3-D EMITTER LOCALIZATION USING INHOMOGENEOUS BISTATIC SCATTERING

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ABSTRACT

The purpose of this work is to establish how a moving emitter can be localized by a passive receiver using inhomogeneous bistatic scattering. This is a novel localization technique that assumes no *a priori* knowledge of the location of the reflecting sources. The emitter parameters of range, heading, velocity, and altitude are estimated and the variances of the estimates are determined. The proposed estimator is successfully demonstrated using field data collected at White Sands Missile Range during the DARPA/Navy Mountaintop program

1. INTRODUCTION AND BACKGROUND

There are numerous passive techniques that can be used to localize an emitting source. One of the simplest forms of localization is triangulation. Two or more physically separated but cooperating receivers obtain angle-of-arrival estimates (AOA) by measuring the angle to the emitter using some angle measurement technique. Hyperbolic localization systems use time difference of arrival (TDOA) measurements instead of angle measurements to localize an emitter. TDOA estimates are obtained by cross-correlating the emitter signals received at the various receiving stations. If there is relative motion between the emitter and the sensors, differential Doppler measurements may also be utilized. Hybrid localization systems combine multiple discriminants, such as TDOA, AOA, or differential Doppler.

Single-sensor localization techniques usually rely on virtual sensors to perform localization estimates. The virtual sensors are formed by the presence of multiple propagation paths (*i.e.*, specular multipath) or by sensor motion. Once the virtual sensors are realized and their positions determined, localization is performed using a multi-sensor technique such as triangulation. The localization technique described here is a new single-sensor technique that uses out-of-plane multipath to estimate the range, heading, velocity, and altitude of a moving emitter.

2. LOCALIZATION IN INHOMOGENEOUS CLUTTER

In this section an estimator is designed that is based on the assumption that bistatic clutter is inhomogeneous. The inhomogeneous assumption means that the bistatic clutter is assumed to be characterized by a number of discrete scatterers and that the multipath signals are primarily due to these scatterers. Sources for discrete scattering can be man-made, such as water towers or buildings, or natural terrain features, such as hills, mountains, cliffs, etc. A hypothetical discrete scattering scenario is shown in Figure 2. The ML (maximum likelihood) estimator for emitter location that was designed based on a homogeneous clutter assumption was presented earlier in [1].

2.1 Statistical Formulation

The statistical formulation and design of the estimator is comprised of two parts. The first part consists of estimating the parameters of relative time delay, differential Doppler, azimuth, and amplitude for a given number of dominant scatterers. This step is described in *Part 1* below. The second part consists of using these scattering parameters to estimate the emitter parameters of range, heading, velocity, and altitude through a set of nonlinear equations. The equations are linearized via Taylor series truncation and the *best linear unbiased estimator* (BLUE) [2] technique is employed. This step is described in *Part 2*.

Part 1: Estimation of Scattering Parameters

Each discrete scatterer that reflects the emitter signal is described by four parameters. These parameters are relative delay, D, Doppler, ω, azimuth, ϕ, and amplitude, a, which, for the i^{th} scatterer, are written as $[D_i, \omega_i, \phi_i, a_i]$.

An estimator for scatterer parameters that is based on cross-correlation is shown in Figure 1 for a single scatterer. This structure is the maximum likelihood estimator for narrowband signals where both the signal and the additive noise are white. We search over the independent parameters of $[D, \omega, \phi]$ for the maximum cross-correlation $|\mathrm{R}_{sz}|$. The joint estimation of differential time delay and differential Doppler shift was shown to be uncorrelated in [3] when the observation time is sufficiently long and the signal to noise ratios are sufficiently high. Under these assumptions the joint estimator is efficient in the sense that it is unbiased and achieves the Cramer-Rao lower bound. We assume that these conditions hold here. The amplitude estimate a, can be derived from the cross-correlation $\mathbf{r}_{sz}$ as $a_i = r_{sz}(i) / r_{ss}(0)$.

If there are P scatterers in the data, there will be P peaks in the delay-Doppler-azimuth space covered by the estimator. We choose the parameters at these peaks to be our scatterer estimates. To simplify the analysis of the scatterer estimates, we assume that each scatter can be treated as a separate estimation problem with no interaction between neighboring scatterers. This may be justified somewhat by the fact that there are three factors working to decorrelate the scattered signals: delay, Doppler, and azimuth. However, this idealization may produce bounds that are slightly optimistic as we shall see in Section 3.

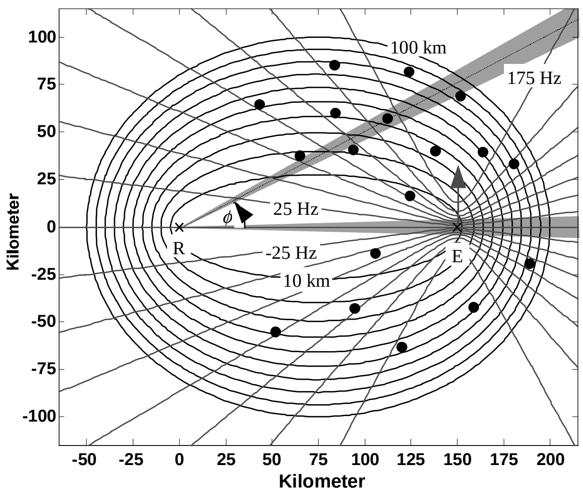

Figure 2. Emitter localization scenario showing discrete scatterers for the inhomogeneous scattering formulation. A reference beam samples the emitter waveform, and multipath beams collect attenuated, delayed, and Doppler shifted replicas reflected by the discrete scatterers.

With the above assumptions, the variances of the scatterer parameter estimates for this estimator are taken directly from the Cramer-Rao lower bounds (CRLB) for point targets. We are assuming that discrete scatterers have the same bounds as conventional radar point targets. The CRLB for delay, Doppler, angle, and amplitude for point targets can be found in [2], [5]. The bounds depend upon receiver and signal parameters such as bandwidth, signal-to-noise ratio, integration time, etc.

Part 2: Estimation of Emitter Parameters

In this section we design an estimator for emitter parameters $\theta = [R, H, V, A]$ based on the scatterer parameter vectors $\left[\hat{\mathbf{D}}, \hat{\phi}, \hat{\omega}, \hat{\mathbf{a}}\right]$ that were determined above. We begin by writing the measured Doppler shift for the i^{th} scatterer as a function g of the *true* emitter parameters θ and the remaining *true* scattering parameters $\alpha_i = [D_i, \phi_i]$ as

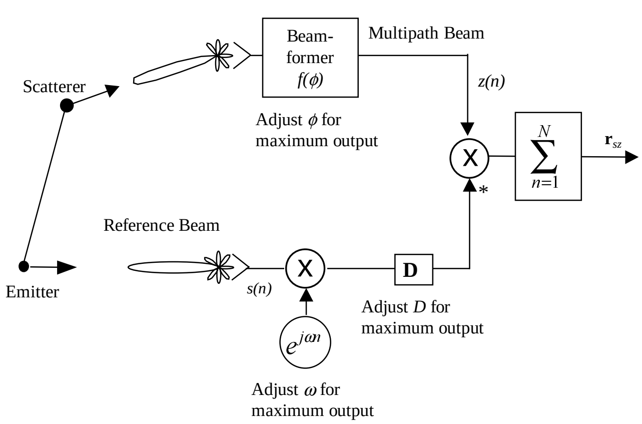

Figure 1. Maximum likelihood estimator for independent scattering parameters [D, ω, ϕ].

$$\hat{\omega}_i = g(\theta, \alpha_i) + e_{\omega_i} \tag{1}$$

where e_{ω_i} is the error in the i^{th} Doppler frequency estimate. The function g is a highly nonlinear function that is described in [4] for several different geometrical approximations.

We do not know the true scatterer parameters α_i so we expand (1) in a truncated Taylor series about the estimates $\hat{\alpha}_i$ as

$$\hat{\omega}_i \cong g(\theta, \hat{\alpha}_i) + \delta_{\alpha_{i1}} g'_{\alpha_{i1}}(\theta, \hat{\alpha}_i) + \delta_{\alpha_{i2}} g'_{\alpha_{i2}}(\theta, \hat{\alpha}_i) + e_{\omega_i} \tag{2}$$

where $\hat{\alpha}_{i1}$ and $\hat{\alpha}_{i2}$ denote the i^{th} delay and azimuth estimates respectively. We are assuming that the errors in scattering parameters are small, and have retained only the zero- and first-order terms. This linear truncation of the Taylor series permits the use of a best linear unbiased estimator (BLUE) [2]. Note that $\delta_\alpha = \alpha - \hat{\alpha}_i$ and $g'_{\alpha_{i1}}$ denotes the partial derivative of g with respect to the variable α_{i1}.

We next assume that we have an initial guess that is close to the true emitter parameters. This starting point is denoted as $\theta_\mathbf{0}$ and will initially be obtained using a coarse least-squares search. We further expand (2) in a Taylor series about $\theta_\mathbf{0}$ as

$$\begin{aligned}\hat{\omega}_i \cong\ & g(\theta_0, \hat{\alpha}_i) + \delta_{\theta_1} g'_{\theta_1}(\theta_0, \hat{\alpha}_i) + \delta_{\theta_2} g'_{\theta_2}(\theta_0, \hat{\alpha}_i) + \delta_{\theta_3} g'_{\theta_3}(\theta_0, \hat{\alpha}_i) + \\ & \delta_{\theta_4} g'_{\theta_4}(\theta_0, \hat{\alpha}_i) + \left[\delta_{\alpha_{i1}} g'_{\alpha_{i1}}(\theta_0, \hat{\alpha}_i) + \delta_{\alpha_{i2}} g'_{\alpha_{i2}}(\theta_0, \hat{\alpha}_i) + e_{\omega_i}\right]\end{aligned} \tag{3}$$

which linearizes the problem in δ_θ. The term in square brackets denotes the noise caused by errors in estimating the scatterer delay, azimuth, and Doppler, and will be referred to as observation noise, n_i. To determine δ_θ we rewrite (3) as

$$\begin{aligned}& \delta_{\theta_1} g'_{\theta_1}(\theta_0, \hat{\alpha}_i) + \delta_{\theta_2} g'_{\theta_2}(\theta_0, \hat{\alpha}_i) + \delta_{\theta_3} g'_{\theta_3}(\theta_0, \hat{\alpha}_i) + \\ & \qquad \delta_{\theta_4} g'_{\theta_4}(\theta_0, \hat{\alpha}_i) = \hat{\omega}_i - g(\theta_0, \hat{\alpha}_i) - n_i\end{aligned} \tag{4}$$

which can be written in vector form as

$$\mathbf{A}\delta = \mathbf{z} - \mathbf{n} \tag{5}$$

with

$$\mathbf{A} = \begin{bmatrix} g'_{\theta_1}(\theta_0, \hat{\alpha}_1) & g'_{\theta_2}(\theta_0, \hat{\alpha}_1) & g'_{\theta_3}(\theta_0, \hat{\alpha}_1) & g'_{\theta_4}(\theta_0, \hat{\alpha}_1) \\ g'_{\theta_1}(\theta_0, \hat{\alpha}_2) & g'_{\theta_2}(\theta_0, \hat{\alpha}_2) & g'_{\theta_3}(\theta_0, \hat{\alpha}_2) & g'_{\theta_4}(\theta_0, \hat{\alpha}_2) \\ \vdots & \vdots & \vdots & \vdots \\ g'_{\theta_1}(\theta_0, \hat{\alpha}_P) & g'_{\theta_2}(\theta_0, \hat{\alpha}_P) & g'_{\theta_3}(\theta_0, \hat{\alpha}_P) & g'_{\theta_4}(\theta_0, \hat{\alpha}_P) \end{bmatrix},$$

$$\delta = \begin{bmatrix} \delta_{\theta_1} \\ \delta_{\theta_2} \\ \delta_{\theta_3} \\ \delta_{\theta_4} \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} \hat{\omega}_1 - g(\theta_0, \hat{\alpha}_1) \\ \hat{\omega}_2 - g(\theta_0, \hat{\alpha}_2) \\ \vdots \\ \hat{\omega}_P - g(\theta_0, \hat{\alpha}_P) \end{bmatrix},$$

and

$$\mathbf{n} = \begin{bmatrix} \delta_{\alpha_{11}} g'_{\alpha_{11}}(\theta_0, \hat{\alpha}_1) + \delta_{\alpha_{21}} g'_{\alpha_{21}}(\theta_0, \hat{\alpha}_1) + e_{\omega_1} \\ \delta_{\alpha_{12}} g'_{\alpha_{12}}(\theta_0, \hat{\alpha}_2) + \delta_{\alpha_{22}} g'_{\alpha_{22}}(\theta_0, \hat{\alpha}_2) + e_{\omega_2} \\ \vdots \\ \delta_{\alpha_{1P}} g'_{\alpha_{1P}}(\theta_0, \hat{\alpha}_P) + \delta_{\alpha_{2P}} g'_{\alpha_{2P}}(\theta_0, \hat{\alpha}_P) + e_{\omega_P} \end{bmatrix}.$$

Equation (5) is in the form of a BLUE (best linear unbiased estimator), which is solved for δ as

$$\delta = \left[\mathbf{A}^{\mathrm{T}} \mathbf{R}_n^{-1} \mathbf{A}\right]^{-1} \mathbf{A}^{\mathrm{T}} \mathbf{R}_n^{-1} \mathbf{z} \qquad (6)$$

which has minimum variance of

$$\mathrm{var}(\hat{\theta}_i) = \mathrm{var}(\hat{\delta}_i) = \left[\left(\mathbf{A}^{\mathrm{T}} \mathbf{R}_n^{-1} \mathbf{A}\right)^{-1}\right]_{ii} \qquad (7)$$

and a parameter covariance matrix of

$$\mathbf{R}_{\hat{\theta}} = \left(\mathbf{A}^{\mathrm{T}} \mathbf{R}_n^{-1} \mathbf{A}\right)^{-1} \qquad .(8)$$

We proceed with the estimation by updating the initial guess

$$\theta_0 \longleftarrow \theta_0 + \delta \qquad (9)$$

and repeating the computation in (6). We repeat the iterations until the norm of δ is sufficiently small. When this occurs, the error in the emitter parameters is described by (7) above.

The covariance matrix of the observation noise $\mathbf{R}_n$ is

$$\mathbf{R}_n = \mathbf{R}_\omega + \mathbf{H}^{\mathrm{T}} \mathbf{R}_\alpha \mathbf{H} \qquad (10)$$

with

$$\mathbf{R}_\alpha = \begin{bmatrix} \mathbf{R}_D & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_\phi \end{bmatrix} \qquad (11)$$

and

$$\mathbf{H}^{\mathrm{T}} = \begin{bmatrix} g'_D(\theta, \hat{D}_1) & & \mathbf{0} & g'_\phi(\theta, \hat{\phi}_1) & & \mathbf{0} \\ & \ddots & & & \ddots & \\ \mathbf{0} & & g'_D(\theta, \hat{D}_P) & \mathbf{0} & & g'_\phi(\theta, \hat{\phi}_P) \end{bmatrix}$$

The covariances for the scatterer parameter estimates $\mathbf{R}_\omega$, $\mathbf{R}_D$, and $\mathbf{R}_\phi$ are diagonal matrices with the diagonal elements equal to the variances of the scatterer estimates. These variances are assumed to be known *a priori* for this development. This is a standard assumption for localization problems [6]. As noted in Section 2.1, these variances are obtained by computing the CRLB for each scatterer as if it were a radar point target. Equation (7) specifies the CRLB for the emitter parameters when the observation noise is Gaussian, since the BLUE estimator is also the ML estimator for that case.

3. APPLICATION TO MOUNTAINTOP DATA

In this section we demonstrate the estimators developed in Section 2 using field data collected at White Sands Missile Range (WSMR) during the DARPA/Mountaintop program [7]. The UHF radar, referred to as RSTER (Radar Surveillance Technology Experimental Radar), was installed on North Oscura Peak, NM, where the terrain is a mixture of desert and mountains. The antenna was configured to support 14-channel digital beamforming in the azimuthal dimension (with a peak gain of 29 dBi). Thus, once the data for each channel have been recorded, 8-degree receive beams can be formed in any or all directions simultaneously. The airborne emitter was a Lear jet containing dipole antennas (with broad radiation patterns) in the nose and tail portions of the aircraft. A 700-Watt noise waveform was radiated that filled the 200-kHz bandwidth of the RSTER receiver.

A practical implementation of the ML estimator for scatterer parameters that was shown in Figure 1 is given in Figure 3. Cross-correlations are performed for the direct path and thirty-three beams ranging from -64 to 64 degrees. In this manner, a three-dimensional cube of scattered energy verses delay, Doppler, and azimuth is formed. These correlation cubes have been generated in the past by Gabel [8]. The cube is passed to the "Select and Centroid Scatterers" function which applies a threshold and centroids the results to a set of discrete scatterer parameter vectors $\hat{\mathbf{D}}$, $\hat{\omega}$, $\hat{\phi}$, and $\hat{\mathbf{S}}/\hat{\mathbf{N}}$. The threshold is set significantly high so that false alarms are essentially nonexistent. In addition, this function must reject sidelobe signals that may occur in all three dimensions of delay, Doppler, and azimuth. This is accomplished by selecting only the largest scatterer for each dimension.

Data set HOT-6067 was processed in this manner and 67 scatterers were selected. The relative delay-Doppler map for one of the beams is shown in Figure 4 with the position of each scatterer denoted by an "x". The solid line denotes the center of the beam and the dashed lines denote the 3-dB beam edges.

With the scatterer parameters estimated, we are now ready to estimate the emitter parameters. A coarse least-squares search using Equation (7) is performed over emitter range, heading, and velocity for a nominal fixed altitude. The range-heading slice is shown in Figure 5. The coarse search is performed to provide an initial guess at the emitter parameters, $\theta_{\mathbf{0}}$. This initial guess is used as the starting point for the iterative procedure described in Section 2. We found that it was not necessary to include emitter

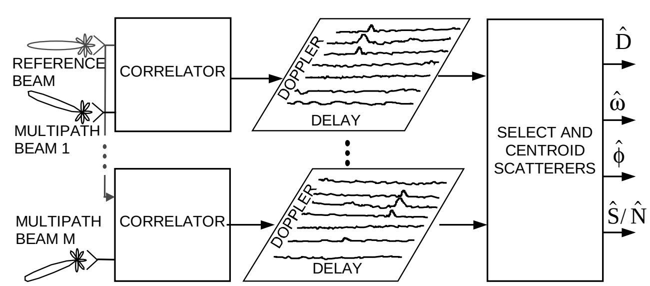

Figure 3. Estimator for scatterer parameters of delay, Doppler, azimuth, and SNR.

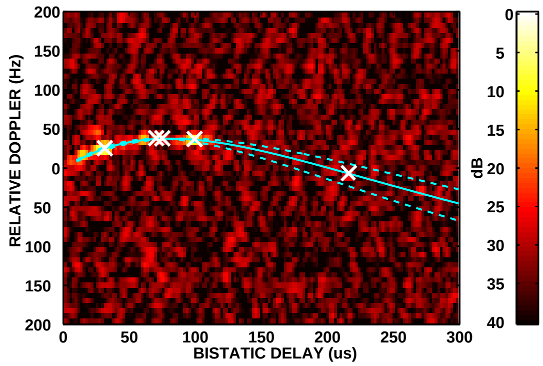

Figure 4. Correlation vs. delay and Doppler for a single beam at -40 degrees. The position of each scatterer is denoted by an "x".

altitude in the coarse search. The iterative procedure converged to the same point regardless of the initial starting altitude.

The iterative procedure converged rapidly to the estimates listed in Table 1. Only five iteration steps were required. The results are excellent and are fairly close to the CRLB for all parameters except the emitter velocity. The range error is only 0.7 km and the heading error is only 0.2 degrees.

4. SUMMARY

In this paper we localize passively a moving emitter using out-of-plane multipath signals reflected by the terrain. The emitter parameters of range, heading, velocity, and altitude are estimated by exploiting the correlation between the direct-path signal and the delayed and Doppler modulated multipath signals for the case of inhomogeneous bistatic scattering. A more detailed discussion containing additional examples and covering both homogeneous and inhomogeneous scattering is contained in [4].

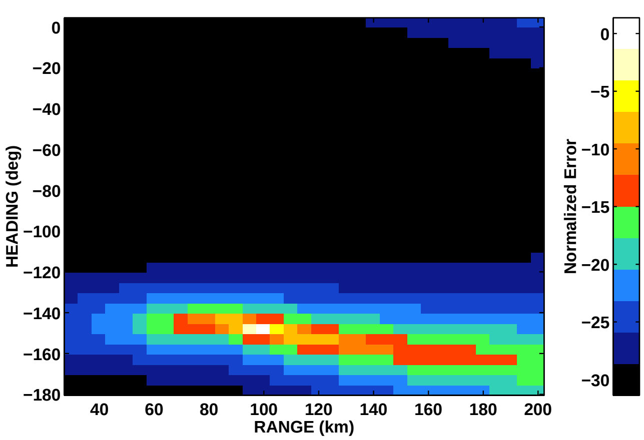

Figure 5. Least-squares search through parameter space for HOT-6067.

Table 1. Results for Data Set HOT-6067 using the estimator designed for inhomogeneous clutter.

Emitter Parameter	True Value	Est. Value	Error Value	Error CRLB	CRLB σ
Range (km)	101.7	101.0	-0.7	2.3σ	0.31
Heading (deg)	147.2	147.0	0.2	4σ	0.05
Velocity (m/s)	184.5	179.3	5.2	11.1σ	0.47
Altitude (km)	8	9.4	1.4	4.5σ	0.31

5. REFERENCES

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