FAST RANGE AND DOPPLER ESTIMATION FOR NARROWBAND ACTIVE SONAR

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ABSTRACT

In this paper, we present a computationally simple algorithm suitable for fast, high resolution estimation of time delays and doppler shifts (which are necessary for target localization and tracking) between narrowband signals in an active sonar system. The algorithm uses a modulated Lagrange interpolation filter and an LMS-type algorithm. The problem of delay and doppler estimation is reduced to a linear regression problem. Convergence and performance analysis of the method is studied both analytically and through simulation. It is demonstrated that the method provides estimates close to the Cramer-Rao Lower Bound.

1. INTRODUCTION

An active sonar system basically transmits an acoustic signal into the ocean and from the reflected echo attempts to extract information about the target. Often, the parameters of interest are the range (location) and velocity of the target, estimates of which may be determined from the differential time delay (DTD) and differential frequency offset (DFO), also termed doppler shift, between the transmitted and received signals [1]. In a conventional active sonar system, block data segments are obtained and the narrowband pulse is detected in one of these segments, hence rendering a coarse range estimate. Commonly, the DTD and DFO is then estimated more accurately from a computationally expensive ambiguity function calculation comprising of acquisition and tracking modes [2]. To localize the target with a very high degree of precision, estimation of non-integral sample time delays is required and hence interpolation by a sinc function is often used [2]. There are two disadvantages with this approach, the computational cost (which can impair ability to perform real-time target tracking) and the ranging and doppler errors incurred by non-optimal interpolation.

Here, we consider (comparatively) fast but high resolution estimators suitable for DTD and DFO estimation using narrowband signals (and hence perform high resolution localization and real-time target tracking at a low computational cost). We develop and generalize estimators originally proposed for on-line time delay estimation (TDE) only.

The least mean square time delay estimator (LMSTDE) algorithm [3] is a well-known technique but can only render integer delay estimates—closer estimates are obtained by subsequent interpolation. Again, the "postprocessing" interpolation has no optimality properties. A technique which directly updates the delay estimate such as the explicit time delay estimator (ETDE), using the same sinc interpolator in an adaptive system identification configuration, has been shown to give superior performance [4]. The advantage of the ETDE method is that the time delay estimate is adapted directly on a sample by sample basis. This removes the phenomenon of "false peaks" using standard LMSTDE [5]. The ETDE has the potential to exploit *a priori* signal information by incorporating a more suitable interpolator than the sinc. From now on, we use the term ETDE to refer to the general explicit TDE algorithm with arbitrary interpolator, and the sinc-ETDE (SETDE) to refer to the ETDE with the specific sinc interpolator of [4]. Previous work has resulted in a computationally efficient and accurate delay estimation algorithm for narrowband signals, the Lagrange-ETDE (LETDE) method which uses a Lagrange interpolator to approximate the delay [6]. Significant estimation mean-squared error reduction can be achieved by modulating the Lagrange interpolator to the signal frequency ω_c [6]. Performance of DTD and DFO estimation for different ω_c is very much of interest, as there exists an optimum transmission frequency related to the target range [1]. Here, it is shown that the modulated LETDE can be applied successfully to the joint DTD and DFO estimation problem.

In this paper, the LETDE algorithm is used to determine the delay between a reference signal and a delayed, frequency offset but otherwise identical signal in noise. The effect of the frequency offset is to cause the delay between the two signals to be linearly-time varying. The proposed method herein estimates off-line the DTD and DFO via linear regression analysis on the delay estimates time series obtained from the LETDE. Linear regression over a data segment of typical length is computationally simpler than the ambiguity function computation of [2]. The performance of the proposed method is assessed through numerical examples.

2. PROBLEM FORMULATION

2.1. Signal model

Consider the two-signal model where $x(k)$ is the reference (transmitted signal) and $y(k)$ is a data segment containing the reflected echo, delayed by the DTD τ and doppler shifted by DFO ν:

$$x(k) = s(k)e^{j\omega_c k} + \theta(k) \tag{1}$$

$$y(k) = \beta s(k-\tau)e^{j(\omega_c+\nu)(k-\tau)} + \phi(k). \tag{2}$$

In the above model, $s(k)$ is a narrowband signal of bandwidth $W << \omega_c$, where the center frequency ω_c is known; k is the time index; the sampling period is assumed to be unity and $\{\theta(k), \phi(k)\}$ are complex white, zero-mean noises with unknown variances $\{\sigma_\theta^2, \sigma_\phi^2\}$. The attenuation β is assumed to be unity throughout this paper to simplify the analysis. The method presented herein is easily extended to also estimate β if required without affecting the relative performances of the methods. Here, it is sufficient to estimate the DTD τ and DFO ν.

2.2. Fractional delay approximation

The accuracy of delay estimation by adaptive techniques such as the ETDE hinges on the approximation of $s(k-D)$, where D is a fractional sample delay:

$$s(k-D) \approx \sum_{m=0}^{N} h(m,D)s(k-m) \tag{3}$$

which depends naturally very heavily on the choice of interpolation function $h(m,D)$ [7]. The ideal interpolation function, the sinc function, is infinite in length and hence unrealizable [7]. In lieu of this, the truncated sinc filter was proposed [4] but the Lagrange interpolation filter has since been shown to be superior for narrowband signals [6] in terms of delay estimation mean squared error.

The Lagrange interpolation filter (LIF) is defined as

$$h^0(m,D) = \prod_{\substack{i=0\\ i\neq m}}^{N} (D-i)/(m-i) \tag{4}$$

and has some very useful properties for TDE. Firstly, The LIF is equivalent to a filter maximally flat at zero frequency [7], where maximally flat means that derivatives up to Nth order at a point in the frequency domain approximation error are forced to zero [7]. The LIF can be made maximally flat at a frequency ω_0 by applying a complex modulation [8]. Secondly, The LIF is particularly useful as the derivative of the coefficients with respect to D can be computed exactly [6]. This follows as Lagrange interpolation is a polynomial in D, which is differentiable. Thirdly, the coefficients are easily computed via (4) or a Farrow approximation structure (an efficient implementation of interpolation filters) [7] or a high resolution lookup table can be used.

2.3. The LETDE algorithm

The LETDE algorithm was proposed in [6] and is quickly summarized here. The LETDE is an extension of the SETDE algorithm [4]. Its system block diagram is depicted in figure 1. The interpolator filter coefficients $h(m,\hat{D}(k))$ are constrained to be the truncated sinc filter in the SETDE algorithm or the Lagrange interpolator filter (4) for the LETDE algorithm. Both ETDE algorithms use the (complex) LMS algorithm and the updated delay is [6]

$$\hat{D}(k+1) = \hat{D}(k) + 2\mu \mathrm{Re}\{e^*(k)\sum_{m=0}^{N} x(k-m) f(m,\hat{D}(k))\} \tag{5}$$

where

$$e(k) = y(k) - \sum_{m=0}^{N} x(k-m)h(m,\hat{D}(k)) \tag{6}$$

and $f(m,\hat{D}(k)) = \frac{\partial h(m,\hat{D}(k))}{\partial \hat{D}(k)}$ and μ is the stepsize. Note that we use the notation $\hat{D}(k)$ to mean the estimated delay between $x(k)$ and $y(k)$ at time k and should not be confused with the DTD τ. For the LETDE algorithm, the LIF is modulated to the signal centre frequency, *i.e.* $\omega_0 = \omega_c$, and hence $h(m,\hat{D}(k)) = e^{j\omega_c(m-\hat{D}(k))}h^0(m,\hat{D}(k))$, or a complex modulated version of (4). This implies $f(m,\hat{D}(k)) = e^{j\omega_c(m-\hat{D}(k))}\,[f^0(m,\hat{D}(k)) - j\omega_c h^0(m,\hat{D}(k))]$ where $f^0(m,\hat{D}(k))$ is the derivative of $h^0(m,\hat{D}(k))$ obtainable from [6] or by forming the Nth order Farrow approximation (a polynomial approximation) [7] to $h^0(m,\hat{D}(k))$ and differentiating this.

3. DTD AND DFO ESTIMATORS

In this section, DTD and DFO estimators are proposed, derived from using the LETDE algorithm to estimate the delay between $x(k)$ and $y(k)$. Through our narrowband assumption, it can be shown that this delay is linearly time-varying; equivalently, the delay can be expressed as

$$D(k) = A + Bk + \epsilon(k) \tag{7}$$

for some constants $\{A, B\}$ and zero-mean perturbation $\epsilon(k)$ due to noise. It is shown in the next section that $\hat{D}(k)$ converges to such a straight line, from whose estimated gradient $\hat{B}$ and estimated intercept $\hat{A}$ one may obtain DTD and DFO estimates.

3.1. Convergence of the LETDE

By taking the expectation of (5) and simplifying, it can be shown that (using the narrowband approximation $s(k) = 1$ for simplicity)

$$\begin{aligned} E\{\hat{D}(k+1)\} &= E\{\hat{D}(k)\} - 2\mu\omega_c \sin(\omega_c(E\{\hat{D}(k)\} - \tau) \\ &\quad +\nu(k-\tau)) - 2\mu\sigma_\theta^2 b(E\{\hat{D}(k)\}) \end{aligned} \tag{8}$$

where $b(x)$ is the function $b(x) = \sum_{m=0}^{N} h^0(m,x)f^0(m,x)$. Assuming that the LETDE tracks the delay such that $\omega_c\hat{D}(k) + \nu k - (\omega_c+\nu)\tau$ is small, or that the delay estimate does not fall out of lock [5], one can perform the following analysis:

$$\begin{aligned} E\{\hat{D}(k+1)\} &\approx \lambda E\{\hat{D}(k)\} + 2\mu\omega_c(\omega_c+\nu)\tau \\ &\quad -2\mu\omega_c\nu k - 2\mu\sigma_\theta^2 b(E\{\hat{D}(k)\}) \end{aligned} \tag{9}$$

$$\begin{aligned} &= \alpha\tau + \lambda(E\{\hat{D}(k)\} - \tau) \\ &\quad -2\mu\omega_c\nu k - 2\mu\sigma_\theta^2 b(E\{\hat{D}(k)\}) \end{aligned} \tag{10}$$

where

$$\lambda = 1 - 2\mu\omega_c^2 \tag{11}$$

$$\alpha = 1 + (1-\lambda)\frac{\nu}{\omega_c} \tag{12}$$

This can be further simplified with straightforward algebra to

$$\begin{aligned} E\{\hat{D}(k+1)\} &= \tau(\alpha + (\alpha-1)\sum_{i=1}^{k}\lambda^i) + \lambda^{k+1}(D(0)-\tau) \\ &\quad +(1-\alpha)k\sum_{i=0}^{k}\lambda^i - (1-\alpha)\sum_{i=1}^{k} i\lambda^i \\ &\quad -2\mu\sigma_\theta^2\sum_{i=0}^{k}\lambda^{k-i} b(E\{\hat{D}(i)\}) \end{aligned} \tag{13}$$

where $D(0)$ is an initial estimate. Substituting for α and evaluating the summations, this simplifies to

$$\begin{aligned} E\{\hat{D}(k+1)\} &= \tau[1 + \frac{\nu}{\omega_c}(1-\lambda^{k+1})] + \lambda^{k+1}(D(0)-\tau) \\ &\quad +\frac{\nu}{\omega_c}\frac{\lambda}{1-\lambda}(1-\lambda^{k+1}) + \frac{\nu}{\omega_c}k \\ &\quad -2\mu\sigma_\theta^2\sum_{i=0}^{k}\lambda^{k-i} b(E\{\hat{D}(i)\}) \end{aligned} \tag{14}$$

Clearly, a necessary condition for stability is that $|\lambda| < 1$, which in turn specifies the bounds on the stepsize parameter: $0 < \mu < 1/\omega_c^2$. If μ is chosen in this range, then as $k \to \infty$, one arrives at:

$$\begin{aligned} E\{\hat{D}(k+1)\} &= \tau(1+\frac{\nu}{\omega_c}) + \frac{\nu}{\omega_c}(\frac{\lambda}{1-\lambda}) + \frac{\nu}{\omega_c}k \\ &\quad -2\mu\sigma_\theta^2 \sum_{i=0}^{k} \lambda^{k-i} b(E\{\hat{D}(i)\}). \end{aligned} \quad (15)$$

3.2. Choice of μ or λ

The selection of the parameter μ—or equivalently, λ (see (11)) is critical to algorithm performance. A small value of μ implies a value of $\lambda \approx 1$ which causes large bias, and could mean the algorithm fails to track a fast moving delay. A large value of μ would solve these problems but estimates would be sensitive to noise. Simulations have shown for high SNR, there is little difference among the resulting mean square errors for different λ.

3.3. Regression analysis

Noting that (15) is (ignoring the final term) a straight line such as (7), whose parameters $\{A, B\}$ can be estimated by a standard linear regression technique [9] between starting point $\hat{D}(M)$ and end point $\hat{D}(K-1)$ (and hence over a data length $K-M$). From the line-fitting paramater estimates $\{\hat{A}, \hat{B}\}$ we have the proposed DTD and DFO estimators

$$\hat{\tau} = \frac{\hat{A} - c\hat{B}}{1+\hat{B}} \quad (16)$$

$$\hat{\nu} = \omega_c \hat{B} \quad (17)$$

where $c = \lambda/(1-\lambda)$.

4. ESTIMATION BIAS

It can be seen that (15) is not quite of the same form as (7). The final term in (15) contains an unknown parameters σ_θ^2 and $b(\hat{D}_k)$. If we denote the term by $T(\hat{D}(k), \lambda)$, we have

$$T(\hat{D}(k), \lambda) = \frac{\sigma_\theta^2(\lambda-1)}{\omega_c^2} \sum_{i=0}^{k} \lambda^{k-i} b(E\{\hat{D}(i)\}) \quad (18)$$

and the function $b(\hat{D}(k))$ for the LIF is depicted in figure 2 for various filter lengths $L = N+1$ (note LIFs are usually chosen to be of even lengths [7]) where it can be seen that the bias reduces as L increases and is symmetrical about the point $\hat{D}(k) = 0.5$, where it is zero. It should also be noted that in the situation of interest where $x(k)$ is the reference signal and $\theta(k)$ is quantization/round-off noise, σ_θ^2 will be typically small ($\approx 10^{-4}$ for quantization to 5 bits, corresponding to SNR≈ 35 dB). Hence, with this assumption, the contribution of $T(\hat{D}(k), \lambda)$ is negligible.

5. NUMERICAL RESULTS

Simulation tests have been conducted to compare the performance of the LETDE and SETDE algorithms. Two variants of each method are used—one method when the filter is modulated to $\omega_0 = \omega_c$ as in Section 2.3, and one where the filter is left unmodulated (or $\omega_0 = 0$). Hence we have four algorithms: modulated and unmodulated LETDE, and the modulated and unmodulated SETDE. In our experiments, $\tau = 0.3$, $\nu = 0.00003$, $L = 4$, SNR$= 40$ dB. Also, $\lambda = 0$ which implies $\mu = 1/(2\omega_c^2)$ and $s(k) = 1$ was assumed. The regression analysis was performed on 2000 data points from iterations 2981 to 4980. Results are the average of 50 runs. The performance was measured by the mean squared error (MSE) of the estimates, defined below:

$$\text{MSE}(\hat{\tau}) = \frac{1}{50}\sum_{i=1}^{50}(\hat{\tau}(i) - \tau)^2 \quad (19)$$

$$\text{MSE}(\hat{\nu}) = \frac{1}{50}\sum_{i=1}^{50}(\hat{\nu}(i) - \nu)^2. \quad (20)$$

In figure 3(a), we have the MSE for the time delay estimate $\hat{\tau}$ as ω_c changes. It can be seen that the modulated filters significantly outperform the unmodulated filters. For $\omega_c > 0.3\pi$, both unmodulated filters perform relatively poorly. However when $\omega_c < 0.3\pi$, the unmodulated LETDE has comparable performance with the modulated filters and appears much better than the unmodulated SETDE; this is due to the superior interpolation qualities of the LIF at low frequencies.

Figure 3(b) compares the MSE for the DFO estimate $\hat{\nu}$ and the corresponding CRLB. Again, the modulated algorithms yield estimates close to the CRLB (and the unmodulated LETDE but just for small ω_c) and appear superior to the unmodulated algorithms.

Finally, the signal $s(k)$ was chosen to be $\cos(Wk)$ and hence has a finite bandwidth W. Figure 4 shows just the DTD estimates MSE from this scenario (with $\lambda = 0.5$ and hence $\mu = 1/4\omega_c^2$) and the estimation quality of the modulated LETDE becomes apparent as it clearly outperforms the other algorithms. This is due to the significantly better interpolation properties of the LIF for frequencies in the vicinity of the maximally flat frequency [7]. This suggests for narrowband signals with finite bandwidth, the LETDE will render range and doppler estimates of significantly less MSE than the SETDE.

6. CONCLUSIONS

A new, computationally simple, method for computationally simple estimation of DTD and DFO has been presented. The performance of the well-known ETDE algorithm incorporating the modulated Lagrange interpolation filter has been studied by deriving its convergence dynamics and bias in addition to numerical examples. It is found through simulation that the new algorithm is able to render DTD and DFO estimation MSE comparable to the CRLB for all ω_c. In addition, similar performance is demonstrated using a signal of bandwidth $W < 0.1\pi$.

7. ACKNOWLEDGMENT

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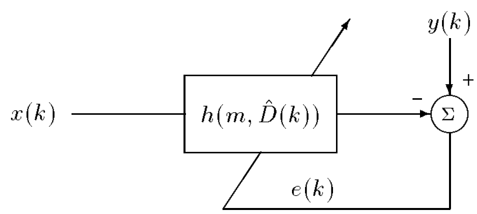

Figure 1: The ETDE configuration.

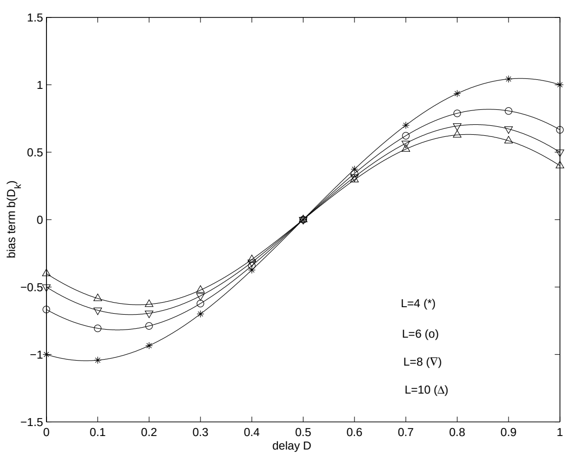

Figure 2: Bias term $b(\hat{D}(k))$ as a function of LIF length L.

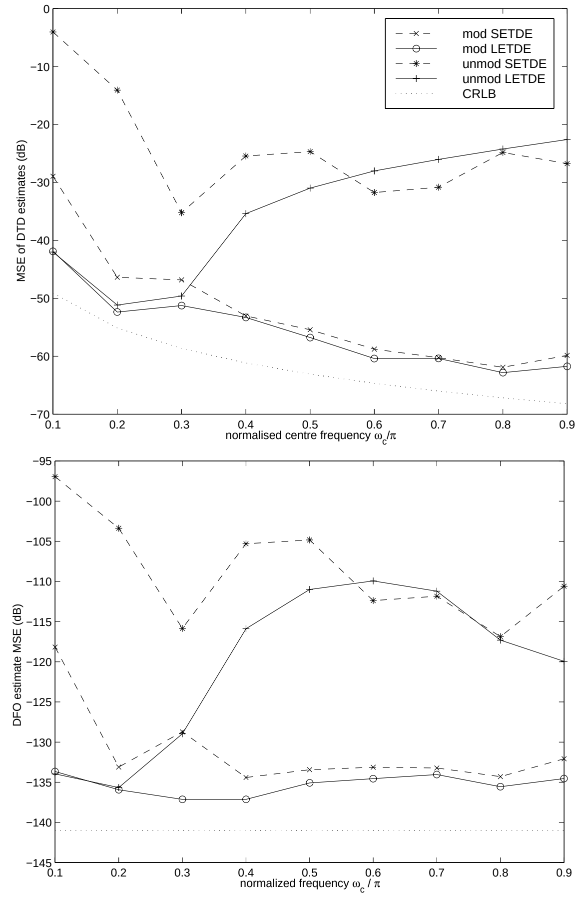

Figure 3: Estimation MSE for (a) DTD $\hat{\tau}$ and (b) DFO $\hat{\nu}$ for modulated Lagrange (-o-), modulated sinc (- -x- -), unmodulated Lagrange (-+-) and unmodulated sinc (- -*- -) ETDE algorithms; CRLBs are shown ($\cdots$).

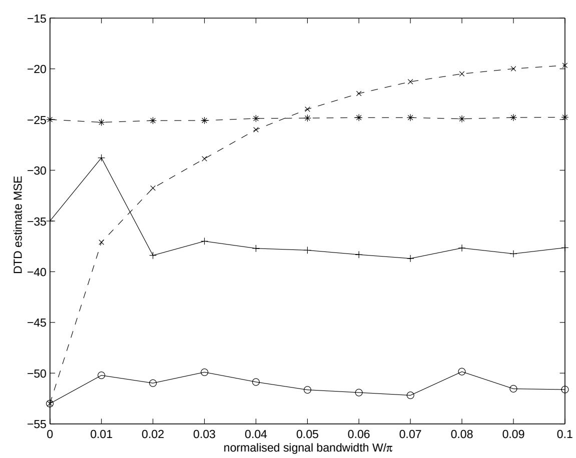

Figure 4: Estimation MSE for DTD $\hat{\tau}$ as W changes for modulated Lagrange (-o-), modulated sinc (- -x- -), unmodulated Lagrange (-+-) and unmodulated sinc (- -*- -) ETDE algorithms.