# A NON STATIONARY RLS ALGORITHM FOR ADAPTIVE TRACKING OF MARKOV TIME VARYING CHANNEL

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## Abstract

In this paper we propose a new adaptive algorithm designed to track system presented by a filter that has markovian time evolution. As the Non Stationary LMS (NSLMS) algorithm [1], the Non Stationary RLS (NSRLS) algorithm performs better than the LMS and is able to identify the unknown order and parameters of the markov model. However in the case of the NSRLS algorithm, the convergence speed of the markovian parameter is very high compared to that of the NSLMS algorithm. Moreover, the NSRLS algorithm has a better tracking capacity than the NSLMS, especially when the filter poles that characterize time variations of the channel are close to the unit circle.

## 1 Introduction

Basically the LMS algorithm is designed to recursively estimate the value of a fixed unknown filter. However, in a non stationary context this algorithm has interesting tracking capacity. This steady state property has been extensively analyzed in the literature for random walk variations (see for example [2]).
In fact, the adaptive identification offered by the LMS is blind in regard to the nature of the time evolution model of the channel.
In order to guarantee better results than those realized by the classical LMS, we proposed respectively in [1] and [3] the Non Stationary LMS algorithm (NSLMS), and the Finite Memory NSLMS (FM-NSLMS) algorithm. They are designed for a markovian non stationary context. The NSLMS and the FM-NSLMS can identify the markovian time evolution of the real filter, encountered in transmission systems.
The memorization capabilities built into the FM-NSLMS algorithm, enhances its performances in some particular situations. For instance, in the case where the filter poles that characterize time variations of the channel are close to unit circle.

The major inconvenience of these algorithms is the lower convergence speed of the adaptive markovian parameters to their real values. Hence, they can not be used in some particular applications. For instance, in the case of radio-mobile channel equalization.
To overcome this important problem, we propose the NSRLS algorithm witch is based on the theory of the RLS algorithm known for its good convergence proprieties.
This algorithm is superior to the NSLMS algorithm. Indeed, it presents a better tracking capacity than the NSLMS. In addition, the convergence speed of the adaptive markovian parameter is much higher.

Contrarily to the Kalman approaches, the proposed algorithm does not require a prior knowledge of the non stationarity structure and the unknown statistics of the observation noise and the filter noise ([4], [5]).

The paper is organized as follows. In section 2, we present the adaptive problem and some background on the NSLMS. Section 3 describes the design of the NSRLS. In section 4, we study the performance of the proposed algorithm and compare it to the NSLMS.

## 2 Background: NSLMS algorithm and markovian non stationary context

### 2.1 Non stationary context

Here, we are interested in the adaptive identification of markovian time-varying channel. The classical formulation of such filtering problem is depicted in Figure (1). The

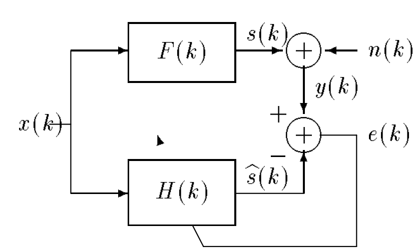


Figure 1: Adaptive identification of time-varying channel

noisy input/output equation of the channel is,

$$y_k = F_k^T X_k + n_k \tag{1}$$

where, $X_k = (x_k, x_{k-1}, \cdots, x_{k-N+1})^T$ is the known stationary input vector and $n_k$ is an unknown i.i.d. observation noise. The filter parameter vector is assumed to be $P$-order markov time-varying,

$$F_k = \sum_{i=1}^{P} a_i F_{k-i} + \Omega_k \tag{2}$$

where the $(a_i)_{i=1,P}$ ensure the stability of the channel, and $\Omega_k$ called non stationary noise, is an unknown zero−mean, i.i.d. process independent of $\{X_k\}$ and $\{n_k\}$.

This general model represents different types of non stationarity such as variations of mobile transmission channels or underwater acoustic channels.

The evolution of the parameter vector $H_k$ of the adaptive filter is governed by the estimate error, $e_k = y_k - H_k^T X_k$, in order to minimize a criterion, such as the mean square error $E(e_k^2)$, for the LMS.
The tracking capacity of the adaptive algorithm is measured by the normalized misadjustement, $M = \lim_{k \to \infty} (E(e_k^2) - P_n)/P_n$, where $P_n$ is the power of $n_k$.
It is important to note that the LMS algorithm is not designed to track time variations of a channel whereas both the NSLMS and the proposed NSRLS are tailored approaches for tracking stochastic non stationarities.

## 2.2 The Non Stationary LMS algorithm

The Non Stationary (NSLMS) algorithm is designed in a way to take into account the prior knowledge of the non stationarity model, (2). We keep the structure of the classical LMS, and include the constraints on the nature of the non stationarity. Without loss of generality, we consider a first order markovian non stationarity,

$$F_k = aF_{k-1} + \Omega_k \tag{3}$$

Therefore, the time evolution of $H_k$) is as follows:

$$H_{k+1} = \widehat{a} H_k + \mu X_k e_k \tag{4}$$

where $\widehat{a}$ is adaptive estimate of the unknown markov model parameter.
The adaptive estimation of the parameter $\widehat{a}$ is also made by minimizing $E(e_k^2)$. The true gradient of $e_k^2$ is given by,

$$\frac{de_k^2}{d\widehat{a}}|_{\widehat{a}=a(k)} = 2\left[e_k\left(H_{k-1}^T X_k + \widehat{a}\frac{dH_{k-1}}{d\widehat{a}}\right)\right]_{\widehat{a}=a(k)} \tag{5}$$

This complexity is due to the recursive nature of the markovian structure 3.
The classical approximation of the true gradient considered in the extended LMS ([6]) is used in order to update recursively the parameter $a$ of the algorithm. Therefore the NSLMS algorithm is described by,

$$\begin{aligned} e_k &= y_k - H_{k-1}^T X_k && (6)\\ a_{k+1} &= a_k + \mu_1 (H_{k-2}^T X_k) e_k && (7)\\ H_{k+1} &= a_{k+1} H_k + \mu X_k e_k && (8) \end{aligned}$$

where $\mu_1 > 0$ are (small) step sizes that control the adaptive identification of $a$. Even if the step size $(\mu_1) > 0$ is large, the convergence of the parameter to its true value may take a considerable time. An adaptation by the RLS algorithm, as proposed in this paper, will overcome this problem.

In [1] [3] we show by a new formulation, the recursive aspect of the problem. Such aspect introduces a memory related to the update of $H_k$. However, the use of the RLS algorithm to update $H_k$, will help to take into account the recursivity of the problem without introducing some memorization capability as in the case of the FM-NSLMS.

Note that in [3], we exhibit the tracking superiority of both the NSLMS and the FM-NSLMS over the LMS. However we pointed out some bias in parameter estimation.

# 3 Design of the Non Stationary RLS algorithm

The Non Stationary RLS algorithm is designed in such a way that takes into account the prior knowledge of the structure of non stationary (3). As in the NSLMS, we keep the classical RLS algorithm (with forgetting factor) described by,

$$\begin{aligned} e_{k+1} &= y_{k+1} - H_k^T X_{k+1} && (9)\\ H_{k+1} &= H_k + G_{k+1} e_{k+1} X_{k+1} && (10)\\ G_{k+1} &= \frac{1}{\lambda} G_k \left( I_N - \frac{X_{k+1} X_{k+1}^T G_k}{\lambda + X_{k+1}^T G_k X_{k+1}} \right) && (11) \end{aligned}$$

and we include the constraints on the nature of the non stationarity as follows:

$$H_{k+1} = \widehat{a} H_k + G_{k+1} e_{k+1} X_{k+1}$$

As noted previously, we decide to use the RLS algorithm to identify the filter $F_k$, since the RLS algorithm has better convergence speed than the LMS algorithm, and gives a better performance when the filter is recursive (3).

To accelerate the convergence of the adaptive parameter, we propose to adjust its time evolution by minimizing the least square error.

Considering the error given by (6), a classical approximation value of the true gradient of $J_k = \left(\sum_{i=1}^{k} e_i^2\right)$ is given by,

$$\frac{dJ_k}{d\widehat{a}} = -2\sum_{i=1}^{k}\left(y_i - X_i^T (G_{i-1} e_{i-1}) X_{i-2}\right) H_{i-2}^T X_i$$

$$+2\sum_{i=1}^{k}\widehat{a}\left(H_{i-2}^{T}X_i\right)^2 \quad (12)$$

Therefore, the NSRLS algorithm is described by,

$$\begin{aligned}
e_k &= y_k - H_{k-1}^T X_k && (13)\\
m_{k+1} &= m_k + \left(y_{k+1} - \mu e_{k-1} X_{k-1}^T X_{k+1}\right) H_{k-1}^T X_{k+1} &&\\
d_{k+1} &= d_k + \left(H_{k-1}^T X_{k+1}\right)^2 &&\\
a_{k+1} &= \frac{m_{k+1}}{d_{k+1}} && (14)\\
H_{k+1} &= a_{k+1} H_k + G_{k+1} e_{k+1} X_{k+1} && (15)\\
G_{k+1} &= \frac{1}{\lambda} G_k \left(I_N - \frac{X_{k+1} X_{k+1}^T G_k}{\lambda + X_{k+1}^T G_k X_{k+1}}\right) && (16)
\end{aligned}$$

These equations can be easily generalized to a $P$ order markovian non stationarity. It is important to notice that for the NSRLS, we must optimize only the forgetting factor $\lambda$; while for the NSLMS, we have to optimize the two step sizes, $\mu$ and $\mu_1$.

# 4 Steady state performance : Superiority of the NSRLS over NSLMS

The performance analyses of the NSRLS are conducted by simulations. We consider an i.i.d. input with power $P_x = 1$, and a correlated input generated by a first order AR model,

$$x_k = \alpha x_{k-1} + b_k$$

where $b_k$ is an i.i.d. noise and $\alpha = 0.8$. The power of the non stationary noise $P_\omega$ is fixed through $\delta = \frac{N P_x P_\omega}{P_n}$. For all the presented results, we use $\delta = 1$, and $N = 3$.

- **An i.i.d. input.**

In this case, we compare the two algorithms when $a = 0.9$ and $a = 0.5$. As expected, the NSRLS has a better tracking capacity than the NSLMS. This important result is illustrated in Figure (2) showing the graph of the misadjustement $M$ versus the forgetting rate $\mu P_x = 1 - \lambda$. For the NSLMS algorithm, the step size $\mu_1 = 0.001$ for $a = 0.9$ and $a = 0.5$. As expected, Figure  shows that when $a$ is closer to 1, the superiority of the NSRLS over the NSLMS is much more pronounced.

- **A correlated input.**

As indicated in Figure (3), the NSRLS presents a better tracking capacity than the NSLMS. The graphs shown in Figure (3) correspond to $a = 0.9$, and $\mu_1 = 0.001$ for the NSLMS.

# 5 Convergence of the markov parameter

Here the focus is on the speed and the quality of convergence of the markov parameter to its true value.

- **Convergence speed.**

In Figure (4), we show the evolution of the adaptive parameter $a_k$ versus time ($k$), when the forgetting rates $\mu$ and $1-\lambda$ are fixed to their optimal values. The graphs (1),. . . ,(5) are respectively relative to the real value of $a = 0.9$, the NSLMS (i.i.d. input), the NSRLS (i.i.d. input), the NSLMS (correlated input), and the NSRLS (correlated input). This figure illustrates two important results:
1- The NSRLS has much higher convergence than the NSLMS for both i.i.d. input and correlated input.
2- When the input is correlated the convergence speed of the NSLMS is lower than in the case of i.i.d. input. However as known the convergence speed of the NSRLS is not sensitive to the correlation of the input.

- **Convergence quality.**

The bias $E(\hat{a}) - a$ is computed for different values of the forgetting rate $\mu P_x = 1 - \lambda$. We illustrate some results in Figure (5) relative to $a = 0.9$. Simulation shows that the NSRLS presents better convergence quality for both i.i.d. and correlated data. In fact the bias does not vary remarkably when the forgetting rate changes. This result is very important for the implantation of the algorithm.

# 6 Conclusion and extension

We present in this paper a new adaptive algorithm (NSRLS) that takes into account the structure of time variations of the channel. We show that the NSRLS has better tracking capacity than the NSLMS. In addition, the NSRLS has much higher convergence speed than the NSLMS. Hence, it is an algorithm that can be used in many real applications such as radio-mobile channel equalization. We note that the implantation of the NSRLS requires the generalization, for non stationary markovian context, of fast RLS version that ensures numerical stability. We note that the extension of the NSRLS for a $P$ order markovian non stationarity is described by the following equations.

$$\begin{aligned}
e_k &= y_k - H_{k-1}^T X_k\\
m_j(k+1)|_{j=1,P} &= m_j(k)\\
&\quad + H_{k-j}^T X_{k+1}\left(y_{k+1} - \mu e_{k-1} X_{k-1}^T X_{k+1}\right)\\
&\quad - \mathrm{H}_{k-j}^{\mathrm{T}} \mathrm{X}_{k+1}\left(\sum_{l=1,(l\neq j)}^{\hat{P}} a_l(k) \mathrm{H}(k-l)^{\mathrm{T}} \mathrm{X}_{k+1}\right)
\end{aligned}$$

$$
\begin{aligned}
d_j(k+1)|_{j=1,P} &= d_j(k) + \left(H_{k-j}^T X_{k+1}\right)^2 \\
a_j(k+1)|_{j=1,P} &= \frac{m_j(k+1)}{d_j(k+1)} \\
H_{k+1} &= \sum_{j=1}^{\hat{P}} a_j(k+1) H_{k-j} + G_{k+1} e_{k+1} X_{k+1} \\
G_{k+1} &= \frac{1}{\lambda} G_k \left( I_N - \frac{X_{k+1} X_{k+1}^T G_k}{\lambda + X_{k+1}^T G_k X_{k+1}} \right)
\end{aligned}
$$

The evaluation of this generalized version of the NSRLS requires some simulation.

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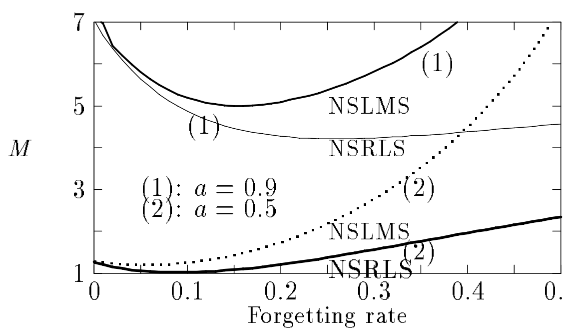


Figure 2: Superiority of NSRLS over NSLMS (i.i.d. input)

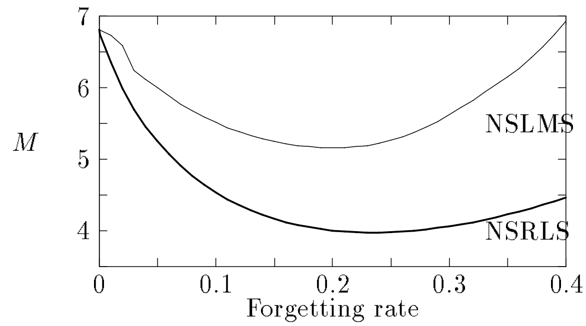


Figure 3: Superiority of NSRLS over NSLMS (Correlated input)

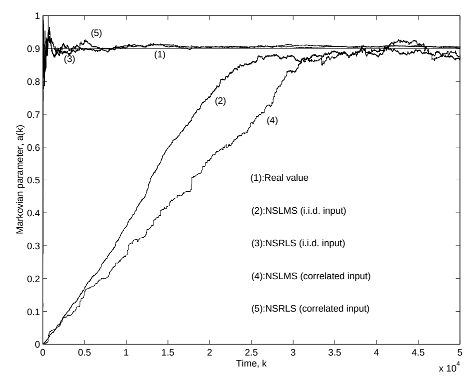


Figure 4: The NSRLS has higher convergence speed than the NSLMS

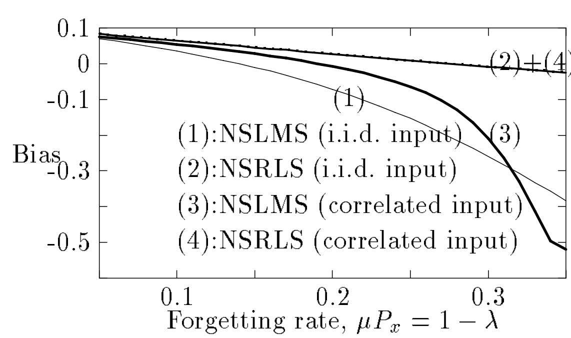


Figure 5: The NSRLS has better convergence quality than the NSLMS