}\,. \tag{11}$$

Note that (10) contains all derivation operators but no losses, while (11) is an algebraic equation without derivatives.

3.1.2. decoupling of the reactive part

The reactive part requires a special handling before partial derivatives with respect to time and space can be discretized. Not only the components of $\boldsymbol{i}$ but also the independent coordinates are decoupled by suitable transformations. At first, we introduce a transformation of the dependent coordinates to diagonalize the operator matrix $\boldsymbol{Z}_l$

$$\tilde{\boldsymbol{u}} = \boldsymbol{Q}^{-1}\boldsymbol{u},\ \tilde{\boldsymbol{i}} = \boldsymbol{Q}^{-1}\boldsymbol{i},\ \tilde{\boldsymbol{Z}}_l = \boldsymbol{Q}^{-1}\boldsymbol{Z}_l\boldsymbol{Q},\ \boldsymbol{Q} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}. \tag{12}$$

The reactive part in diagonal form results as

$$\tilde{\boldsymbol{Z}}_l \tilde{\boldsymbol{i}} = \tilde{\boldsymbol{u}}, \quad \tilde{\boldsymbol{Z}}_l = \begin{bmatrix} \rho_0 D_t - r_0 D_x & 0 \\ 0 & \rho_0 D_t + r_0 D_x \end{bmatrix}. \tag{13}$$

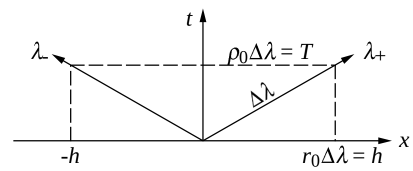

Figure 1: transformation of space and time coordinates

Next, we change the independent coordinates space and time to λ_+ and λ_- as shown in Fig. 1. The choice of the transformation

$$\begin{bmatrix} x \\ t \end{bmatrix} = \begin{bmatrix} r_0 & -r_0 \\ \rho_0 & \rho_0 \end{bmatrix} \begin{bmatrix} \lambda_+ \\ \lambda_- \end{bmatrix} = \rho_0 \begin{bmatrix} c & -c \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_+ \\ \lambda_- \end{bmatrix} \tag{14}$$

aligns the new coordinates with the propagation direction of sound. In terms of the transformed variables

$$\hat{\boldsymbol{i}}(\lambda_+, \lambda_-) = \tilde{\boldsymbol{i}}(x,t), \quad \hat{\boldsymbol{u}}(\lambda_+, \lambda_-) = \tilde{\boldsymbol{u}}(x,t), \tag{15}$$

the reactive part takes the form

$$\hat{\boldsymbol{Z}}_l \hat{\boldsymbol{i}} = \hat{\boldsymbol{u}}, \qquad \hat{\boldsymbol{Z}}_l = \begin{bmatrix} D_{\lambda_-} & 0 \\ 0 & D_{\lambda_+} \end{bmatrix}, \tag{16}$$

or for the elements of $\hat{\boldsymbol{u}}$ and $\hat{\boldsymbol{i}}$

$$\hat{u}_1 = D_{\lambda_-}\hat{i}_1 = \frac{\partial \hat{i}_1}{\partial \lambda_-}, \qquad \hat{u}_2 = D_{\lambda_+}\hat{i}_2 = \frac{\partial \hat{i}_2}{\partial \lambda_+}. \tag{17}$$

Eqn. (17) shows that the transformations (12) for the dependent and (14) for the independent variables have decoupled the reactive part completely. Not only is $\hat{\boldsymbol{Z}}_l$ diagonal, also each diagonal element contains a partial derivation with repect to only one coordinate. This means the numerical treatment of the differentiation operators in (17) can be handled independently.

3.1.3. numerical integration

It is thus sufficient to show the integration only for one component in (17). We solve the first equation for $\hat{i}_1$

$$\hat{i}_1(\lambda_+, \lambda_-) = \int_{\infty}^{\lambda_-} \hat{u}_1(\lambda_+, \xi)\, d\xi \tag{18}$$

and apply the trapezoidal rule of integration with step size $\Delta\lambda$ (λ_+ omitted)

$$\hat{i}_1(\lambda_-) = \hat{i}_1(\lambda_- - \Delta\lambda) + \frac{\Delta\lambda}{2}\Big[\hat{u}_1(\lambda_-) + \hat{u}_1(\lambda_- - \Delta\lambda)\Big]. \quad (19)$$

A similar relation follows for $\hat{i}_2$. Evaluation of these equations on a uniform grid in the (λ_+, λ_-)-plane with $R_0 = 2/\Delta\lambda$ and $\hat{i}_{1/2}[\mu,\nu] = \hat{i}_{1/2}(\mu\Delta\lambda, \nu\Delta\lambda)$ gives

$$\hat{u}_1[\mu,\nu] - R_0\hat{i}_1[\mu,\nu] = -\hat{D}_\nu[\hat{u}_1[\mu,\nu] + R_0\hat{i}_1[\mu,\nu]] \quad (20)$$

$$\hat{u}_2[\mu,\nu] - R_0\hat{i}_2[\mu,\nu] = -\hat{D}_\mu[\hat{u}_2[\mu,\nu] + R_0\hat{i}_2[\mu,\nu]] \quad (21)$$

with the shift operators

$$\hat{D}_\mu\hat{w}[\mu,\nu] = \hat{w}[\mu-1,\nu], \quad \hat{D}_\nu\hat{w}[\mu,\nu] = \hat{w}[\mu,\nu-1] \quad (22)$$

Unfortunately, eqns. (20,21) are not computable, since $\hat{u}_{1/2}$ and $\hat{i}_{1/2}$ occur both inside and outside the shift operators. This situation can be circumvented with another change of variables.

3.1.4. wave quantities

Transforming $\hat{u}_{1/2}$ and $\hat{i}_{1/2}$ into new variables $\hat{a}_{1/2}$ and $\hat{b}_{1/2}$ according to

$$\begin{bmatrix} \hat{a}_\kappa \\ \hat{b}_\kappa \end{bmatrix} = \boldsymbol{R}_0 \begin{bmatrix} \hat{u}_\kappa \\ \hat{i}_\kappa \end{bmatrix}, \ \kappa = 1,2, \quad \boldsymbol{R}_0 = \begin{bmatrix} 1 & R_0 \\ 1 & -R_0 \end{bmatrix} \quad (23)$$

turns eqns. (20,21) into a computable set of decoupled equations

$$\begin{bmatrix} \hat{b}_1[\mu,\nu] \\ \hat{b}_2[\mu,\nu] \end{bmatrix} = -\begin{bmatrix} \hat{D}_\nu & 0 \\ 0 & \hat{D}_\mu \end{bmatrix} \begin{bmatrix} \hat{a}_1[\mu,\nu] \\ \hat{a}_2[\mu,\nu] \end{bmatrix}. \quad (24)$$

The new variables $\hat{a}$ and $\hat{b}$ are also called wave quantities [1]. Eqns. (24) provide a very simple way to compute wave propagation phenomena. However, they are formulated in terms of wave quantities depending on samples in a (λ_+, λ_-)-plane. To make (24) compliant with our initial problem, we have to convert back to the elements velocity and pressure in $\boldsymbol{i}(x,t)$. This will again introduce the interdependence removed in section 3.1.2 and result in more involved shift operators than $\hat{D}_\mu$ and $\hat{D}_\nu$ from (22). But — in contrast to the differential operators encountered earlier — shift operators can be easily implemented on a digital computer and pose no fundamental problem. This will be shown now.

When converting back from the (λ_+, λ_-)-coordinates to (x,t), it has to be insured that the sampling points $(\lambda_+ = \mu\Delta\lambda, \lambda_- = \nu\Delta\lambda)$ and $(x = mh, t = kT)$ coincide. This is achieved by the conditions (see Fig. 1)

$$h = cT, \quad \Delta\lambda = \frac{h}{r_0} = \frac{T}{\rho_0}. \quad (25)$$

From (14) or by inspection of Fig. 1 follows that the change in the discrete coordinate systems $[\mu,\nu] \to [m,k]$ results in the shift operators

$$\hat{D}_\mu\hat{w}[\mu,\nu] = \tilde{D}_+\tilde{w}[m,k], \quad \tilde{D}_+\tilde{w}[m,k] = \tilde{w}[m-1,k-1]$$

$$\hat{D}_\nu\hat{w}[\mu,\nu] = \tilde{D}_-\tilde{w}[m,k], \quad \tilde{D}_-\tilde{w}[m,k] = \tilde{w}[m+1,k-1]$$

With (24) results the equation set for the wave quatities $\tilde{a}[m,k]$ and $\tilde{b}[m,k]$ in discrete x,t-coordinates

$$\underbrace{\begin{bmatrix} \tilde{b}_1[m,k] \\ \tilde{b}_2[m,k] \end{bmatrix}}_{\tilde{\boldsymbol{b}}} = -\underbrace{\begin{bmatrix} \tilde{D}_- & 0 \\ 0 & \tilde{D}_+ \end{bmatrix}}_{\tilde{\boldsymbol{D}}_\pm} \underbrace{\begin{bmatrix} \tilde{a}_1[m,k] \\ \tilde{a}_2[m,k] \end{bmatrix}}_{\tilde{\boldsymbol{a}}}. \quad (26)$$

Note, that (26) corresponds to (13). Inversion of the decoupling process in (12) with $\boldsymbol{a} = \boldsymbol{Q}\tilde{\boldsymbol{a}}$, $\boldsymbol{b} = \boldsymbol{Q}\tilde{\boldsymbol{b}}$ gives a wave quantity representation of the elements of $\boldsymbol{i}$ and $\boldsymbol{u}$ similar to (23)

$$\begin{bmatrix} a_\kappa \\ b_\kappa \end{bmatrix} = \boldsymbol{R}_0 \begin{bmatrix} u_\kappa \\ i_\kappa \end{bmatrix}, \quad \kappa = 1,2\ . \quad (27)$$

3.1.5. state space representation

The final step is the formulation of the simulation algorithm as a state space representation. The choice of $\boldsymbol{z} = \tilde{\boldsymbol{b}} = \boldsymbol{Q}^{-1}\boldsymbol{b}$ as state vector turns (26) into

$$\boldsymbol{z} = -\tilde{\boldsymbol{D}}_\pm \boldsymbol{Q}^{-1}\boldsymbol{a}\ . \quad (28)$$

To obtain the state equation, we require further information from the reactive part in (11). Using (27) to represent $\boldsymbol{i}$ and $\boldsymbol{u}$ in (11) by the wave quantities $\boldsymbol{a}$ and $\boldsymbol{b}$ and solving for $\boldsymbol{a}$ results in

$$\boldsymbol{a} = \boldsymbol{S}_\varrho\, \boldsymbol{b} + \boldsymbol{S}_e\, \boldsymbol{e} \quad (29)$$

with

$$\boldsymbol{S}_\varrho = \begin{bmatrix} S_{\varrho,1} & 0 \\ 0 & S_{\varrho,2} \end{bmatrix}, \quad \boldsymbol{S}_e = \begin{bmatrix} S_{e,1} & 0 \\ 0 & S_{e,2} \end{bmatrix}, \quad (30)$$

$$S_{\varrho,\kappa} = \frac{r_\kappa - R_0}{r_\kappa + R_0}, \quad S_{e,\kappa} = \frac{2R_0}{r_\kappa + R_0}, \quad \kappa = 1,2. \quad (31)$$

Inserting (29) into (28) gives the state equation

$$\boldsymbol{z} = \tilde{\boldsymbol{D}}_\pm\, [\boldsymbol{A}\, \boldsymbol{z} + \boldsymbol{B}\, \boldsymbol{e}] \quad (32)$$

with

$$\boldsymbol{A} = -\boldsymbol{Q}^{-1}\boldsymbol{S}_\varrho\, \boldsymbol{Q}, \quad \boldsymbol{B} = -\boldsymbol{Q}^{-1}\boldsymbol{S}_e. \quad (33)$$

The state equation (32) represents the state in terms of wave quantities rather than by i_1 and i_2 as required by (8). The conversion of wave quantities to $\boldsymbol{i}$ gives the output equation of the state space representation.

The relation between wave quantities $\boldsymbol{a}$, $\boldsymbol{b}$ and the original variables $\boldsymbol{i}$ follows from (27) as

$$\boldsymbol{i} = \frac{1}{2R_0}\, [\boldsymbol{a} - \boldsymbol{b}]\, . \quad (34)$$

Elimination of $\boldsymbol{a}$ by (29) gives the output equation

$$\boldsymbol{i} = \boldsymbol{C}\, \boldsymbol{z} + \boldsymbol{F}\, \boldsymbol{e} \quad (35)$$

with

$$\boldsymbol{C} = \frac{1}{2R_0}\, [\boldsymbol{S}_\varrho - \boldsymbol{I}]\, \boldsymbol{Q}, \quad \boldsymbol{F} = \frac{1}{2R_0}\boldsymbol{S}_e\ . \quad (36)$$

where $\boldsymbol{I}$ is the identity matrix.

Eqns. (32) and (35) are the state space representation of a discrete model for the wave propagation process described by (8). It computes the variables v_x and p in $\boldsymbol{i}$ from the given source terms in $\boldsymbol{e}$. Choosing wave quantities as state vector results in a simple form of the discrete operator $\tilde{\boldsymbol{D}}_\pm$ for the shift and delay operations.

3.2. three space dimensions

The derivation of a discrete model for wave propagation in three space dimensions follows the same lines as for one space dimension. The key point is that the matrix derivation operator in (6) (possibly with additional loss terms) can be decomposed into three terms, one for each space dimension. The discretization of these terms can be performed separately as explained in section 3.1.

The result is a state space representation of a discrete system of the same general form as (32) and (35) for one space dimension

$$\boldsymbol{z} = \tilde{\boldsymbol{\mathcal{D}}}_{\pm}\left[\boldsymbol{\mathcal{A}}\boldsymbol{z} + \boldsymbol{\mathcal{B}}\boldsymbol{e}\right], \tag{37}$$

$$\boldsymbol{i} = \boldsymbol{\mathcal{C}}\boldsymbol{z} + \boldsymbol{\mathcal{F}}\boldsymbol{e}. \tag{38}$$

The variables of the source terms $\boldsymbol{e}$, the output $\boldsymbol{i}$ and the state vector $\boldsymbol{z}$ depend on time and all three space dimensions. The state space matrices $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{B}}$, $\boldsymbol{\mathcal{C}}$, and $\boldsymbol{\mathcal{F}}$ follow from the original PDE in a similar way as in (33) and (36). However, due to couplings between the three space dimensions, the expressions are more involved. Space does not permit to report the derivation in detail.

4. BOUNDARY CONDITIONS

The operators $\tilde{\boldsymbol{D}}_{\pm}$ and $\tilde{\boldsymbol{\mathcal{D}}}_{\pm}$ in the state equations contain shifts in both directions of each spatial dimension. This requires the knowledge of the previous states in all adjacent points. However, if a point is at the boundary of the spatial domain, e.g. at the wall of an enclosure, then one or more of the adjacent points are beyond the boundary, where the PDE is no more valid. In this case, the state of these points has to be determined from the boundary conditions (7) rather than from the PDE (1, 2). (See also [5].)

The idea is to split the state vector $\boldsymbol{z}$ into two components: the interior states $\boldsymbol{z}_i$ and the boundary states $\boldsymbol{z}_b$. The interior states follow from a state equation similar to (37). The boundary states follow from the interior states and the boundary conditions. The state space representation has to consider both types of states appropriately. Its general form is given by

$$\boldsymbol{z}_i = \left(\boldsymbol{T}_i^T \tilde{\boldsymbol{\mathcal{D}}}_{\pm}\right)\left[\boldsymbol{\mathcal{A}}\boldsymbol{z} + \boldsymbol{\mathcal{B}}\boldsymbol{e}\right], \tag{39}$$

$$\boldsymbol{z}_b = \boldsymbol{\mathcal{A}}_b \boldsymbol{z}_e + \boldsymbol{\mathcal{B}}_b \boldsymbol{e}, \tag{40}$$

$$\boldsymbol{z} = \boldsymbol{T}_i \boldsymbol{z}_i + \boldsymbol{T}_b \boldsymbol{z}_b, \tag{41}$$

$$\boldsymbol{i} = \boldsymbol{\mathcal{C}}\boldsymbol{z} + \boldsymbol{\mathcal{F}}\boldsymbol{e}. \tag{42}$$

The matrices $\boldsymbol{T}_i$ and $\boldsymbol{T}_b$ contain only ones and zeros. They depend on the geometry and describe whether a state is an interior state $\boldsymbol{z}_i$ or a boundary state $\boldsymbol{z}_b$. Eq. (39) is very similar to the state equation (37), except that it delivers only the interior states. The boundary states are computed in (40) from the interior states and the boundary conditions, which determine $\boldsymbol{\mathcal{A}}_b$ and $\boldsymbol{\mathcal{B}}_b$. Both interior and boundary states are merged into the complete state vector $\boldsymbol{z}$ in (41). It is used to deliver the output quantities in (42) and to update the interior states in (39). Note, that the boundary state equation (40) holds for memoryless wall impedances R_w. If the boundary has temporal or spatial memory, then also (40) contains a delay or shift operator similar to $\tilde{\boldsymbol{\mathcal{D}}}_{\pm}$.

5. EXAMPLE

As an example, we consider a simple setup for active noise control. Fig. 2 (top) shows a section of a flow duct. The source S_1 represents a disturbing fan noise. Its sound field interferes with a second sound field emanating from the source S_2. Typically, S_2 is controlled by an adaptive system to minimize the micropone signal M_1. In this example, S_1 and S_2 are sine waves with an appropriate phase shift. The resulting sound pressure has been computed from (39-42). Fig. 2 (bottom) shows a horizontal cross section through the 3D pressure field. The effects of constructive and destructive interference upstream and downstream of S_2 are clearly visible.

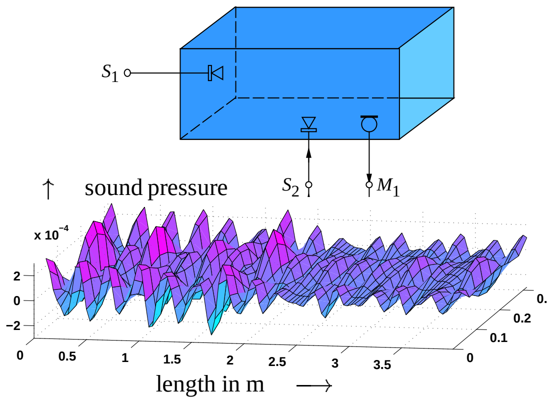

Figure 2: Example: active noise control in a flow duct

6. CONCLUSION

We have presented an algorithm for the simulation of dynamic wave propagation in three space coordinates. It is based on the wave digital principle and shares its advantages. However, unlike in previous publications on this topic, no reference to multidimensional network theory and complex analysis has been made. Instead, a straightforward derivation from the basic laws of physics to a multidimensional discrete state space formulation has been made. This justifies to call it a direct method to computational acoustics. Although presented and implemented for the simulation of acoustical phenomena, the method can also be applied to other technical wave propagation effects.

7. REFERENCES

[1] A. Fettweis. Multidimensional wave-digital principles: From filtering to numerical integration. In *Proc. Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP 94)*, pages VI–173 – VI–181. IEEE, April 1994.

[2] Th. Schetelig and R. Rabenstein. Simulation of three-dimensional sound propagation with multidimensional wave digital filters. In *Proc. Int. Conf. Acoustics, Speech, and Signal (ICASSP 98)*, pages 3537–3540. IEEE, 1998.

[3] R. Rabenstein and Th. Schetelig. A multidimensional wave digital filter algorithm for the 3D simulation of room acoustics. In *Proc. Image and Multidimensional Digital Signal Processing Workshop (IMDSP 98)*, pages 255–258. IEEE, infix, 1998.

[4] L. J. Ziomek. *Fundamentals of Acoustic Field Theory and Space-Time Signal Processing*. CRC Press, Boca Raton, 1995.

[5] H. Krauß. Simulation of coupled transmission lines by multidimensional wave digital filters. In *Proc. Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP 96)*, pages III–1747– III–1750, 1996.