A PRACTICAL METHOD TO REDUCE A NUMBER OF REFERENCE SIGNALS FOR THE ANC SYSTEM

Masaichi Akiho, Miki Haseyama** and Hideo Kitajima***

* Alpine Electronics, Inc.

20-1 Yoshima Kogyo-danchi, Iwaki, Fukushima 970-1192 Japan

** Faculty of Engineering, Hokkaido University

Kita 13, Nishi 8, Kita-ku, Sapporo, Hokkaido 060-0812 Japan

ABSTRACT

In this paper, we propose a practical method to reduce a number of reference signals for the active noise cancellation (ANC) system and the filter characteristics to generate the reduced number of reference signals, which maintain the original value of the coherence function. This method finds the number of independent noise sources and provides the filter characteristics based on SVD (singular value decomposition) of the power spectrum matrix of the reference signals. Then, we also use the multiple coherence function analysis to select dominant components in the reference signals. The method contributes greatly in reducing the number of reference signals for the ANC system that uses the large number of reference signals. We also discuss the characteristics of the filters that synthesis the new set of reference signals. And an experimental test is performed to confirm the theory.

1. INTRODUCTION

Active Noise Cancellation technology is one of the solutions to improve noisy vehicle cabin environments. When we implement an LMS (Least Mean Square) based active road noise cancellation system, we need to find the exact noise sources to detect the reference signals. It is well known that road noise is mainly caused by the vibrations of the suspension mechanism which stimulate the body shell of the vehicle. Therefore in most of cases, the significant components of the road noise will be the combination of the resonance noise of the vehicle body and the structure borne noise. If we have prior information about the direction of the stimulating force at the exact mounting point of the suspension to the vehicle body, and it is time invariant, a single-axis accelerometer is sufficient to detect the stimulating force for each mounting point. But in general, the direction of stimulating force is varied by the driving condition. Therefore, a tri-axis accelerometer will be installed, and the reference signals may contain information about fewer numbers of noise sources compared to the number of accelerometer outputs. This is one of the major reasons why we have a greater number of reference signals than the number of noise sources. For the LMS based ANC system implementation, it is necessary to have an individual adaptive signal processing unit for each reference signal. Hence, the number of the reference signals will significantly affect to the system configuration.

2. REFERENCE SIGNALS FOR ANC SYSTEM

Figure 1 shows the noise power spectrum in a vehicle cabin, which is stimulated by the vibration of the suspension mechanism on the chassis dynamometers (CH/DY). We placed the test vehicle on a simulated brick surface roller CH/DY to shake only one wheel (rear left wheel) for this particular measurement. The cabin noise has a relatively broad band spectrum that is similar to colored random noise. This noise is commonly known as the road noise in vehicle cabin caused by the vibration of the suspension mechanism.

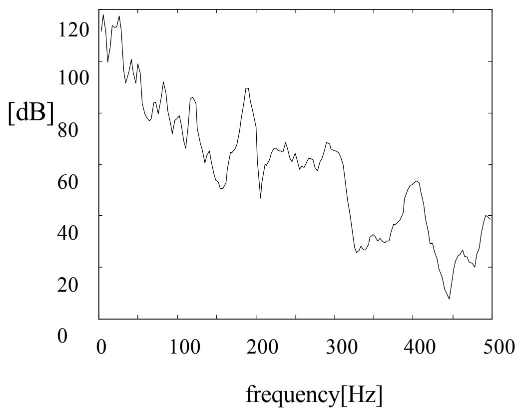

Figure 1 A road noise in a vehicle cabin.

For the ANC system, we need to find the proper reference signals to achieve the optimal performance of the adaptive cancellation. The reference signals are detected by accelerometers placed on several points of the vehicle suspension. To evaluate the quality of the reference signals, the multiple coherence function γ^2 is used. For maximum noise reduction performance of the active noise cancellation system, which is using adaptive filter technique, the performance can be estimated as follow [1]:

$$S_{yy} = S_{xx} (1 - \gamma^2), \quad (1)$$

where S_{yy} is the power spectral density of the noise with optimal control of the ANC system and S_{xx} is for the original power spectral density of the noise. In general, more reference signals will provide the better value of the multiple coherence function. But on the other hand, the number of reference signals will require the same number of adaptive digital filter units. Therefore more reference signals will make the system larger. From this standing point, a smaller number of reference signals will reduce the size of the total system and will be cost effective.

For the practical application, it will be acceptable to have good coherent reference signals only for the specific frequency bands that are the significant noise frequency bands. For example, let us assume the range of 50Hz ~ 500Hz and focus on several frequency components for the road noise cancellation in figure 1. The multiple coherence function of figure 2 indicates for the three output signals. And those three outputs are practically sufficient to use as the reference signals for the active road noise cancellation system. We also use these reference signals as simulation data for the further discussion.

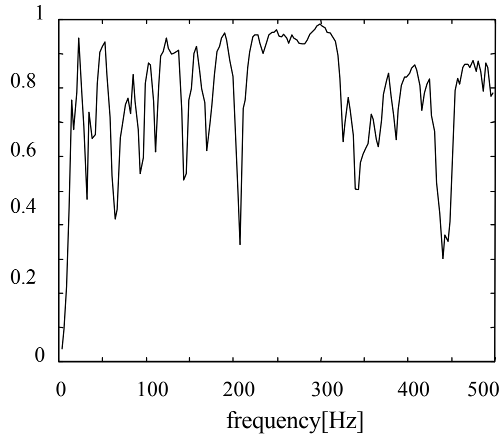

Figure 2 A multiple coherence function

3. VIRTUAL NOISE SOURCE THEORY

The virtual noise source estimation technology will be provided to estimate the orthogonal reference signals by the following set of calculations. The power spectrum density matrix S_{nn} for inputs of multiple inputs - multiple outputs system is as follows:

$$S_{nn} = \begin{bmatrix} S_{11} & S_{12} & \bullet & S_{1n} \\ S_{21} & S_{22} & \bullet & S_{2n} \\ \bullet & \bullet & \bullet & \bullet \\ S_{n1} & S_{n2} & \bullet & S_{nn} \end{bmatrix} \quad (2)$$

where S_{11} is the auto power spectrum of the first input signal X_1. S_{ij} is the cross power spectrum between ith input X_i and jth input signal X_j . Therefore we get the equation, $S_{ij} = S_{ji}^*$, where * denotes the complex conjugate. We then get the power spectra density matrix of output signals N_{nn} as:

$$N_{nn} = \begin{bmatrix} N_{11} & N_{12} & \bullet & N_{1n} \\ N_{21} & N_{22} & \bullet & N_{2n} \\ \bullet & \bullet & \bullet & \bullet \\ N_{n1} & N_{n2} & \bullet & N_{nn} \end{bmatrix} \quad (3)$$

N_{ij} is the cross power spectrum between ith output Y_i and jth output signal Y_j. Let the frequency response function (FRF) matrix be H_{nn}, where H_{ij} is the frequency response function from ith input x_i to jth output Y_j. That size is $n \times n$ also. Using the matrix H_{nn}, N_{nn} can be expressed as equation (4) with variable (f) as frequency:

$$N_{nn}(f) = H_{nn}(f)\, S_{nn}(f)\, H_{nn}^H(f) . \quad (4)$$

Superscript H expresses the Hermitian transpose of the matrix.

We notice that the matrices S_{nn} and N_{nn} are the Hermitian matrices. Therefore, those two matrices could be diagonalized to the $n \times n$ diagonal matrices by the proper unitary matrices [2] regardless of the ranks of both matrices. Our objective is the proposition of the method to reduce the number of the reference signals. Assuming the S_{nn} has the rank of m (m□n). Hence, we apply the Singular Value Decomposition (SVD) [2] to the S_{nn} matrix. Making a new matrix U_{nn} which columns are the eigenvectors of $S_{nn}S_{nn}^H$, and making a matrix V_{nn} which columns are the eigenvectors of $S_{nn}^H S_{nn}$. Both U_{nn} and V_{nn} are unitary matrices, S_{nn} can be rewritten as follows:

$$S_{nn} = U_{nn}\, \Sigma\, V_{nn}^H . \quad (5)$$

Σ is the $n \times n$ diagonal matrix, which has the same number of positive and real diagonal elements as the rank of S_{nn}. S_{nn} is the matrix, which is formed by the auto and cross power spectral density of the n signals. Therefore it is clear that S_{nn} equals S_{nn}^H. This condition also makes $S_{nn} S_{nn}^H$ and $S_{nn}^H S_{nn}$ equal. Hence, U_{nn} equals V_{nn}. Rewriting (5), we will get a new equation for Σ as follows:

$$\Sigma = U_{nn}^H\, S_{nn}\, (U_{nn}^H)^H . \quad (6)$$

We note that Σ is the diagonal matrix as:

$$\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_m, 0, \ldots, 0) \,. \quad (7)$$

The σ's are ordered as $\sigma_1 \geq \sigma_2 \geq \cdots \sigma_m > 0$ ($m \leq n$) [3]. Hence, comparing the equations (4) and (6), we can assume $\boldsymbol{U}_{nn}^{H}$ as the frequency response function matrix. Then, the equation (6) gives the power spectrum matrix of the output signals, which is the diagonal matrix. Realizing the filters, which represent the $U,_{nn}^{H}$ will provide a new set of signals, which are independent together (: virtual noise sources). From this set of calculations, it is clear that the number of non zero diagonal elements of Σ is identical to the number of independent noise sources. And those virtual noise source signals can be fed to the adaptive processing units as the reference signals.

4. REDUCING THE NUMBER OF THE REFERENCE SIGNALS

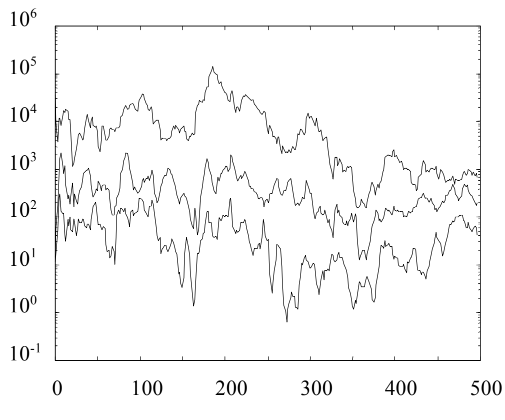

Figure 3 Singular values of $\boldsymbol{S}_{nn}$

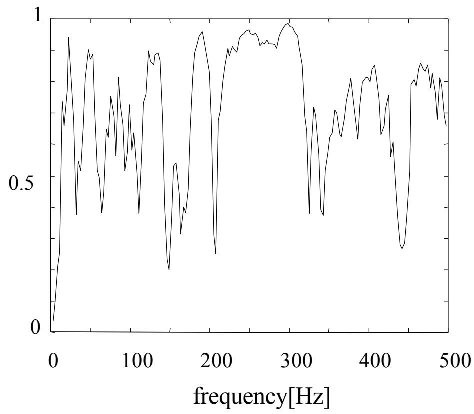

Figure 4 A multiple coherence for 2 virtual noise sources.

To reduce the number of reference signal, we should define the number of the dominant independent noise sources from the independent noise sources. In chapter 3, we proposed a method to provide the frequency response functions which will reduce the number of reference signals, when the power spectrum matrix for those reference signals has the rank of m ($m < n$). Here, we discuss the method to define the virtual reference signals, which perform as the dominant noise sources. For this particular purpose, the multiple coherence function is also used. Figure 3 shows the singular values of the power spectrum matrix $\boldsymbol{S}_{nn}$ for the three reference signals, which are the same as in Chapter 2. According to the equation (6), we can assume that figure 3 is the auto power spectrums of the individual noise sources. By selecting the two of highest powered signal components, we have new multiple coherence function, which is shown in figure 4. Comparing figure 2 and figure 4, we have very slight loss of the coherency by selecting two of the highest dominant virtual noise sources. In this case, we can say that we have two significant noise sources in the three accelerometer outputs. This means it is possible to converge these three signals into two virtual noise sources. Those virtual noise sources can be synthesized and expressed by the following equation:

$$Y'i \quad \sum_{t}^{n} Uit * Xt \,, \qquad (8)$$

where Y'_i is the ith virtual reference signal, and X_t is the tth original reference signal respectively. For the m virtual reference signals calculation, i will be $i = 1, 2, \bullet \bullet \bullet, m$.

5. FILTER CHARACTERISTICS FOR THE VIRTUAL SOURCES

Our objective in this paper is to optimize the virtual reference signals that can be substituted for the original reference signals for the ANC system. For this particular purpose, we need to realize the fixed coefficients digital filters, which represent the $\boldsymbol{U}_{nn}^{H}$ matrix of the SVD. Generally, the frequency response functions of $\boldsymbol{U}_{nn}^{H}$ are not guaranteed to be minimum phase conditions, hence we should assume that they are nonminimum phase conditions. For the virtual reference signal synthesis, we should consider minimizing the delay for the signal generation to maintain the causality of the control and the capability of the process. Standing on this issue, we discuss the effects of the minimum phase filters, which are substituting the nonminimum phase filters for the virtual reference signals synthesis.

We are assuming that all the elements of $\boldsymbol{U}_{nn}^{H} = \boldsymbol{Q}_{nn}$ is the nonminimum phase condition. Let the each element of the $\boldsymbol{Q}_{nn}$ factorize into a minimum phase part $(Q_{kl})_{min}$ and an allpass part $(Q_{kl})_{all}$ [4].

$$Q_{kl} = (\, Q_{kl} \,)_{min} \, (\, Q_{kl} \,)_{all} \,, \qquad (9)$$

where

$$(\, Q_{kl} \,)_{min} = |\, Q_{kl} \,| \exp(j\phi_{mkl})$$

$(Q_{kl})_{all} = \exp(j\phi_{a\,kl})$.

The ith diagonal element σ_i of the matrix Σ will be written as:

$$\sigma_i = \sum_r^n \sum_s^n Q_{ir}^* Q_{is} S_{sr} \quad . \qquad (10)$$

For the case of n=2, the σ_1 will be:

$$\sigma_1 = |Q_{11}|^2 S_{11} + |Q_{12}|^2 S_{22} + C_1 , \qquad (11)$$

where C_1 is:

$$C_1 = |Q_{11}||Q_{12}||S_{12}|\{\exp j\Phi + \exp(-j\Phi)\}, \qquad (12)$$

$$\Phi = \phi m11 + \phi a11 - \phi m12 - \phi a12 + \alpha_{12},$$

where α_{12} is an angle of the S_{12}.

Therefore, C_1 is always real. The first and second terms of the equation (11) show that we have no difference, even if the minimum phase filters substitute the nonminimum phase filters for these terms. The minimum phase filter will affect only to the third term of the equation (11). According to the equation (12), it is obvious that the term performs to subtract the overlapped components. The overlapped components will be identical for both the minimum phase and nonminimum phase conditions as long as they have the same gain characteristics. Hence, the C will be unchanged, even if we use the minimum phase filters instead of the nonminimum phase filters. Hence, the minimum phase filters are sufficient to synthesis the virtual reference signals.

6. EXPERIMENTAL TEST

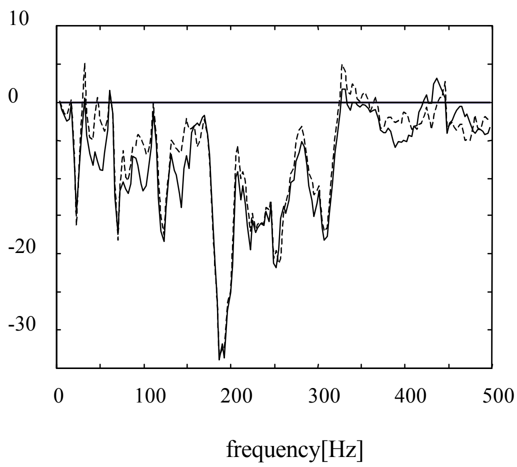

Figure 5 An ANC performance

For the experimental test, we used Yule-walker function of software package MATLAB (The Math-Works, Inc.) that is based on a Modified Yule -Walker Equations (MYW) method [5] to design the digital filter coefficients from the frequency response function. For this experimental test, we used the same time series data, which had been acquired on CH/DY for the multiple coherence evaluation at figure 2. Three accelerometers outputs were fed to a total of 6 ARMA fixed coefficients digital filters to generate two virtual reference signals. The ARMA direct form filters of 50th order of the AR model and 50th order for the MA model calculate a new set of time series data. Figures 5 indicates the cancellation performance of the filtered - X LMS ANC system after 75000 iterations for both three reference signals and only two virtual reference signals. A solid line indicates the original three reference system performance and a shaded line indicates the performance of the proposed method. We observe a very slight degradation of canceling performance in figure. That is predicted by the evaluation of the multiple coherence function in chapter 4.

7. CONCLUSIONS

Several conclusions seem to be apparent from our experiment. Firstly, the virtual reference signal technology can be applied to the active noise and vibration cancellation system to reduce the number of reference signals, when the reference signals provide information about the fewer number of the noise sources than the number of reference signals. Secondly, fixed coefficients digital filters with minimum phase conditions will be suitable to converge the reference signals into the virtual reference signals. In our experiment, we added a total of 6 ARMA digital filters to converge 3 reference signals to 2 virtual reference signals. It may increase the size of the system configuration, but the virtual reference technology proves the possibility to reduce the number of the reference signals for any ANC system when we have a large number of reference signals to expect sufficient performance of the cancellation.

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