GAUSSIAN MODELING FOR CHANNEL ERRORS DIAGNOSIS IN IMAGE TRANSMISSION

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ABSTRACT

In this paper we propose an original study of the reconstruction of subband compressed images impaired by channel transmission errors. The method proceeds in two steps : first a detection scheme is applied to determine which coefficients of the subband decomposition have been affected by transmission, and then an estimation step tries to evaluate the erroneous coefficients. In our model, subband coefficients are considered to be drawn from jointly gaussian random processes. Based on this assumption, conditional statistics can be computed which enable to test the likelihood of a given set of received coefficients with respect to the rest of the image. The detection and estimation processes are derived from these statistics. The method is validated through simulation and visual results are provided. The drawbacks of the method are outlined and explained through the discrepancies between the gaussian assumption and real world images, namely around image edges.

1. INTRODUCTION

Nowadays, the field of image restoration has been studied in many papers, combating different types of noises and degradations. In the case of transmission of coefficients of an image over noisy digital channels, the type of degradation at hand is a *salt and pepper* noise, but with non constant impulse values : since a pixel value is transmitted as its binary representation, any of its bits can be corrupted, and yield an other decimal value in reconstruction. Classical image restoration methods used against impulsive noise include markov random fields regularized reconstruction [1], order statistics filters [2, 3], with median filters as a particular case [4, 5, 6]. Those methods work in the pixel domain, with the general assumption that an image is made of a set of uniform regions and edges. In the particular case of subband images [7, 8] corrupted by channel noise, the salt and pepper noise appears on each subband coefficient, and translates into modulations of the filters basis functions at the synthesis stage, appearing randomly on the reconstructed image. It seems interesting to try to use this knowledge to correct the received image, in the subband domain. If we want to detect and correct the errors occuring on the subband coefficients, we cannot rely on the same assumption for modeling the relationship between coefficients inside and between subbands as in the pixel domain. Moreover, rather than simply use a classical "uniform regions plus edges" assumption, we can try to exploit the spatial and frequential redundancy inside and between subbands of an image to get information about the coefficients. In order to obtain a sufficiently versatile and flexible assumption regarding the subband decomposition of an image, we drop in this paper the classical high-level assumptions, and model the subband coefficients as jointly gaussian random variables, which have between them relationships modeling the redundancy of the subband decomposition. This model is described in section 2. Section 3 reviews the application of the model to the detection and reconstruction of the subband coefficients affected by channel noise : to decide whether a coefficient (or a set of coefficients) contains a channel error, we use the statistic of the given coefficients conditioned on their neighbours, both in the spatial and frequential domains. The computation of these statistics is made easier thanks to the gaussian assumption. In the same section, we add some knowledge of the noise generation process (i.e. the channel) to the model, giving rise to a discrete conditioned probability model. Section 4 presents simulations and visual results, and reviews the achievements and drawbacks of the method, namely a general reconstruction gain of nearly 5 dB in PSNR at 1% error rate, but a performance that is limited by the poor edge recognition capabilities of the gaussian probability density function (pdf).

2. NOTATIONS AND THEORETICAL BASIS

The original image is decomposed by a 2-D $M \times M$ critically-sampled filter bank, giving rise to M^2 subband signals $y^{(k,l)}(i,j)$, indices (k,l) referring to the subband number, and (i,j) to the spatial coordinates. These subbands are scalar quantised with appropriate quantisers. Let $Q^{(k,l)}$ be the number of quantisation levels of subband (k,l), and let us denote by $q_r^{(k,l)}$, $r = 0,\dots,Q^{(k,l)}-1$ the reproduction levels of these quantisers. The binary representations of the indices of the quantisation levels are sent on a binary symmetric channel, with error probability P_e.

The receiver only sees the values $y^{(k,l)}(i,j)$, some of them corrupted by channel noise. The main idea of the model is to assume that each of the coefficients has gaussian probability density function. To take into account the spatial (intra-band) and frequential (inter-band) redundancy, we will compute the statistics of a set of coefficients $\mathcal{S}$ (the *conditioned set*), conditioned on an other set of coefficients $\mathcal{S}'$ (the *conditioning set*). This conditional pdf is also gaussian. If we denote by $(\mu_{\mathcal{S}}, \mathbf{R}_{\mathcal{S}})$ and $(\mu_{\mathcal{S}'}, \mathbf{R}_{\mathcal{S}'})$ the mean and covariance matrices of $\mathcal{S}$ and $\mathcal{S}'$ respectively, and by $\mathbf{R}_{\mathcal{S}\mathcal{S}'}$ their cross-covariance matrix, the mean of $\mathcal{S}$ given $\mathcal{S}'$ is

$$\mu_{\mathcal{S}|\mathcal{S}'} = \mu_{\mathcal{S}} + \mathbf{R}_{\mathcal{S}\mathcal{S}'}\mathbf{R}_{\mathcal{S}'}^{-1}\left(\mathcal{S}' - \mu_{\mathcal{S}'}\right), \qquad (1)$$

and its conditional covariance matrix is

$$\mathbf{R}_{\mathcal{S}|\mathcal{S}'} = \mathbf{R}_{\mathcal{S}} - \mathbf{R}_{\mathcal{S}\mathcal{S}'}\mathbf{R}_{\mathcal{S}'}^{-1}\mathbf{R}_{\mathcal{S}\mathcal{S}'}^{T}. \qquad (2)$$

Now if we want to decide whether the set $\mathcal{S}$ contains an error induced by the channel, we only have to verify its coherence with $\mathcal{S}'$, by computing its likelihood under the conditional statistics $\left(\mu_{\mathcal{S}|\mathcal{S}'}, \mathbf{R}_{\mathcal{S}|\mathcal{S}'}\right)$. One can show that $\left|\mathbf{R}_{\mathcal{S}|\mathcal{S}'}\right| \le \left|\mathbf{R}_{\mathcal{S}}\right|$, so that the operation of conditioning can only be informative, by decreasing the entropy[1] of $\mathcal{S}$. Then, if we have decided that $\mathcal{S}$ contains an error, we can try to replace it by an estimate. We can for instance use the conditional mean[2] $\mu_{\mathcal{S}|\mathcal{S}'}$ to replace $\mathcal{S}$.

The next section will describe two possible implementations of the detection method outlined here.

3. IMPLEMENTATIONS FOR ERRORS DIAGNOSIS AND CORRECTION

We have tested two different configurations for the error detection step of our reconstruction algorithm.

3.1. Gaussian Conditional Probability (GCP)

The GCP model is the straightforward implementation of the conditional probabilities given above. The method is as follows : first, one has to select sets $\mathcal{S}$ and $\mathcal{S}'$, for instance $\mathcal{S} = \left\{y^{(0,0)}(i,j)\cdots y^{(M-1,M-1)}(i,j)\right\}$, i.e. all the local subband coefficients, and $\mathcal{S}' = \left\{y^{(0,0)}(i\pm1,j\pm1)\cdots y^{(M-1,M-1)}(i\pm1,j\pm1)\right\}$, in which case we test the coherence of each block of subband coefficients $\mathcal{S}$ with respect to its neighbours. Then, one evaluates the first and second order moments of $\mathcal{S}$ and $\mathcal{S}'$ on the received subband coefficients. Finally, the detection can take place : one simply scans the subband images, line by line, constructs local sets $\mathcal{S}$ and $\mathcal{S}'$, and computes the probability p that any realization of $\mathcal{S}$ is closer to the conditional mean $\mu_{\mathcal{S}|\mathcal{S}'}$ than the received one. If p exceeds some predefined threshold T, then the realization of $\mathcal{S}$ is considered too unprobable, and is marked as false. The replacement of $\mathcal{S}$ by its conditional mean can be done after each detection, or only after a pass over the whole image.

The choice of sets $\mathcal{S}$ and $\mathcal{S}'$ is of obvious importance. The greater the dimension of the sets, the more their moments will be difficult to evaluate with precision. Also, with great dimensions, matrix inversions implied by (1) and (2) will be more costly, and less accurate. On the other hand, including lots of coefficients in the sets ensures the capability to catch more long-range dependencies between them. So it is clear that a trade-off is to be found. Experiments show that coefficients belonging to the same subband in a relatively small spatial regions are very correlated, whereas coefficients of different subbands in the same spatial location are nearly decorrelated. Some correlation also exists between coefficients of different subbands in small spatial regions.

3.2. Discrete Probabilities of Quantised Gaussians (DPQG)

This variant of the method takes into account the model of noise production. Since the coefficients are quantised before being sent on the channel, received coefficients can only take a discrete set of values, namely the quantisation levels $q_r^{(k,l)}$. Knowing the error probability P_e of the channel, one can easily compute the probabilities $P_{(k,l)}(r|s)$ that a coefficient of subband (k,l) takes the value $q_r^{(k,l)}$ at the output of the channel if $q_s^{(k,l)}$ was actually sent. Now we can compute discrete probabilities of receiving an erroneous coefficient, using the gaussian source production model. Imagine for instance that we receive the value $q_r^{(k,l)}$ for the coefficient $y^{(k,l)}(i,j)$. We constitute the two sets $\mathcal{S} = \left\{y^{(k,l)}(i,j)\right\}$, and $\mathcal{S}'$, with relevant neighbours. Once $\mu_{\mathcal{S}|\mathcal{S}'}$ and $\mathbf{R}_{\mathcal{S}|\mathcal{S}'}$ are computed by (1) and (2), we completely know a relevant pdf of $y^{(k,l)}(i,j)$ before quantisation; the discrete probabilities of each reproduction level $P(q_s^{(k,l)}|\mathcal{S}')$, $s = 0,\dots,Q^{(k,l)}-1$, are simple one-dimensional integral of this pdf over the quantisation intervals. Then the probability

[1] Recall that the entropy of a K-dimensional gaussian random vector $\mathbf{V}$ is $H(\mathbf{V}) = \frac{K}{2}\log_2 2\pi e|\mathbf{R_V}|^{\frac{1}{K}}$.

[2] Note that this estimation by $\mu_{\mathcal{S}|\mathcal{S}'}$ is equivalent to the minimum mean squared error prediction of $\mathcal{S}$ by $\mathcal{S}'$.

that the received level is correct is

$$P(r) = \frac{P(q_r^{(k,l)}|\mathcal{S}')\, P_{(k,l)}(r|r)}{\sum_{s=0}^{Q^{(k,l)}-1} P(q_s^{(k,l)}|\mathcal{S}')\, P_{(k,l)}(r|s)}. \quad (3)$$

If this probability is less than 0.5, then $y^{(k,l)}(i,j)$ is rejected, and thus replaced by either its conditional mean, or the most probable level. This method has the advantage of taking into account more knowledge than the GCP method, namely the source quantisers and the channel characteristics. On the other hand, DPQG method is restricted to sets $\mathcal{S}$ of dimension 1, due to the integrals computations necessary for each pixel.

4. SIMULATION RESULTS

First, the GCP method has been tested. The figure of merit retained for reconstruction is the peak signal-to-noise ratio (PSNR). The test image used was *Lena*, with a channel bit error rate of 10^{-2}. Tests were conducted via Monte-Carlo simulations, and the results given are means of 20 simulations. The detection performance is measured by the percentage of channel errors detected, as well as the number of false alarms, i.e. the coefficients correctly received, but that the algorithm has marked as erroneous.

The first conclusion of the study is that the dimension of $\mathcal{S}$ giving the best results is simply one : it is always better to analyse and replace each coefficient separately. This comes from the inherent difficulty to estimate precisely multidimensional models from a limited amount of data samples.

A second conclusion is that the model fails in general to account for the presence of edges. To see this, table 1 shows the difference of performance between the theoretically optimal (TO) choice of $\mathcal{S}'$ and an heuristically (H) chosen $\mathcal{S}'$. Both refer to $\mathcal{S} = \left\{y^{(k,l)}(i,j)\right\}$. The optimal choice has been designed so as to minimise the residual variance $\left|\mathbf{R}_{\mathcal{S}|\mathcal{S}'}\right|$, with a size of $\mathcal{S}'$ of 20. The heuristic $\mathcal{S}'$ is designed to take into account the presence of edges as preferential directions : the coherence of $\mathcal{S}$ is tested against four different $\mathcal{S}'$ sets, each containing all neighbouring subband coefficients in a particular direction (horizontal, vertical, and both diagonals). One can see from table 1 that the real problem of setup TO is the number of false alarms. In fact, these are mainly located around edges, but do not appear in setup H. Even if setup H finds less errors than setup TO, its reconstruction quality is far better.

One last series of simulations for the GCP method discusses the choice of the rejection threshold T. If T is high, we will not detect many errors, but if T is too low, we will mark all edges as errors. Figure 1 shows the optimal value for setup H, around $T = 0.9$, which is clearly a compromise between the percentage of error detection and the number of false alarms.

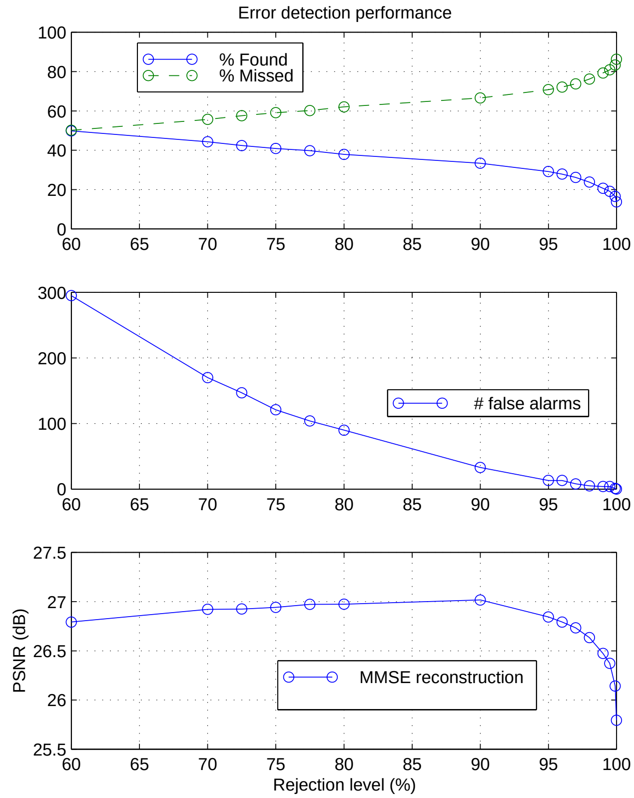

Figure 1: Effect of the rejection threshold on detection and reconstruction performance.

	PSNR	% found	false alarms
TO	26.38 dB	45 %	409
H	28.29 dB	31 %	10

Table 1: Performance of theoretically optimal (TO) set $\mathcal{S}'$ vs. heuristically chosen (H) set $\mathcal{S}'$. The PSNR without any processing is 24 dB.

Figure 2 shows visual results for the GCP method.

Regarding the DPQG method, a setup with $\mathcal{S}'$ of dimension 32 has been found to be the best. It is mainly constituted by neighbouring coefficients of all subbands. The PSNR reached by this method is 28.76 dB, with a recursive implementation, i.e. replacement of erroneous coefficients right after detection. The corresponding visual results are given in figure 2.

5. CONCLUSIONS AND FURTHER WORK

We have studied in this paper a novel method of diagnostic and correction of transmission errors in images, working in the subband domain. A variant of this method, exploiting the discrete nature of the signal has been shown to be ef-

(a) **(b)** **(c)**

Figure 2: Example of reconstructions. **(a)** reconstruction from noisy subbands ($P_e = .01$), without processing (23.6 dB), **(b)** after processing by the GCP model (28.0 dB) and **(c)** by the DPQG model (28.6 dB)

ficient and promising. Gains in PSNR of nearly 5 dB have been reached for bit error rates of 10^{-2}. The main drawback of the method is its lack of built-in recognition of edges. Further work will thus be accomplished towards a richer and more appropriate image source model, such as for instance a mixture model.

6. REFERENCES

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