CRITERA FOR DIRECT BLIND DECONVOLUTION OF MIMO FIR SYSTEMS DRIVEN BY WHITE SOURCE SIGNALS

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ABSTRACT

This paper addresses the blind deconvolution of multi-input–multi-output (MIMO) FIR systems driven by white non-Gaussian source signals. First, we present a weaker condition on source signals than the so-called i.i.d. condition so that blind deconvolution is possible. Then, under this condition, we provide a necessary and sufficient condition for blind deconvolution of MIMO FIR systems. Finally, based on this result, we propose two maximization criteria for blind deconvolution of MIMO FIR systems. These criteria are simple enough to be implemented by adaptive algorithms.

1. INTRODUCTION

Consider a set of received signals that are linear (convolutive) mixtures of a set of source signals. The objective of blind deconvolution is to recover the source signals from the set of received signals without the knowledge of the linear mixtures or the LTI systems. The case when the mixture is instantaneous has been well studied. For example, see [1] and the references therein. Blind deconvolution has received increasing attention in the past few years [2]–[7]. One way for achieving blind deconvolution is first to blindly identify the channel system from the channel outputs [8], and then to design an equalizer accordingly. The other way of achieving blind deconvolution is to directly design an equalizer from the equalizer outputs. For example, see [2]–[7] and the references therein. This *direct approach* is preferable, because it bypasses the process of blind system identification and the order estimation of the channel system that is usually needed for blind system identification. Moreover, the computation becomes simpler, because the dimension of the channel output is usually larger than that of the equalizer output.

Most of the approaches assume that the sequence of source signals is temporally *i.i.d.* For example see [2]–[5] and the references therein. However, the condition of *i.i.d.* for source signals is too restrictive for some applications. For example, in digital communications, the information bearing sequences are coded and hence are unlikely *i.i.d.* On the other hand, these coded sequences are usually interleaved to encounter burst errors and are usually considered to be uncorrelated. Therefore, it is vitally important to weaken the temporary *i.i.d.* condition to the temporary white condition.

In this paper, we first present a *weaker* condition on the source signals than the *i.i.d.* condition so that blind deconvolution is possible. Then under this weaker condition we provide a necessary and sufficient condition for blind deconvolution of MIMO FIR systems. Finally, based on this result, we propose two maximization criteria for blind deconvolution of MIMO FIR systems. These criteria are simple enough to be implemented by adaptive algorithms.

This paper uses the following notation. Matrices are denoted by boldface uppercase letters. Column vectors are denoted by boldface lowercase letters. All others are scalars. The superscript T denotes the transpose of a matrix. The superscript $*$ denotes the complex conjugate of a scalar or a matrix. The (i,j)th component of a matrix $\boldsymbol{A}$ is denoted by a_{ij}. Let $\boldsymbol{I}$ denote an identity of an appropriate size. Let $\mathrm{cum}\{x_1,\cdots,x_n\}$ denote the nth-order (or joint) cumulant of random variables $x_1,\cdots,x_n$, which is defined as a coefficient of the Taylor expansion of the natural logarithm of the joint characteristic function of $x_1,\cdots,x_n$ [11].

2. PROBLEM FORMULATION

We consider an MIMO FIR system shown in Figure 1,

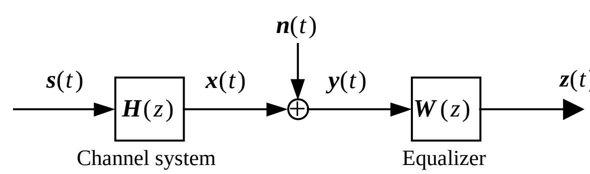

Figure 1. A model for blind deconvolution.

where,

$$\boldsymbol{y}(t) = \sum_{k=0}^{K-1} \boldsymbol{H}(k)\boldsymbol{s}(t-k) + \boldsymbol{n}(t), \tag{1}$$

$$\boldsymbol{z}(t) = \sum_{k=0}^{L-1} \boldsymbol{W}(k)\,\boldsymbol{y}(t-k), \tag{2}$$

Note that no condition is imposed on $\boldsymbol{H}(0)$ nor $\boldsymbol{H}(K-1)$.

The equalizer output $\boldsymbol{z}(t)$ and the input $\boldsymbol{s}(t)$ are related by

$$\boldsymbol{z}(t) = \boldsymbol{G}(z)\boldsymbol{s}(t) + \boldsymbol{W}(z)\boldsymbol{n}(t), \tag{3}$$

where $\boldsymbol{G}(z) := \boldsymbol{W}(z)\boldsymbol{H}(z)$.

The objective of blind deconvolution or source separation is to design an equalizer $\boldsymbol{W}(z)$ that recovers the original source signals only from the observations of the system outputs $\boldsymbol{y}(t)$'s.

Definition 1: A scalar function $g(z)$ of a complex variable z is said to be *monomial* if it can be represented as $g(z) = cz^d$. An LTI system with n inputs and n outputs is said to be *transparent* (or *decoupled)* if its transfer function matrix $\boldsymbol{G}(z)$ has a single nonzero monomial entry in each row and each column.

Note that an LTI system is transparent if and only if $\boldsymbol{G}(z)$ has a decomposition of the form

$$\boldsymbol{G}(z) = \boldsymbol{\Lambda}(z)\boldsymbol{D}\boldsymbol{P}, \tag{4}$$

where $\boldsymbol{\Lambda}(z)$ is a diagonal matrix with diagonal entries $\lambda_{ii}(z) = z^{l_i}$ (where l_i is a non-negative integer), $\boldsymbol{D}$ a regular constant diagonal matrix, and $\boldsymbol{P}$ a permutation matrix. Then, the blind deconvolution problem is formulated as follows: Design an equalizer $\boldsymbol{W}(z)$, so that the condition

$$\boldsymbol{W}(z)\boldsymbol{H}(z) = \boldsymbol{\Lambda}(z)\boldsymbol{D}\boldsymbol{P}. \tag{5}$$

is satisfied, i.e., $\boldsymbol{G}(z)$ is transparent.

A channel system $\boldsymbol{H}(z)$ is said to be *deconvolvable* if there exists an equalizer $\boldsymbol{W}(z)$ so that the composite system $\boldsymbol{G}(z)$ is transparent. A necessary and sufficient condition for the existence of such equalizer $\boldsymbol{W}(z)$ is given by Massey and Sain [10].

Theorem 1 (Massey-Sain Theorem): Let $\boldsymbol{H}(z)$ be an $m \times n$ matrix transfer function of an FIR channel system. Then a necessary and sufficient condition for $\boldsymbol{H}(z)$ to be deconvolvable is that the *greatest common divisor* (GCD) of all the minors of order n in $\boldsymbol{H}(z)$ is nonzero monomial, that is,

$$\text{GCD}\,\{d_i(z) \mid i = 1, 2, \cdots, {}_mC_n\} = z^l \tag{6}$$

for some integer $l \geq 0$, where $d_i(z)$'s denote all the minors of order n in $\boldsymbol{H}(z)$.

To specify the source signals, we introduce some preliminary definitions and notions for stationary non-Gaussian vector-valued random processes as shown in [11].

Definition 2: Let $\{\boldsymbol{s}(t)\}$ be a complex-valued stationary random vector process with components $\{s_i(t)\}$, $i = 1, \cdots, n$. Then the family of *second-order cumulant sequences* of $\{\boldsymbol{s}(t)\}$ is defined by $c_{s_i,s_j}(\tau) := \text{cum}\,\{s_i(t), s_j^*(t+\tau)\}$ for $i, j = 1, \cdots, n$ and $t \in Z$. In particular, the sequence $\{c_{s_i,s_j}(\tau)\}$ is also called the *cross-correlation* of $\{s_i(t)\}$ and $\{s_j(t)\}$ for $i \neq j$ and the *auto-correlation* of $\{s_i(t)\}$ for $i = j$, and $c_{s_i,s_j}(0)$ (denoted by $\sigma_{s_i}^2$) is called the *variance* of $\{s_i(t)\}$. The family of *fourth-order cumulant sequences* is defined by $c_{s_{i_1},s_{i_2},s_{i_3},s_{i_4}}(t_1,t_2,t_3) :=$ $\text{cum}\{s_{i_1}(t), s_{i_2}^*(t+t_1), s_{i_3}(t+t_2), s_{i_4}^*(t+t_3)\}$ for $i_1, i_2, i_3, i_4 =$ $1, \cdots, n$ and $t_1, t_2, t_3 \in Z$. In particular, the sequence $\{c_{s_i,s_j,s_k,s_l}(t_1, t_2, t_3)\}$ is called the *fourth-order auto- or cross-correlation* of $\{s_i(t)\}$, $\{s_j(t)\}$, $\{s_k(t)\}$, $\{s_l(t)\}$ depending on whether all the indices *i, j, k, l* are the same or not. Furthermore, $c_{s_i,s_i,s_i,s_i}(0, 0, 0)$ is called the *fourth-order cumulant* or *kurtosis* of $s_i(t)$ and denoted by κ_{s_i}.

For notational simplicity, we denote the second- and fourth-order cumulants, $c_{s_i,s_i}(t)$ and $c_{s_i,s_i,s_i,s_i}(t_1,t_2,t_3)$, by $c_{s_i}(t)$ and $c_{s_i}(t_1,t_2,t_3)$, respectively, if it is clear from the context.

Definition 3: A process $\{\boldsymbol{s}(t)\}$ is said to be *temporally uncorrelated* if all the auto-correlations $c_{s_i,s_i}(\tau)$, $i = 1, \cdots, n$ are zero except at the origin $\tau = 0$ and it is said to be *spatially uncorrelated* if all the cross-correlations $c_{s_i,s_j}(\tau)$, $i \neq j$ are zero. It is said to be *second-order white* if it is both temporally and spatially uncorrelated. Furthermore, it is *temporally fourth-order uncorrelated* if all the fourth-order auto-correlations $c_{s_i,s_i,s_i,s_i}(t_1,t_2,t_3)$, $i = 1, \cdots, n$ are zero except at the origin $t_1 = t_2 = t_3 = 0$. It is *spatially fourth-order uncorrelated* if all the fourth-order cross-correlations $c_{s_{i_1},s_{i_2},s_{i_3},s_{i_4}}(t_1,t_2,t_3)$ (where i_1, i_2, i_3 and i_4 are not all the same.) are zero. It is said to be *fourth-order white*, if it is temporally and spatially fourth-order uncorrelated.

Henceforth, we assume throughout the paper that

1) The matrices $\boldsymbol{H}(z)$ and $\boldsymbol{W}(z)$ are transfer functions representing FIR systems.
2) The vector sequence $\{\boldsymbol{s}(t)\}$ is a zero-mean random process satisfies the cumulant summability conditions of orders 2 and 4. In addition, assume that the kurtoses κ_{s_i}, $i = 1, \cdots, n$, of all the components of $\boldsymbol{s}(t)$ are nonzero, which implies that it is a non-Guassian process.
3) For the purpose of analysis, the noise is assumed to be zero, i.e., $\boldsymbol{n}(t) \equiv 0$, although the criteria presented in Section 4 can be applied to noisy cases.

We also consider the following two conditions:

(A1) The sequence $\{\boldsymbol{s}(t)\}$ is second-order white, and spatially fourth-order uncorrelated. In addition, assume that the kurtoses κ_{s_i}, $i = 1, \cdots, n$, of all the components of $\boldsymbol{s}(t)$ are nonzero.

(A2) The sequence $\{\boldsymbol{s}(t)\}$ is second-order and fourth-order white.

3. NECESSARY AND SUFFICIENT CONDITIONS

Consider a composite system described by (1)–(3) with (A1). We present a necessary and sufficient condition for blind deconvolution.

Theorem 2: Let $\boldsymbol{H}(z)$ be deconvolvable, and let $\{\boldsymbol{s}(t)\}$ satisfy **(A1)**. Suppose an equalizer $\boldsymbol{W}(z)$ is used to make a composite system $\boldsymbol{G}(z)$. Then the composite system $\boldsymbol{G}(z)$ is transparent if and only if the output sequence $\{\boldsymbol{z}(t)\}$ is a second-order white and spatially fourth-order uncorrelated random process with nonzero variances $\sigma_{z_i}^2 \neq 0,\ i = 1, \cdots, n$.

See [12] for the proof.

Remark 2.1: The condition on the signal source $\{\boldsymbol{s}(t)\}$ can hardly be further weakened. It is known that the second-order statistics alone is not sufficient for blind deconvolution. We have to use high order statistics. In **(A1)**, only the fourth-order spatial statistics are added.

4. CRITERIA FOR MULTICHANNEL BLIND DECONVOLUTION

In this section, based on Theorem 2, we present optimization criteria for blind deconvolution of MIMO FIR systems. The assumption made on the source sequence $\{\boldsymbol{s}(t)\}$ is specified by **(A1)** or **(A2)** in the previous sections.

We shall present two criterion functions for multichannel blind deconvolution below according to **(A1)** and **(A2)**, respectively. Under **(A1)**, we consider the following maximization criterion (A):

Maximize J_A subject to the constraints $c_{z_i, z_j}(\tau) = \delta(i-j)\delta(\tau)$ for all $\tau \in Z$ and all $i, j = 1, \cdots, n$, where J_A is defined by

$$J_A := \sum_{i=1}^{n} \sum_{t_1, t_2, t_3 \in Z} \left| c_{z_i}(t_1, t_2, t_3) \right|^2 \tag{7}$$

Under **(A2)** we consider the following maximization criterion (B):

Maximize J_B subject to the constraints $c_{z_i, z_j}(\tau) = \delta(i-j)\delta(\tau)$ for all $\tau \in Z$ and all $i, j = 1, \cdots, n$, where J_B is defined by

$$J_B := \sum_{i=1}^{n} \left| \kappa_{z_i} \right|^2 \tag{8}$$

We note that the constraints of the criterion (A) are the same as those of the criterion (B), and both of them require the equalizer output process $\{\boldsymbol{z}(t)\}$ to be normalized-white in the second-order sense. This is equivalent to the condition

$$E\{\boldsymbol{z}(t+k)\boldsymbol{z}^{*T}(t)\} = \boldsymbol{I}\delta(k). \tag{9}$$

In order to show the validity of the criteria (A) and (B), we require the following lemma. To this end, we present first the definition of paraunitary systems.

Definition 4: Let $\boldsymbol{H}(z)$ be an $n \times n$ transfer function matrix representing an FIR system. Then it is said to be *paraunitary* if $\boldsymbol{H}(e^{-j\omega})$ is unitary, that is, $\boldsymbol{H}(e^{-j\omega})\boldsymbol{H}(e^{-j\omega})^{*T} = \boldsymbol{I}$ for any real ω. Also, the system itself is called paraunitary for simplicity.

Lemma 1: Let $\boldsymbol{x}(t)$ be a complex vector-valued stationary random process described by

$$\boldsymbol{x}(t) = \boldsymbol{H}(z)\boldsymbol{s}(t), \tag{10}$$

where $\boldsymbol{H}(z)$ is an $n \times n$ transfer function matrix of a paraunitary system and $\{\boldsymbol{s}(t)\}$ is a complex vector-valued stationary random process with zero mean. Then, it holds true that

$$K_{\boldsymbol{x}} = K_{\boldsymbol{s}}, \tag{11}$$

where,

$$K_{\boldsymbol{s}} := \sum_{i_1, i_2, i_3, i_4 = 1}^{n} \sum_{t_1, t_2, t_3 \in Z} \left| c_{s_{i_1}, s_{i_2}, s_{i_3}, s_{i_4}}(t_1, t_2, t_3) \right|^2$$

$$K_{\boldsymbol{x}} := \sum_{i_1, i_2, i_3, i_4 = 1}^{n} \sum_{t_1, t_2, t_3 \in Z} \left| c_{x_{i_1}, x_{i_2}, x_{i_3}, x_{i_4}}(t_1, t_2, t_3) \right|^2$$

See [12] for the proof.

It may be interesting to note from the above lemma that although paraunitary is a condition based on second statistics, it imposes a condition on higher-order statistics.

Using Theorem 2 and the above lemma, we can establish the following theorem, which shows the validity of the maximization criteria (A) and (B).

Theorem 3: Let $\boldsymbol{H}(z)$ be deconvolvable, and let $\{\boldsymbol{s}(t)\}$ satisfy **(A1)** (or **(A2)**). Suppose an equalizer $\boldsymbol{W}(z)$ is used to make a composite system $\boldsymbol{G}(z)$. Then under **(A1)** (or **(A2)**) on $\{\boldsymbol{s}(t)\}$, the maximization criterion (A) (or the maximization criterion (B)) makes the composite system $\boldsymbol{G}(z)$ transparent.

Remark 3.1: Although criterion J_A is not found elsewhere, the criterion J_B has appeared in [3], but under a stronger condition on source signals, i.e., the *i.i.d.* assumption is presumed.

Remark 3.2: Numerical algorithms based on criterion (B) under the *i.i.d.* assumption have been proposed in [13]. It can be shown that these algorithms can also be applied to the case when source signals satisfy **(A2)**.

Remark 3.2: The criteria (A) and (B) may be regarded as extensions of the Salvi-Weinstein (SW) criterion [14] for the single channel case to the multichannel case. It is widely known that the SW criterion has close connections with the constant modulus (CM) criterion and the mean square error (MSE) criterion. For a simple case, some results on their connections are reported by Gu and Tong [15].

Proof of Theorem 3: Without loss of generality, we may assume that the input process $\{\boldsymbol{s}(t)\}$ is normalized-white, and because **(A1)** or **(A2),** $\{\boldsymbol{z}(t)\}$ is normalized-white. Hence, $\boldsymbol{G}(z)$ is paraunitary. Thus applying Lemma 1 to $\boldsymbol{G}(z)$, we have

$$K_{\boldsymbol{z}} = K_{\boldsymbol{s}}, \tag{12}$$

On the other hand, K_z and K_s can be decomposed as

$$K_{\boldsymbol{z}} = K_{\boldsymbol{z}}^{(a)} + K_{\boldsymbol{z}}^{(c)}, \tag{13}$$

$$K_{\boldsymbol{s}} = K_{\boldsymbol{s}}^{(a)} + K_{\boldsymbol{s}}^{(c)}, \tag{14}$$

where $K_{\boldsymbol{z}}^{(a)}$ and $K_{\boldsymbol{s}}^{(a)}$ are the sums of all the absolute squares of all the fourth-order *auto*-cumulant sequences of $\{\boldsymbol{z}(t)\}$ and $\{\boldsymbol{s}(t)\}$, respectively, and $K_{\boldsymbol{z}}^{(c)}$ and $K_{\boldsymbol{s}}^{(c)}$ are the sums of all the fourth-order *cross*-cumulant sequences $\{\boldsymbol{z}(t)\}$ and $\{\boldsymbol{s}(t)\}$, respectively. Under **(A1)** or **(A2)**, it holds true that $K_{\boldsymbol{s}}^{(c)} = 0$, which implies from (12), (13), and (14),

$$K_{\boldsymbol{z}}^{(a)} + K_{\boldsymbol{z}}^{(c)} = K_{\boldsymbol{s}}^{(a)} (= \text{constant}) . \tag{15}$$

Now, consider (A1). Since $K_{\boldsymbol{z}}^{(c)}$ is non-negative and $K_{\boldsymbol{z}}^{(a)} = J_A$ from their definitions, the maximization J_A subject to the constraints in the criterion (A) implies $K_z^{(c)} = 0$, which means that the output process $\{\boldsymbol{z}(t)\}$ is spatially fourth-order uncorrelated. Based on Theorem 2, this concludes that the composite system $\boldsymbol{G}(z)$ is transparent, because $\{\boldsymbol{z}(t)\}$ is second-order white. Similarly, we can prove the case under (A2).

5. CONCLUSIONS

We considered the blind deconvolution of MIMO FIR systems driven by white non-Gaussian source signals. First, we found a weaker condition on source signals than the so-called *i.i.d.* condition so that blind deconvolution is possible. It was found that the condition is that the source signals are second-order white and spatially fourth-order uncorrelated. Then, under this condition, we provided a necessary and sufficient condition for blind deconvolution of MIMO FIR systems. It was shown that blind deconvolution is achieved if and only if the composite output signals are second-order white and spatially fourth-order uncorrelated. Finally, based on this result, we proposed two maximization criteria for blind deconvolution of multiuser-multichannel systems.

These criteria use only the second- and fourth-order statistics of the equalizer outputs. Therefore, we can directly use these criteria to recover the source signals without first using a channel identification process. In particular, the maximization criteria (A) and (B) require no information of, and hence are robust to, the order of the channel systems, but only a bound of the order.

Numerical algorithms based on the maximization criterion (A) are being developed and will be reported in future work.

6. REFERENCES

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