ESTIMATING RANGE, VELOCITY, AND DIRECTION WITH A RADAR ARRAY

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ABSTRACT

We present maximum likelihood (ML) methods for active estimation of range (time delay), velocity (Doppler shift), and direction of a point target with a radar array in spatially correlated noise with unknown covariance. We consider structured and unstructured array response models and compute the Cramér-Rao bound (CRB) for the time delay, Doppler shift, and direction of arrival. We derive ambiguity functions for the above models and discuss the relationship between identifiability, ambiguity, and the Fisher information matrix.

1. INTRODUCTION

In active radar, a known waveform is transmitted and the signal reflected from the target of interest is used to estimate its parameters. Typically, the received signal is modeled as a scaled, delayed, and Doppler-shifted version of the transmitted signal, see [1] for the standard model of a slowly fluctuating point target. Estimating the time delay and Doppler shift provides information about the position and relative motion of the target: the maximum likelihood (ML) procedure for the case of a single receiving antenna was derived in [1, Chapters 9 and 10]. However, recent advances in radar signal processing are associated with the use of antenna arrays.

Using a sensor array allows estimation of the target's direction and guarantees higher accuracy of range and velocity estimation. In this paper, we extend the slowly fluctuating point target model of [1] to account for multiple sensors, where the noise is correlated between sensors with unknown covariance. We consider both structured and unstructured array response models, also used in [2]. However, unlike [2] or [3], we use an arbitrary signal waveform, which allows for precise range estimation and waveform design. We develop ML estimation procedures for this model and derive estimation accuracy measures.

In Section 2, we introduce the array models (structured and unstructured) as well as noise model. In Section 3, we present the ML estimation procedures. For the structured array we estimate the target's direction of arrival (DOA), Doppler shift, and time delay. For the unstructured array, we derive ML estimates of the Doppler shift and time delay (since DOA estimation is not possible in this case). These estimates are more robust compared with the structured model, which is sensitive to errors in the array response model.

In Section 4, we compute the Cramér-Rao bound (CRB) for the DOA, Doppler shift, and time delay for the structured model. We show that the CRB for the Doppler shift and time delay under the unstructured model equals the corresponding CRB for the structured model, i.e. the structured and unstructured array ML methods are asymptotically equivalent for these parameters. Thus, when only range and velocity estimation are of interest, using the unstructured ML method has an advantage since it is robust, while guaranteeing the same asymptotic accuracy as the structured ML.

In Section 5, we derive ambiguity functions for the structured and unstructured array models and show how they relate to the Woodward's ambiguity function [1] used in radar processing with a single antenna. We discuss the connections between ambiguity, parameter identifiability, and the Fisher information matrix.

2. MODELS

Suppose an m-element antenna array receives a scaled, time delayed, and Doppler-shifted echo of a known complex bandpass signal $s(t)\exp(j\omega_c t)$. Knowing the time delay τ and Doppler shift ω_{D}, the target's range ρ and radial component of velocity v are determined by $\rho \approx c\tau/2$ and $v \approx c\omega_{\mathrm{D}}/(2\omega_c)$, see e.g. [1, Section 9.1].

In Sections 2.1 and 2.2 we describe the structured and unstructured array response models as well as noise model used to derive the ML estimation procedures and accuracy measures.

2.1. Structured Array Model

In the structured array model, a given model is assumed for the $m \times 1$ array response vector $\boldsymbol{a}(\boldsymbol{\theta})$ to a plane wave reflected from the target. Here, $\boldsymbol{\theta}$ contains DOA parameters

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(and may contain additional parameters, such as scattering and polarization coefficients).

Converting to base-band, the signal received by the array becomes

$$\boldsymbol{y}(t) = \boldsymbol{a}(\boldsymbol{\theta}) \cdot x \cdot \exp[j\omega_{\mathrm{D}} t] s(t-\tau) + \boldsymbol{e}(t), \tag{2.1}$$

where $t = 1, \ldots, N$ and x, τ, and ω_{D} are the complex amplitude, time delay, and Doppler shift, respectively. Note that we have assumed that the signal bandwidth and antenna locations are such that the time delay τ and direction of arrival $\boldsymbol{\theta}$ are common to the whole array. Equation (2.1) generalizes the slowly fluctuating target model in [1, Chapters 9 and 10] to account for multiple sensors, thus enabling DOA estimation. A special case of the above model is used in [2], [3] where $s(t) \equiv 1$ over the whole "range gate" $t = 1, 2, \ldots, N$. However, such a model does not allow estimation of the time delay τ, so the range resolution can be no better than the width of the range gate. Further, our model extends a variety of signal waveform models used in conventional (one-antenna) radar signal processing (see e.g. [1]) to multiple sensors.

The noise term $\boldsymbol{e}(t)$ models interference due to clutter, receiver noise, and jamming. When the noise is dominated by clutter, $\boldsymbol{e}(t)$ is usually both temporally and spatially correlated. Target-free (passive) measurements may be used to temporally pre-whiten the data. Throughout this paper we assume that $\boldsymbol{e}(t)$ is Gaussian, temporally uncorrelated and spatially correlated with unknown positive definite covariance Σ; thus the data has been temporally pre-whitened, if necessary. The same noise model is used in [2], [3].

Define $\boldsymbol{\eta} = [\omega_{\mathrm{D}}, \tau]^T$. The vector of unknown target parameters is then $\boldsymbol{\rho} = [\mathrm{Re}\{x\}, \mathrm{Im}\{x\}, \boldsymbol{\theta}^T, \boldsymbol{\eta}^T]^T$. Define also $\boldsymbol{\mu}(t, \boldsymbol{\rho}) = \boldsymbol{a}(\boldsymbol{\theta}) x \exp[j\omega_{\mathrm{D}} t] s(t-\tau)$, which is the noiseless array response at time t for the structured model. Observe that the dependence of $\boldsymbol{\mu}(t, \boldsymbol{\rho})$ on $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ is nonlinear. Thus, the estimation algorithm for the structured model requires a nonlinear search over the parameter space of dimension $\dim(\boldsymbol{\theta}) + 2$.

2.2. Unstructured Array Model

We describe the unstructured array response model in which the whole array response vector (also known as "spatial signature") is assumed to be unknown [2], [4]. This model is robust compared with an incorrectly structured array model. Also, the parametrization in (2.1) involves the nonlinear dependence of the data model on $\boldsymbol{\theta}$, necessitating a multi-dimensional search, see the previous section.

These problems can be overcome by using an unstructured model. Estimation using this model is robust and requires only a 2-D search, but does not provide an estimate of the DOA parameter vector $\boldsymbol{\theta}$, i.e. the DOA resolution can be no better than the width of the transmit beam. This model can be written as:

$$\boldsymbol{y}(t) = \boldsymbol{a} \exp[j\omega_{\mathrm{D}} t] s(t-\tau) + \boldsymbol{e}(t), \quad t = 1, \ldots, N, \tag{2.2}$$

with the same noise assumptions as for the structured model in the previous section. Thus, $\boldsymbol{a}(\boldsymbol{\theta}) \cdot x$ in the structured array model is simply substituted by an unstructured array response $\boldsymbol{a}$. The vector of unknown target parameters for this model is $\boldsymbol{\rho}_{\mathrm{u}} = [\mathrm{Re}\{\boldsymbol{a}\}^T, \mathrm{Im}\{\boldsymbol{a}\}^T, \boldsymbol{\eta}^T]^T$. The noiseless array response for this unstructured model at time t is then $\boldsymbol{\mu}_{\mathrm{u}}(t, \boldsymbol{\rho}_{\mathrm{u}}) = \boldsymbol{a} \exp[j\omega_{\mathrm{D}} t] s(t-\tau)$.

3. MAXIMUM LIKELIHOOD ESTIMATION

In this section we present the ML estimation procedures for the structured and unstructured models. They can be easily derived using the results in [4], [5], see [6].

3.1. Estimation with Structured Array

The concentrated likelihood function for the unstructured array model is [6]

$$l_{\mathrm{s}}(\boldsymbol{\theta}, \boldsymbol{\eta}) = 1 + \frac{1}{N} \frac{|\boldsymbol{z}(\boldsymbol{\eta})^* \widehat{R}^{-1} \boldsymbol{a}(\boldsymbol{\theta})|^2}{\boldsymbol{a}(\boldsymbol{\theta})^* \widehat{R}^{-1} \boldsymbol{a}(\boldsymbol{\theta}) \cdot [b(\tau) - \frac{1}{N} \boldsymbol{z}(\boldsymbol{\eta})^* \widehat{R}^{-1} \boldsymbol{z}(\boldsymbol{\eta})]}, \tag{3.1}$$

where $\boldsymbol{z}(\boldsymbol{\eta}) = \sum_{t=1}^{N} \boldsymbol{y}(t) s(t-\tau)^* \exp[-j\omega_{\mathrm{D}} t]$, $b(\tau) = \sum_{t=1}^{N} |s(t-\tau)|^2$, and $\widehat{R} = \sum_{t=1}^{N} \boldsymbol{y}(t) \boldsymbol{y}(t)^* / N$. If $s(t) \equiv 1$ then $\boldsymbol{z}(\boldsymbol{\eta})$ becomes the discrete Fourier transform (DFT) of $\boldsymbol{y}(t)$ and the concentrated likelihood in (3.1) reduces to the expression in [2, equation (16)], [3, equation (19)]. As commented earlier, under this assumption, the time delay τ cannot be estimated.

In the following, we present conditions for equivalence between the discrete-time (see above) and continuous-time processing of radar signals, widely used in radar literature [1]. Continuous-time results are often easier to interpret, at the cost of neglecting the finite sampling effects.

Assumption A. The reflected signal $s(t-\tau)$ is completely received within the "range gate" i.e. $t = 1, 2, \ldots, N$, and the sampling is dense, i.e. $N \to \infty$ but the duration of the range gate is fixed.

Under Assumption A, it follows that $b(\tau) \approx \int_{-\infty}^{\infty} |s(t)|^2 dt$ which is the energy of $s(t)$ and does not depend on τ. Also, the summation in $\boldsymbol{z}(\boldsymbol{\eta})$ can be replaced by an integral which then makes $\boldsymbol{z}(\boldsymbol{\eta})$ equal to to the *short-time Fourier transform* (STFT) [7, p. 94] of $\boldsymbol{y}(t)$ with a sliding window $s(t)$ and parameters τ and ω_{D}. Interestingly, for wideband Doppler, the received signal is of the form $\boldsymbol{\phi}(t, \boldsymbol{\eta}) = s((t-\tau)/\sigma)$, and thus the integral form of $\boldsymbol{z}(\boldsymbol{\eta})$ reduces to the *continuous wavelet transform* (CWT) of $\boldsymbol{y}(t)$.

3.2. Estimation with Unstructured Array

The concentrated likelihood function for the unstructured array can easily be obtained by maximizing $l_{\mathrm{s}}(\boldsymbol{\theta}, \boldsymbol{\eta})$ in (3.1) with respect to an arbitrary $\boldsymbol{a}(\boldsymbol{\theta})$ using the Cauchy-Schwartz inequality. The concentrated likelihood function simplifies to

$$l_{\mathrm{u}}(\boldsymbol{\eta}) = \frac{\boldsymbol{z}(\boldsymbol{\eta})^* \widehat{R}^{-1} \boldsymbol{z}(\boldsymbol{\eta})}{b(\tau)}. \tag{3.2}$$

Under Assumption A, the denominator of the above expression is equal to the signal energy and, therefore, independent of τ. Further, the term in the numerator can be interpreted as a weighted matched filter and reduces, for one sensor, to the *spectrogram*, i.e. squared magnitude of the STFT [7, p. 95]. Thus, the expression in (3.2) can be viewed as a multivariate extension of the spectrogram.

4. CRAMÉR-RAO BOUND

In [6] we derive the Cramér-Rao bounds for $\boldsymbol{\eta}$ (i.e. the time delay and Doppler shift) and show that they are equal for the unstructured and structured array models. Under the structured model, we further derive the CRB for the DOA parameters $\boldsymbol{\theta}$ and show that the bounds on $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ are uncoupled.

4.1. Structured Array

In [6] we derive the CRB for $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$:

$$\mathrm{CRB}_{\theta\theta} = \frac{1}{2b(\tau)|x|^2}\left[\mathrm{Re}\left\{D_a(\boldsymbol{\theta})^* \Sigma^{-\frac{1}{2}} \Pi^{\perp}(\boldsymbol{\theta}) \Sigma^{-\frac{1}{2}} D_a(\boldsymbol{\theta})\right\}\right]^{-1},$$
$$\mathrm{CRB}_{\eta\theta} = 0,$$
$$\mathrm{CRB}_{\eta\eta} = \frac{b(\tau)}{2|x|^2 \boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta})} \cdot \begin{bmatrix} \beta(\tau) & -\delta(\tau) \\ -\delta(\tau) & \xi(\tau) \end{bmatrix}^{-1},$$

where

$$\Pi^{\perp}(\boldsymbol{\theta}) = I - \frac{1}{\boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta})} \Sigma^{-\frac{1}{2}} \boldsymbol{a}(\boldsymbol{\theta}) \boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-\frac{1}{2}}, \quad (4.1a)$$
$$\beta(\tau) = b(\tau) b_2(\tau) - b_1(\tau)^2, \quad (4.1b)$$
$$\xi(\tau) = b(\tau) l(\tau) - |c(\tau)|^2, \quad (4.1c)$$
$$\delta(\tau) = b(\tau)\,\mathrm{Im}\left\{c_1(\tau)\right\} - b_1(\tau)\,\mathrm{Im}\{c(\tau)\}, \quad (4.1d)$$

and $b_1(\tau) = \sum_{t=1}^{N} t|s(t-\tau)|^2$, $b_2(\tau) = \sum_{t=1}^{N} t^2|s(t-\tau)|^2$, $c(\tau) = \sum_{t=1}^{N} s(t-\tau)^* d(t-\tau)$, $c_1(\tau) = \sum_{t=1}^{N} t s(t-\tau)^* d(t-\tau)$, $l(\tau) = \sum_{t=1}^{N} |d(t-\tau)|^2$, $d(t) = ds(t)/dt$, $D_a(\boldsymbol{\theta}) = \partial \boldsymbol{a}(\boldsymbol{\theta})/\partial \boldsymbol{\theta}^T$. Note that the above CRB expressions do not depend on the Doppler frequency ω_{D}. Thus, the estimation accuracy of the target parameters is independent of the Doppler shift.

The CRB for $\boldsymbol{\theta}$ and $\boldsymbol{\eta}$ is block-diagonal. Hence, $\mathrm{CRB}_{\eta\eta}$ remains the same regardless of whether or not $\boldsymbol{\theta}$ is known. Also, under Assumption A, $\mathrm{CRB}_{\theta\theta}$ depends neither on $\boldsymbol{\eta}$ (since $b(\tau) = b$) nor on the signal shape: it is just inversely proportional to the received signal energy $b \cdot |x|^2$.

The $\mathrm{CRB}_{\eta\eta}$ matrix depends on the array response $\boldsymbol{a}(\boldsymbol{\theta})$ only through the term $\boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta})$ via an inversely proportional relation. Furthermore, for spatially white noise (i.e. $\Sigma = \sigma^2 I$) we have that $\boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta}) = m/\sigma^2$. This implies that the bound on asymptotic accuracy of the Doppler shift and time delay is proportional to $1/m$, where m is the number of antennas. Therefore, if the noise is spatially white, the accuracy of the Doppler shift and time delay estimation does not depend on the array configuration, and the $\mathrm{CRB}_{\eta\eta}$ above can be achieved by merely averaging the single-sensor ML estimates over all sensors.

Under Assumption A, the $\mathrm{CRB}_{\eta\eta}$ is independent of the time delay τ, as shown in [6]. Also, β and ξ in equations (4.1b) and (4.1c) become proportional to the effective duration and effective bandwidth of $s(t)$, respectively, as defined in [8, equations (1.21) and (1.22)]. For normalized signal energy (i.e. $b = 1$), these definitions are the same as those in [7, Sections 1.2 and 1.3]. Also, δ can be viewed as the covariance of a signal, i.e. a measure of how time is correlated with the instantaneous frequency, see also [7, equation (1.124)].

4.2. Unstructured Array

In [6] we prove that, for the unstructured model

$$\mathrm{CRB}_{\eta\eta} = \frac{b(\tau)}{2\boldsymbol{a}^* \Sigma^{-1} \boldsymbol{a}} \cdot \begin{bmatrix} \beta(\tau) & -\delta(\tau) \\ -\delta(\tau) & \xi(\tau) \end{bmatrix}^{-1}, \quad (4.2)$$

which is equal to the expression in equation (4.1) after substituting $\boldsymbol{a}(\boldsymbol{\theta}) \cdot x$ by $\boldsymbol{a}$. Thus, the bound on accuracy of the Doppler shift and time delay is the same for both the structured and unstructured array model (i.e. the structured and unstructured ML estimation procedures are asymptotically equally efficient for these parameters), which is an extension of the result in [2] for the bound on accuracy of Doppler shift estimation and signal model $s(t) = 1$. Therefore, the unstructured ML method should be used to estimate the Doppler shift and time delay due to its robustness to modeling inaccuracies compared with the structured ML, see also Sections 2.1 and 2.2.

We use the CRB expressions presented in Sections 4.1 and 4.2 for optimizing the radar system parameters, e.g. the sensor locations and signal waveform, see [6].

5. GLOBAL ACCURACY, AMBIGUITY AND FISHER INFORMATION

In this section we discuss global accuracy as characterized by ambiguity functions, and relations to identifiability and Fisher information. Unlike the FIM, which is a local measure of estimation accuracy, the ambiguity function is used to assess the global resolution and large error properties of the estimates [1]. We derive ambiguity functions for the sensor array models discussed in this paper using a generalized ambiguity function in [9].

5.1. Woodward's Ambiguity Function

In a single-antenna radar system, the system parameters reduce to the signal waveform, which should be chosen to attain good resolution of range (time delay) and velocity (Doppler shift).

Assume that we have two targets with parameters $\boldsymbol{\eta} = [\omega_{\mathrm{D}}, \tau]^T$ and $\boldsymbol{\eta}_0 = [\omega_{\mathrm{D}0}, \tau_0]^T$. The standard ambiguity function for a single antenna is [1, p. 279]: $|\int_{-\infty}^{\infty} s(t - \frac{\delta\tau}{2}) s(t + \frac{\delta\tau}{2})^* \exp(j\delta\omega_{\mathrm{D}} t) dt|^2$, where $\delta\tau = \tau - \tau_0$ and $\delta\omega_{\mathrm{D}} = \omega_{\mathrm{D}} - \omega_{\mathrm{D}0}$ denote shifts in time and angular frequency between the two targets, respectively. It is a measure of the degree of similarity between the complex envelope and its replica which is shifted in time and frequency. Its main use is to measure the range-velocity resolution attainable for a given waveform, see below. The above expression is sometimes referred to as Woodward's ambiguity function because of his pioneering work in [10].

Suppose that the ambiguity function has an effective width of $\Delta\eta = [\Delta\tau, \Delta\omega_{\mathrm{D}}]$. This roughly means that two point targets spread by less than $c\Delta\tau/2$ in range and less than $c\Delta\omega_{\mathrm{D}}/(2\omega_c)$ in radial velocity cannot be distinguished by the radar. Two such targets are said to be ambiguous; hence the name ambiguity function.

5.2. Sensor Array Ambiguity Functions

We start by defining a distance measure between two multivariate Gaussian distributions. Then, we use this measure to derive a generalized ambiguity function, following the definition in [9].

For the structured array we use the following measure of separation between the probability density functions of the measurements corresponding to two targets with parameters $\boldsymbol{\rho}_0$ and $\boldsymbol{\rho}$ (assuming fixed covariance Σ):

$$d(\boldsymbol{\rho}, \boldsymbol{\rho}_0) = \sum_{t=1}^{N} [\boldsymbol{\mu}(\boldsymbol{\rho}, t) - \boldsymbol{\mu}(\boldsymbol{\rho}_0, t)]^* \Sigma^{-1} [\boldsymbol{\mu}(\boldsymbol{\rho}, t) - \boldsymbol{\mu}(\boldsymbol{\rho}_0, t)]. \tag{5.1}$$

The corresponding distance measure for the unstructured model follows by replacing $\boldsymbol{\mu}$, $\boldsymbol{\rho}$, and $\boldsymbol{\rho}_0$ in the above equation by $\boldsymbol{\mu}_{\mathrm{u}}$, $\boldsymbol{\rho}_{\mathrm{u}}$ and $\boldsymbol{\rho}_{\mathrm{u}0} = [\mathrm{Re}\{\boldsymbol{a}_0\}^T, \mathrm{Im}\{\boldsymbol{a}_0\}^T, {\boldsymbol{\eta}_0}^T]^T$, respectively.

The measure in (5.1) is by definition the square of the Mahalanobis distance; it is also the Kullback-directed divergence, used in [9]. Additionally, the identifiability by distribution of the target parameters $\boldsymbol{\rho}$ reduces to the following requirement: $d(\boldsymbol{\rho}, \boldsymbol{\rho}_0) = 0$ if and only if $\boldsymbol{\rho} = \boldsymbol{\rho}_0$.

As observed earlier, both the structured and unstructured model are special cases of the general model in [6]. Thus, in [6] we first consider the ambiguity function for that general model, and then derive the special cases. We use the ambiguity function with nuisance parameters, see [9, Definition 2].

For the unstructured array, the nuisance parameter vector is $\boldsymbol{a}$ and the ambiguity function follows [6]

$$\mathcal{A}(\boldsymbol{\eta}, \boldsymbol{\eta}_0) = \frac{1}{b(\tau) b(\tau_0)} \left| \sum_{t=1}^{N} s(t-\tau_0) s(t-\tau)^* \exp(-j(\omega_{\mathrm{D}} - \omega_{\mathrm{D}0})t) \right|^2.$$

Again, under Assumption A in Section 3.1, $b(\tau) = b(\tau_0) =$ constant and the above summation can be substituted by a corresponding integral. Then, the above expression becomes proportional to the Woodward's ambiguity function in the previous section. Thus, signal analysis based on the ambiguity in e.g. [1, Chapter 10] is directly applicable to the unstructured array model.

Next, we present the ambiguity function for the structured array model (where the nuisance parameter is x):

$$\mathcal{A}([\boldsymbol{\theta}^T, \boldsymbol{\eta}^T]^T, [\boldsymbol{\theta}_0^T, \boldsymbol{\eta}_0^T]^T) = = \frac{|\boldsymbol{a}(\boldsymbol{\theta}_0)^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta})|^2}{\boldsymbol{a}(\boldsymbol{\theta}_0)^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta}_0) \cdot \boldsymbol{a}(\boldsymbol{\theta})^* \Sigma^{-1} \boldsymbol{a}(\boldsymbol{\theta})} \cdot \mathcal{A}(\boldsymbol{\eta}, \boldsymbol{\eta}_0),$$

see [6]. This ambiguity function is a measure of DOA-range-velocity resolution attainable for a given sensor array configuration and signal waveform. Observe the separation between the DOA and range-velocity parameters in the above expression: this is consistent with the local accuracy result in Section 4.1 stating that the CRB for the DOA and range-velocity parameters is block-diagonal. If we also choose $\boldsymbol{\theta}$ to be a nuisance parameter, then the ambiguity function reduces to $\mathcal{A}(\boldsymbol{\eta}, \boldsymbol{\eta}_0)$.

Local behavior of the distance measure $d(\boldsymbol{\rho}, \boldsymbol{\rho}_0)$ is of interest: it is desirable to have good resolution when the target parameters are close, i.e. $\boldsymbol{\rho} = \boldsymbol{\rho}_0 + \delta\boldsymbol{\rho}$. Then, the distance measure $d(\boldsymbol{\rho}, \boldsymbol{\rho}_0)$ and the Fisher information matrix of the source parameters $\boldsymbol{\rho}$ (or $\boldsymbol{\rho}_{\mathrm{u}}$ for the unstructured model) are related by the following simple formula: $d(\boldsymbol{\rho}_0 + \delta\boldsymbol{\rho}, \boldsymbol{\rho}_0) \approx \frac{1}{2}\delta\boldsymbol{\rho}^T \mathcal{I} \delta\boldsymbol{\rho}$.

6. CONCLUDING REMARKS

We developed ML methods for estimating the range, velocity, and direction of arrival of a moving target by radar arrays where the noise was assumed to be temporally uncorrelated and spatially correlated with unknown covariance. Two array models were used: structured and unstructured. We derived Cramér-Rao bounds for the unknown target parameters. Finally, we derived ambiguity functions for the array models and showed how they relate to Woodward's (single-antenna) ambiguity function.

In [6] we apply the derived bounds to optimal design of system parameters, i.e. sensor array configuration and signal shape. We derive a criterion for optimizing the accuracy of the target parameters.

Further research will include developing the ML method and accuracy measures for a wideband signal model with radar or sonar sensor arrays. Passive estimation of moving sources will also be considered.

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