

# SIGNAL COMPRESSION BY PIECEWISE LINEAR NON-INTERPOLATING APPROXIMATION

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## ABSTRACT

In this paper we present a signal compression scheme based on coding linear segments approximating the signal. Although the approach is useful for many types of signals, we focus in this paper on compression of ElectroCardioGram (ECG) signals. ECG signal compression has traditionally been tackled by heuristic approaches. However, it has recently been demonstrated [1] that exact optimization algorithms outclass these heuristic approaches by a wide margin with respect to reconstruction error. The exact optimization algorithm extracts signal samples from the original signal by formulating the sample selection problem as a graph theory problem. Thus known optimization theory can be applied in order to yield optimal compression. This paper generalizes the exact optimization scheme by removing the interpolation restriction when applying piecewise linear approximation. This guarantees a lower reconstruction error with respect to the number of extracted signal samples. The method shows superior performance compared to traditional ECG compression methods.

## 1. INTRODUCTION

Signal compression is an important issue for many types of signals and applications. Due to the large amount of data involved in storage and transmission of digital signals it is an apparent need to compress such signals in order to keep the amount of data in manageable sizes. The compression must be done in a way that makes accurate reconstruction of the signal possible. In this paper we focus on compression of ECG signals although the method described is a general scheme applicable to many kinds of signals.

Coding by time domain methods is based on the idea of extracting a subset of *significant* signal samples to represent the signal. The key to a successful algorithm is a good rule for determining the most significant samples. Decoding is based on interpolation in this set.

ECG signal compression, using approximation by piecewise linear segments, has traditionally been tackled by heuristic approaches in the sample selection process. Examples of such algorithms is the popular FAN algorithm [2], the well known AZTEC algorithm [3] and recently attempts of improvements to time domain algorithms, such as SLOPE [4] and AZTDIS [5]. These algorithms all extract signal samples based on some heuristic criterion and therefore they are all suboptimal. As opposed to these algorithms, the *Cardinality Constrained Shortest Path* (CCSP) algorithm presented in [6] is based on a rigorous mathematical model of the entire sample selection process. By modeling the signal samples as nodes in a graph, optimization theory may be applied in order to achieve the best compression possible under the given circumstances. In [6] the goal is to minimize the reconstruction error given a bound on the number of samples to be extracted. The samples of the original signal are modeled as nodes in a directed graph. Any pair of nodes are connected with an arc, the direction of which is given from the sample order. Including a particular arc in our solution corresponds to letting the end nodes of the arc constitute consecutive samples in the extracted subset of signal samples. The length of each arc in the digraph can be defined in a variety of ways. In [6] the length of the arc connecting two samples  $i$  and  $j$  is defined as the contribution to reproduction error from eliminating all samples between  $i$  and  $j$ . Defining the problem in this way, minimization of the reconstruction error can be recognized as solving the *cardinality constrained shortest path problem* defined on the graph.

Reconstruction of a signal compressed by any of the time domain algorithms mentioned so far is done by linear interpolation between the elements of the extracted subset of signal samples. This is a simple, but computationally effective way of reconstructing the signal. However, by applying interpolation between the extracted signal samples we insist on exact equality between the reconstructed

and the original signal samples at the points of extraction. This imposes restrictions on the algorithm. Removing this restriction will give us a higher degree of freedom in extraction of signal samples, and thus hopefully a better representation of the original signal. In this paper we therefore demonstrate how the algorithm in [6] can be further developed in order to apply to piecewise linear *non-interpolating* approximation.

Applying linear interpolation in the reconstruction phase, the algorithm in [6] is proven to converge in cubic time. In [7] it is demonstrated how to cope with real time constraints of the algorithm. In this paper it is shown that the idea presented in [6], and roughly sketched here, can be applied to the case where piecewise non-interpolative approximation is used in reconstruction of the signal. This is obtained without increasing the computational complexity of the algorithm.

In the next section the problem is defined in strict mathematical terms. Section 3 is devoted to a discussion of encoding schemes for the extracted sample points. Finally, experimental results are reported, and in the concluding section different aspects of the method along with directions for future work are considered.

## 2. OPTIMIZATION MODEL

Denote the samples taken from an ECG signal at constant interval by  $y(1), y(2), \dots, y(N)$ . Let  $M$  denote the bound on the number of extracted samples and  $S$  denote the *sample set*  $S = \{y(1), y(2), \dots, y(N)\}$ . Let the starting points of the straight lines representing the approximated signal be denoted by  $z(k)$ . In general  $z(k) \neq y(k)$ . We seek an appropriate *compression set*  $C = \{n_1, \dots, n_M\}$  and the corresponding approximated sample values  $z(n_1), \dots, z(n_M)$  to represent  $S$ . Assume  $n_1 = 1$  and  $n_M = N$ . The approximation is then given by  $\hat{y}(n) = z(i) + \frac{\hat{y}(j) - z(i)}{j - i}(n - i)$  for all  $n \in [i, j)$  if  $j < N$ ,  $n \in [i, j]$  if  $j = N$ . In this way we get a piecewise linear non-interpolating approximation to the original signal starting in  $n_1 = 1$  and ending in  $n_M = N$ .

Define the directed graph  $G = (V, A)$  whose *vertex set*  $V = \{1, 2, \dots, N\}$  and *arc set*  $A$  contains node pairs  $(i, j)$  where  $i, j \in V$  and  $i < j$ . The set  $(n_1, n_2, \dots, n_M)$  is said to be a *path* from  $n_1$  to  $n_M$  in  $G$  if  $n_1, \dots, n_M \in V$  are distinct vertices and  $n_1 < n_2 < \dots < n_M$ . Let  $P_n$  denote the path from node 1 up to node  $n$ . The length of each arc  $(i, j)$  in  $A$  is given as the contribution to the total reconstruction error by eliminating all nodes between  $i$  and  $j - 1$  if  $j < N$  and between  $i$  and  $j$  if  $j = N$ . This length can be expressed as  $c_{ij}^2 = \sum_{n=i}^{j-1} (\hat{y}(n) - y(n))^2$  if  $j < N$  and  $c_{ij}^2 = \sum_{n=i}^j (\hat{y}(n) - y(n))^2$  if  $j = N$ . The length of  $P_n$  will thus be the sum of the length of all arcs included in the path up to node  $n$ . Each arc  $(i, j)$  in  $A$  represents the possibility of letting  $i$  and  $j$  be consecutive members of  $C$ .

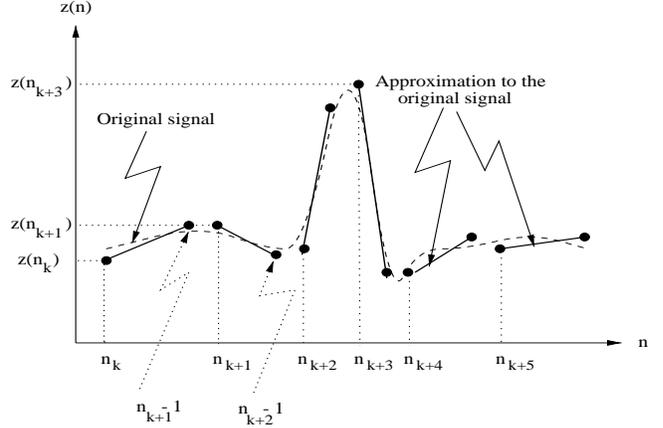


Figure 1: Example of a short sequence of original and reconstructed signal

The parameters describing each straight line segment of the approximation to the original signal is found by minimizing the expression for  $c_{ij}^2$  with respect to these parameters. The graph  $G$  consists of  $\frac{N(N-1)}{2}$  arcs. The expression for  $c_{ij}^2$  is a sum of  $n$  terms in the worst case. Straightforward computation of all the arc lengths will thus result in an algorithm with a complexity of  $O(N^3)$ . Fortunately, this can be avoided by careful computation of the arc lengths [8].

When all arc lengths are available, we are faced with the following problem: Minimize the length of  $P_N$  under the constraint that  $P_N$  contains no more than  $M$  vertices. This problem is solved by a dynamic programming algorithm thoroughly described in [6].

## 3. ENCODING SCHEME

In general we need three parameters for each segment of the signal in order to describe the piecewise linear non-interpolation approximation uniquely. The exact optimization algorithm described in Section 2, represents each linear segment of the signal between nodes  $n_k$  and  $n_{k+1}$  by the approximated sample values  $z(n_k)$  and  $z(n_{k+1} - 1)$  in addition to the run length,  $r(n_k) = n_{k+1} - n_k$ . This is shown in Figure 1. Before entropy coding, we change the representation of the line segments slightly. The approximated sample values representing the end of one line segment and the start of the next one is replaced by its mean, i.e.,  $z_{mid}(n_{k+1}) = \frac{z(n_{k+1} - 1) + z(n_{k+1})}{2}$ , and the distance between the mean and the approximated sample values, i.e.,  $\delta(n_{k+1}) = z(n_{k+1} - 1) - z_{mid}(n_{k+1}) = z_{mid}(n_{k+1}) - z(n_{k+1})$ . This will lower the dynamic range of the parameters to be encoded as compared to encoding all the approximated sample values directly. We thus have three parameters to be encoded for each segment of the signal:  $r(n_k)$ ,  $z_{mid}(n_k)$  and  $\delta(n_k)$ . We apply separate entropy coders for each of these parameters.

The FAN algorithm [2] is a well known time domain compression method. In comparison with other time domain coders, it has been reported to give high compression ratio, in addition to producing reconstructed signals with high fidelity [9]. We compare the results from the FAN algorithm to the linear non-interpolating exact optimization techniques described in the preceding pages. We also compare the results from the method described in this paper to the linear interpolating exact optimization algorithm described in [1, 6]. Both the linear interpolation exact optimization method and the FAN algorithm approximates the signal by linear line segments represented by one extracted sample value and one run per segment of the signal. These parameters are encoded by two separate entropy coders.

#### 4. NUMERICAL EXPERIMENTS

For evaluation of the performance of the coders described in Section 3, the commonly used *Percentage Root Mean Difference* is applied:

$$PRD = \sqrt{\frac{\sum_{l=1}^N [y(l) - \hat{y}(l)]^2}{\sum_{l=1}^N [y(l) - \bar{y}]^2}} \times 100\% \quad (1)$$

where  $\bar{y}$  is the mean value of the original signal  $y$ ,  $\hat{y}$  is the reconstructed signal and  $N$  is the original signal length. We evaluate the PRD as a function of *bit rate = average number of bits used to represent one signal sample* in the original signal. The PRD performance measure is useful for testing the relative performance of the coding techniques. However, as each compression method has its own distortion characteristic, the PRD should be supplemented by visual inspection of the reconstructed signal.

Several recordings taken from the MIT [10] database were applied in the coding experiments. We present results for two test signals: mit100\_1000 and mit207\_1800. Here “mitxxx\_yyyy” denotes record number xxx starting at time yy:yy. Each total record time is ten minutes, corresponding to 216 000 samples. The sampling frequency is 360 Hz with 12 bits per sample. The signal mit100\_1000 is normal sinus rhythm while mit207\_1800 is abnormal ventricular rhythm.

Figure 2 presents obtained PRD’s for the coders for bit rates between 0.2 and 1.8 bits per sample (bps). For both test signals, the FAN method is outperformed by both the CCSP methods by a wide margin. At low bit rates (around 0.6 bps) the FAN algorithm has from 40 % to 130 % higher PRD than the CCSP algorithm based on piecewise linear non-interpolating approximations. At higher rates (around 1.0 bps) the difference is smaller, but still significant.

For test signal mit100\_1000, the CCSP method based on linear interpolation gives slightly lower PRD’s than the non-interpolating approach for bit rates below 0.3 bps. For

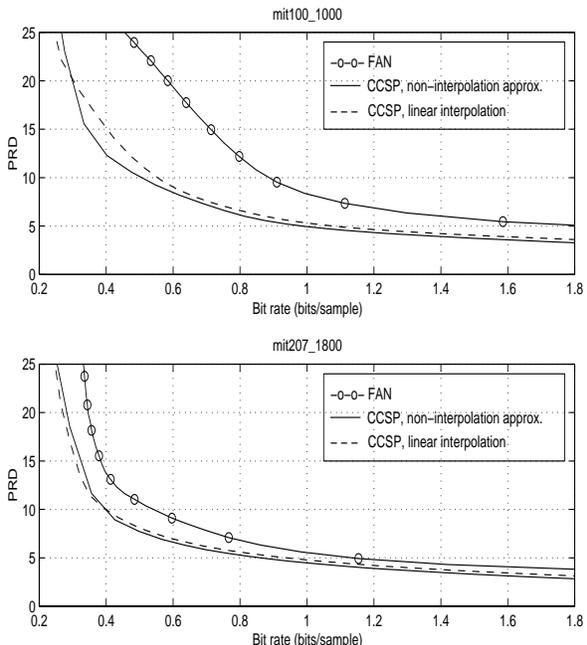


Figure 2: Bit rate versus PRD for test signal mit100\_1000 and test signal mit207\_1800

bit rates between 0.3 and 0.5 bps, the non-interpolating approach performs significantly better than the interpolation approach. For all other bit rates the linear non-interpolating approximation method performs slightly better than the linear interpolation approach in terms of PRD. For test signal mit207\_1800 the CCSP method based on linear interpolation performs slightly better than the non-interpolation approach for bit rates below 0.4 bps. For all other bit rates it is the other way around.

Side information has not been taken into account in the rate-distortion curves of Figure 2. When applying the encoding technique chosen here, we assume that we are either using fixed tables in the encoding and decoding part of the system or coding on a block-adaptive basis.

Evaluation of the performance of the different coders should be accompanied by visual inspection of the reconstructed signals. This is to show coding artifacts as they appear for the different coders. We have chosen a short segment of the mit100\_1000 signal. The reconstructed signal is shown at a bit rate of 1.0 bps in Figure 3. The original signal is also included. We see that all coders smooth out some of the details in the original signal. This is particularly evident for the FAN coder, where the linear line pieces are also most prominent in the reconstructed signal. Between the coders evaluated here, the CCSP coder using linear non-interpolating approximations produces a reconstructed signal with highest fidelity based on visual inspection.

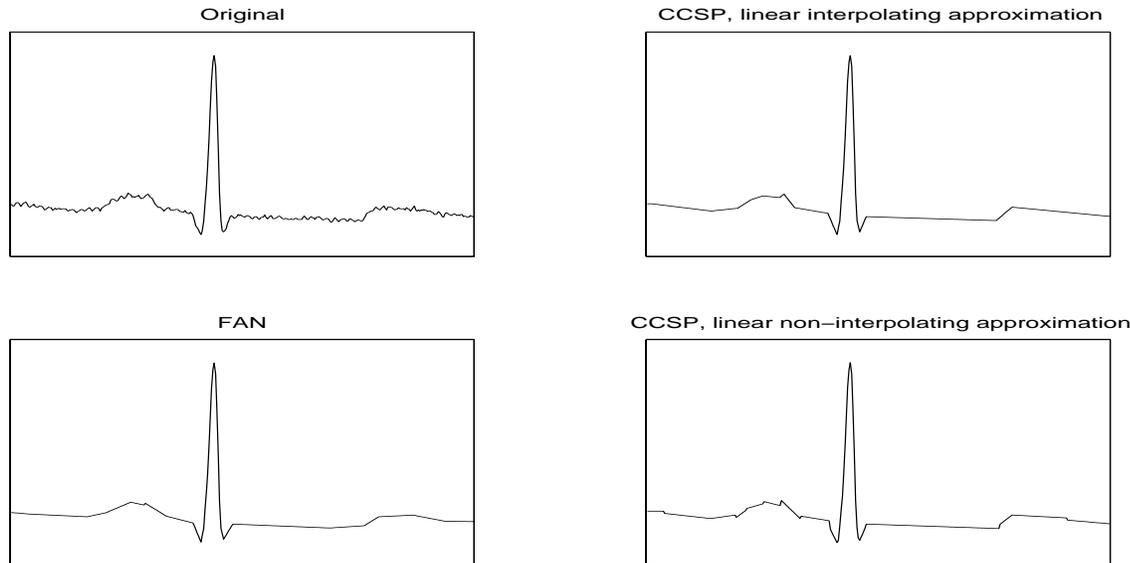


Figure 3: Reconstructed signal segment (taken from mit100\_1000) at 1.0 bps.

## 5. CONCLUSIONS

This paper demonstrates how the optimal time-domain coder presented in [6, 8] can be further developed in order to use piecewise non-interpolating approximation in the reconstruction of the signal. This is done without increase in the computational complexity of the algorithm.

Our compression algorithm is based on combinatorial optimization theory. By the very nature of our approach the distortion is guaranteed to be the smallest possible of all techniques using piecewise linear approximation, given the number of retained curve points.

Coding experiments show that the optimal time domain coders have significantly higher performance than other well known time domain compression methods. This is validated by evaluation of the coders based on PRD and verified by visual inspection of the reconstructed signal. The CCSP algorithm based on piecewise non-interpolating approximations has slightly higher performance than the CCSP algorithm based on linear interpolation for bit rates above 0.3 - 0.4 bps.

In this paper we have considered piecewise linear non-interpolating approximation in compression of ECG signals. The results show that there is a potential for such algorithms. In [8] second order polynomials are applied in reconstruction of the signal. Incorporation of second order polynomials into the non-interpolating approximation approach may result in algorithms with even better results than the ones obtained by the exact optimization algorithms so far. Development of such algorithms is left as a topic for future research.

## 6. REFERENCES

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