A CONSTRAINED OPTIMAL DATA ASSOCIATION FOR MULTIPLE TARGET TRACKING

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ABSTRACT

One of the major problems in multiple target tracking is to obtain an accurate association between targets and noisy measurements. We introduce a new scheme, called *Constrained Optimal Data Association* (CODA), that finds the optimal data association by a MAP estimation method and uses a new energy function. In this scheme, the natural constraints between targets and measurements are defined so that they may contain missed detection and false alarm errors. Most current algorithms involve many heuristic adjustments of the parameters. Instead, this paper suggests an adaptive mechanism for such parameter updation. In this manner, the system automatically adapts to the clutter environment as it continuously changes in time and space, resulting in better association. Experimental results, using PDA, NNF, and CODA, show that the new approach reduces position errors in crossing trajectories by 13.9% on average compared to NNF.

1. INTRODUCTION

Multiple target tracking (MTT) plays an important role in radar, especially in surveillance radar systems that must estimate positions and velocities of moving targets from noisy measurements. In this field, it is well known that the most probable errors are false alarms and missing targets. The other important errors are those due to a single measurement from multiple targets and multiple measurements from a single target. So far there is no method that can deal satisfactorily with all these errors simultaneously.

Among many MTT schemes, the three most important methods are *Joint Probabilistic Data Association* (JPDA) [1], the *Expectation Maximization* approach [9], and the neural net approach [10, 6]. JPDA is a kind of closed loop system consisting of two systems: data association and prediction. For each time frame, Kalman filters predict target centers. Based on the measurements in the gates formed around the target centers, the data association unit tries to associate measurements and targets in some optimal manner. After that, the updated measurements are used in the Kalman filters again. Molnar [9] derived the whole system consisting of prediction and association units in an unified manner with the EM method [3]. Defining the association matrix as missing data, the algorithm can estimate the association matrix for the current measurements. Also the algorithm can determine the association in a time-recursive manner. Like the Kalman filter technique, this scheme computes measurement and time updates in parallel. On the other hand, the neural net approach interprets the constraints as separate terms in an energy function that is easily implemented with a Hopfield-style network.

Our research is confined to the association matrix and the related energy function as a cost function for optimal constraints. By adopting the constraints on missing targets and false alarm errors, the new energy function is more general and natural. Also, it is fully automatic in that all the Lagrangian parameters are updated on-the-fly.

2. THE OVERALL STRUCTURE OF MTT

Fig. 1 shows the overall scheme of our target tracking system. It

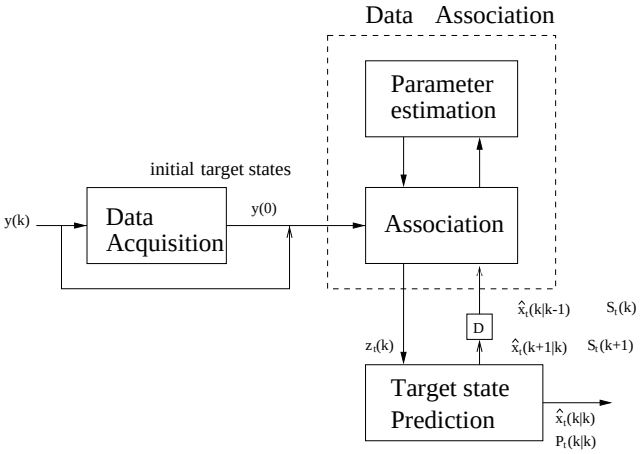

Figure 1: The multiple target tracking system.

consists of three parts: acquisition, association, and prediction. The purpose of the acquisition part is to provide the necessary data when the system starts from the beginning, a target appears or disappears from the field of scope, or intermittent measurements are received from a target. Once detected, the targets should be continuously tracked by the joint cooperation of the association and prediction parts. The prediction part uses Kalman filters to provide the association part with the predicted positions of the targets along with their gate sizes. In the current time frame, the association part counts the measurements that lie in the gates and encodes this information in a validation matrix. Utilizing this information with other additional constraints, the association part decides which measurements correspond to which targets and represents the relationships by an association matrix. This information is supplied to the Kalman filters for a measurement update

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stage for the current time frame. This routine repeats for each time frame. The crucial part of this algorithm is the association part and therefore this paper is restricted to this issue.

To begin with, let us denote the state and measurement of the target t by $\mathbf{x}_t(k)$ and $\mathbf{z}_t(k)$, respectively. These can be represented by the dynamical equation:

$$\begin{cases} \mathbf{x}_t(k) = \mathbf{F}_t(k-1)\mathbf{x}_t(k-1) + \mathbf{G}_t(k-1)\mathbf{w}(k-1), \\ \mathbf{z}_t(k) = \mathbf{H}_t(k)\mathbf{x}_t(k) + \mathbf{v}(k), \end{cases} \tag{1}$$

where $\mathbf{F}_t(k)$, $\mathbf{G}_t(k)$, and $\mathbf{H}_t(k)$ are respectively state transition, process noise coupling, and measurement matrices. The noise processes $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are mutually independent Gaussian with zero mean and variances $\mathbf{Q}(k)$ and $\mathbf{R}(k)$, respectively.

The Kalman filters calculate the target state $\hat{\mathbf{x}}_t(k|k-1)$ and the measurement prediction covariance matrix $\mathbf{S}_t(k)$,

$$\begin{cases} \hat{\mathbf{x}}_t(k|k-1) = \mathbf{F}_t(k-1)\hat{\mathbf{x}}_t(k-1|k-1), \\ \mathbf{S}_t(k) = \mathbf{H}_t(k)\mathbf{P}_t(k|k-1)\mathbf{H}_t^T(k) + \mathbf{R}(k), \end{cases} \tag{2}$$

for each target and send this information to the association unit. Note that the measurement update part of Kalman filter is

$$\hat{\mathbf{x}}_t(k|k) = \hat{\mathbf{x}}_t(k|k-1) + \mathbf{W}_t(k)\{\mathbf{z}_t(k) - \mathbf{H}_t(k)\hat{\mathbf{x}}_t(k|k-1)\}, \tag{3}$$

and

$$\mathbf{P}_t(k|k) = [\mathbf{I} - \mathbf{W}_t(k)\mathbf{H}_t(k)]\mathbf{P}_t(k|k-1), \tag{4}$$

where the Kalman gain $\mathbf{W}_t(k)$ is

$$\mathbf{W}_t(k) = \mathbf{P}_t(k|k-1)\mathbf{H}_t^T(k)\mathbf{S}_t^{-1}(k). \tag{5}$$

Next, consider the association part dealing with m measurements and n targets. The relationships between the targets and measurements are conveniently denoted by the association matrix $\Omega = \{\omega_{jt} | j \in [1,m], t \in [1,n]\}$. Here, $\omega_{jt} \in [0,1]$ denotes the probability of the association between measurement j and target t. Also assume that the measurements are $\mathbf{y}_j(k)$ for $j \in [1,m]$ and the gate centers are $\mathbf{g}_t(k)$ for $t \in [1,n]$. Then, the state $\hat{\mathbf{x}}_t(k|k-1)$ predicted by the filter is utilized as an input to the association part to produce the center position of gate t:

$$\mathbf{g}_t(k) = \mathbf{H}_t(k)\hat{\mathbf{x}}_t(k|k-1), \;\; t \in [1,n]. \tag{6}$$

Next, the measurement prediction covariance matrix $\mathbf{S}_t(k)$ is used to produce the validation matrix Ω^0 in the following way. The distance between a measurement j and the center of gate t is given by the Mahalanobis distance:

$$r_{jt}^2(k) = [\mathbf{y}_j(k) - \mathbf{g}_t(k)]\mathbf{S}_t^{-1}(k)[\mathbf{y}_j(k) - \mathbf{g}_t(k)]^T. \tag{7}$$

If $r_{jt}(k)$ is smaller than the radius g of the gate, then the measurement is considered in the gate. This relationship can be conveniently described by the *validation matrix* $\boldsymbol{\omega}^0 = \{\omega_{jt}^0 | \omega_{jt}^0 \in [0,1], j \in [1,m], t \in [1,n]\}$. Since the noise is Gaussian, the distance from the gate center is directly related with the probability. Therefore, it is natural to define the elements of the matrix as

$$\omega_{jt}^0 = \begin{cases} \frac{1}{|2\pi \mathbf{S}_t(k)|^{1/2}} \exp(-r_{jt}^2/2), & \text{if } r_{jt}^2 \le \gamma, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

Then, the association part generates the measured position of the target t by

$$\mathbf{z}_t(k) = \mathbf{y}_{j'}(k), \;\; t \in [1,n], \tag{9}$$

where $j' = \arg\max_j \omega_{jt}$. This is the quantity to be supplied to the prediction filter for measurement update and for further processing.

3. MAP ESTIMATES FOR DATA ASSOCIATION

The previous section described the overall structure of our scheme for multiple target tracking. This section describes the internal structure of the association unit in detail. One can assume that the probability space consists of the parameter Ω and the observation Ω^0, as illustrated in Fig. 2. The parameter Θ is fixed for the time

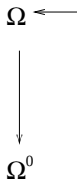

Figure 2: The parameter-measurement spaces.

being and later on will be released from this restriction.

The goal is to find the association matrix $\boldsymbol{\omega}$, given the modified validation matrix $\boldsymbol{\omega}^0$. Specifically, a MAP estimate $\boldsymbol{\omega}^*$ is given by

$$\boldsymbol{\omega}^* = \arg\max_{\boldsymbol{\omega}} \ln p(\boldsymbol{\omega}|\boldsymbol{\omega}^0). \tag{10}$$

In this description, the posterior probability can be derived by the Bayes rule:

$$p(\boldsymbol{\omega}|\boldsymbol{\omega}^0) = \frac{p(\boldsymbol{\omega}^0|\boldsymbol{\omega})p(\boldsymbol{\omega})}{p(\boldsymbol{\omega}^0)} \tag{11}$$

We assume that the conditional $p(\boldsymbol{\omega}^0|\boldsymbol{\omega})$ and the prior $p(\boldsymbol{\omega})$ are all Gibbsian.

$$\begin{cases} p(\boldsymbol{\omega}^0|\boldsymbol{\omega}) & \triangleq \frac{1}{Z_1}\exp\{-E(\boldsymbol{\omega}^0|\boldsymbol{\omega})\}, \\ p(\boldsymbol{\omega}) & \triangleq \frac{1}{Z_2}\exp\{-E(\boldsymbol{\omega})\}, \end{cases} \tag{12}$$

where Z_1 and Z_2 are partition functions. Substituting (12) into (10) yields

$$\boldsymbol{\omega}^* = \arg\min_{\omega}\{E(\boldsymbol{\omega}^0|\boldsymbol{\omega}) + E(\boldsymbol{\omega})\}. \tag{13}$$

From now on, we must specify the energy functions $E(\boldsymbol{\omega}^0|\boldsymbol{\omega})$ and $E(\boldsymbol{\omega})$ in (13). Let us first look into the relationship between Ω and Ω^0. In a parameter-observation model, one can assume that $\Omega - \Omega^0$ is a Gaussian noise process and therefore

$$E(\boldsymbol{\omega}^0|\boldsymbol{\omega}) \triangleq \frac{\beta}{2}\sum_{t=1}^{n}\sum_{j=1}^{m}(\omega_{jt} - \omega_{jt}^0)^2, \tag{14}$$

where β is a positive constant.

In order to model the prior of Ω, we require all the constraints involved in this quantity. First, a target must be associated with at most one measurement. If no measurement is assigned for a target,

then it means a missed detection has occurred. This condition can be nicely represented mathematically by assuming that the sum of a column must be less than or equal to 1. Conversely, a false alarm is treated the other way around. The sum of a row must be less than or equal to 1. If the sum is zero, then it means that the measurement is a false alarm. Combining these facts together, we obtain

$$\begin{cases} \sum_{j=1}^{m} \omega_{jt} \leq 1, & \text{for } t \in [1, n] \,, \\ \sum_{t=1}^{n} \omega_{jt} \leq 1, & \text{for } j \in [1, m] \,. \end{cases} \tag{15}$$

These constraints are different from those in previous work [5, 4] in that it deals with equality constraints. The final constraint is trivial:

$$0 \leq \omega_{jt} \leq 1. \tag{16}$$

The constraints (15) and (16) can be integrated into (14) using the three Lagrangian multipliers λ, ϵ, and μ [8] to obtain the Lagrangian $L(\boldsymbol{\omega})$:

$$\begin{aligned} L(\boldsymbol{\omega}) = \frac{\beta}{2} \sum_{t=1}^{n} \sum_{j=1}^{m} (\omega_{jt} - \omega_{jt}^0)^2 - \sum_{t=1}^{n} \sum_{j=1}^{m} \mu_{jt} \omega_{jt} \\ + \sum_{t=1}^{n} \lambda_t (\sum_{j=1}^{m} \omega_{jt} - 1) + \sum_{j=1}^{m} \epsilon_j (\sum_{t=1}^{n} \omega_{jt} - 1). \end{aligned} \tag{17}$$

Here, $\beta > 0$,$\lambda_t \geq 0$, $\epsilon_j \geq 0$, and $\mu_{jt} \geq 0$.

4. FINDING OPTIMAL SOLUTION

In order to solve (17), we must determine both the Lagrangian multipliers λ, ϵ, μ and the minimizer $\boldsymbol{\omega}^*$. The Lagrangian can be represented by the vector equation

$$L(\boldsymbol{\omega}) = E(\boldsymbol{\omega}) + < G(\boldsymbol{\omega}), \mathbf{z}^* > \tag{18}$$

where $G(\boldsymbol{\omega})$ is a constraint vector including (15) and (16), and $\mathbf{z}^*$ is a Lagrange multiplier vector with the same dimension, $n + m + n \times m$, as $G(\boldsymbol{\omega})$. The symbol$<,>$ denotes an inner product. According to the generalized Kuhn-Tucker theorem, there is a $\mathbf{z}_0^* \in \mathbf{Z}^*$, $\mathbf{z}_0^* \geq \theta$ so that the Lagrangian is stationary at ω^* and $< G(\boldsymbol{\omega}), \mathbf{z}_0^* >= 0$. Following this rule, a necessary condition, that the Lagrangian is stationary at $\boldsymbol{\omega}^*$, is easily obtained as

$$\omega_{jt}^* = \frac{\mu_{jt}^* - \lambda_t^* - \epsilon_j^* + \beta \omega_{jt}^0}{\beta}, \tag{19}$$

where ω_{jt}^* is the (j, t) element of the solution $\boldsymbol{\omega}^*$.

The condition $< G(\boldsymbol{\omega}), \mathbf{z}_0^* >= 0$ is equivalent to

$$\begin{aligned} \sum_{t=1}^{n} \lambda_t^* (\sum_{j=1}^{m} \omega_{jt}^* - 1) + \sum_{j=1}^{m} \epsilon_j^* (\sum_{t=1}^{n} \omega_{jt}^* - 1) \\ + \sum_{j=1}^{m} \sum_{t=1}^{n} (-\mu_{jt}^* \omega_{jt}^*) = 0. \end{aligned} \tag{20}$$

Since all terms in this equation are non-positive, they must be all zero. Therefore, this equation can be separated into the following three equations:

$$\begin{cases} \lambda_t^* (\sum_{j=1}^{m} \omega_{jt}^* - 1) = 0, & \text{for } t \in [1, n] \\ \epsilon_j^* (\sum_{t=1}^{n} \omega_{jt}^* - 1) = 0, & \text{for } j \in [1, m] \\ \mu_{jt}^* \omega_{jt}^* = 0, & \text{for } t \in [1, n], j \in [1, m] \end{cases} \tag{21}$$

Combining (19) and (21), we get

$$\begin{cases} \lambda_t^* = \max(0, \frac{-\beta + \sum_{j=1}^{m} (\mu_{jt}^* - \epsilon_j^* + \beta \omega_{jt}^0)}{m}), \\ \epsilon_j^* = \max(0, \frac{-\beta + \sum_{t=1}^{n} (\mu_{jt}^* - \lambda_t^* + \beta \omega_{jt}^0)}{n}), \\ \mu_{jt}^* = \max(0, \lambda_t^* + \epsilon_j^* - \beta \omega_{jt}^0). \end{cases} \tag{22}$$

Unfortunately, (22) must be solved with some iterative scheme. In particular, we use the *Gauss-Seidel method* to increase the stability of convergence.

Algorithm (Iterative Algorithm) *Given $\boldsymbol{\omega}^0$ and β, compute λ, μ, and ϵ for each $1 \leq k \leq n$, $1 \leq j \leq m$, and $l \geq 0$.*

1. *Set the initial values of all Lagrange multipliers to zero.*
2. *Calculate the Lagrange multipliers sequentially.*

$$\begin{aligned} \mu_{jt}^{(l+1)} &= \max(0, \lambda_t^{(l)} + \epsilon_j^{(l)} - \beta \omega_{jt}^0), \\ \lambda_t^{(l+1)} &= \max(0, \frac{-\beta + \sum_{j=1}^{m} (\mu_{jt}^{(l+1)} - \epsilon_j^{(l)} + \beta \omega_{jt}^0)}{m}), \\ \epsilon_j^{(l+1)} &= \max(0, \frac{-\beta + \sum_{t=1}^{n} (\mu_{jt}^{(l+1)} - \lambda_t^{(l+1)} + \beta \omega_{jt}^0)}{n}). \end{aligned}$$

3. *If the norm of the change of the Lagrange multipliers is larger than a threshold, let $k = k + 1$ and return to the step* 2. *Otherwise, go to the step* 4.
4. *Find the minimum point from the Lagrange multipliers,*

$$\omega_{jt} = \frac{\mu_{jt} - \lambda_t - \epsilon_j + \beta \omega_{jt}^0}{\beta}.$$

One can assign any value to β considering the numerical range of a computer. Here we set $\beta = 1$ without loss of generality.

The computational complexity of the single loop of the algorithm is $\mathcal{O}(mn)$. The complete algorithm require $\mathcal{O}(\bar{k}mn)$, here we assume $\bar{k}$ is the average number of iteration. Therefore, even if the number of the tracks and the measurements are increased, the computational requirement does not increase exponentially.

5. EXPERIMENTAL RESULTS

Experiments were conducted with the new algorithm CODA described in **Algorithm** using the measurement model described in (1). The *Nearest Neighbor Filter* (NNF) [1, 7] and the *Probabilistic Data Association* (PDA) [2, 1] techniques were also used for the same model.

As a typical scenario, two crossing targets with initial positions and velocities as listed in Table 1, were considered. The state

Table 1: Initial Positions and Velocities

target	x (km)	y (km)	$\dot{x}$ (km/s)	$\dot{y}(km/s)$
1	-4.0	1.0	0.20	-0.05
2	-4.0	-1.0	0.20	0.05

transition and process noise coupling matrices in (1) were, respectively,

$$\mathbf{F}_t(k) = \begin{pmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \mathbf{G}_t(k) = \begin{pmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{pmatrix}, \tag{23}$$

where T is a sampling interval. The state vector was represented in two dimensional Cartesian coordinates. Without loss of generality, the sampling interval T was set at $1s$. As a noise model, the covariance matrix of the process noise of target t was

$$\mathbf{Q}(k) = \begin{pmatrix} 1.2106 \times 10^{-5} & 0.0 \\ 0.0 & 1.2106 \times 10^{-5} \end{pmatrix} \quad (24)$$

for all k. Similarly the measurement noise covariance matrix was defined as

$$\mathbf{R}(k) = \begin{pmatrix} 0.0225 & 0.0 \\ 0.0 & 0.0225 \end{pmatrix} \quad (25)$$

for all k. Other assumptions were as follows: the probability of detection P_D was 0.7, a set of clutter densities were 0.2, 0.4, and $0.6\ km^{-2}$, the probability of validation P_G was 0.99. Finally, the threshold used for the validation gate was $\gamma = g^2 = 9.2$.

Monte Carlo simulation was performed for $N = 50$ runs. Fig. 3 is one of the many samples that used the clutter density $C = 0.2km^{-2}$. We have shown only the trajectories of CODA

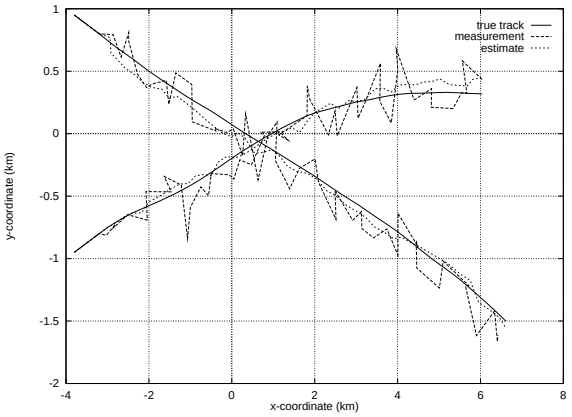

Figure 3: Tracking crossing targets using CODA

because of space limitation. The figure contains three sets of trajectories: actual target trajectories, measurements of target positions, and the traces calculated by CODA. Extensive experimentation showed that CODA generally tends to successfully separate two crossing targets in most clutter cases.

To analyze the performance statistically, the track maintenance rate and RMS position error were observed and are listed in Table 2. There is no unique definition for track maintenance rate. So,

Table 2: The tracking performances based on the crossing targets

Clutter Density $C(/km^2)$	Track Maintenance Rate (%)			RMS Position Error (km)		
	CODA	PDA	NNF	CODA	PDA	NNF
0.2	86	98	86	0.1393	0.1695	0.1592
0.4	88	100	80	0.1755	0.1608	0.2276
0.6	86	100	86	0.2703	0.1776	0.2889

we considered that a track is lost when there was no measurement in the gate of the track for at least the last five sampling times. Track maintenance rate is the percentage of tracks not lost. Our definition for track maintenance concerns radar operator's point of view. RMS position errors is the RMS value of the distances between the actual and the estimated tracks. Although PDA is generally the best, CODA performs comparably.

One of advantages of CODA is that, like NNF, it does not need to know the probability of detection P_D and the clutter density C. However, these parameters of a priori knowledge are mandatory for PDA, and the reason for better performance. Generally CODA displays better performance than NNF, especially in terms of RMS error rate, which is improved by 13.9% on average.

6. CONCLUSION

We have derived a scheme called CODA, that computes the data association by a MAP estimation method and uses a new energy function. Unlike PDA, this algorithm does not need any additional information like the probability of detection and the clutter density. Also our algorithm does not need any parameters like the balance control coefficients used in the neural network approach. All the parameters are recursively updated on-the-fly. As a result, in CODA, there is no need for any trial and error to select values for the necessary coefficients. These properties are important for adapting to unknown environments with good performance and stability.

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