COMBATING CHANNEL DISTORTIONS FOR CHAOTIC SIGNALS

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ABSTRACT

Over the past several years, there have been various proposals for communication with chaotic signals. But the issue of compensating the distortions introduced by the physical channel like noise, time varying fading and multi-path has not been fully addressed. In this paper, we first describe a noise reduction method for chaotic signals corrupted by an additive noise. The method uses the phenomenon of chaos synchronization to approximate the maximum likelihood (ML) decoder for the AWGN channel. Further we use the synchronizing receiver to nullify slowly time varying fading and multi-path. We find the region of operation for such a receiver and show how the time varying parameters characterizing such channels can be tracked at the receiver.

1. INTRODUCTION

Communicating with chaotic signals has been an active area of research for the past several years [1, 2, 3]. The retrieval of information by observing the symbolic dynamic sequence encoded in the chaotic trajectory [1] or by synchronization at the receiver [2, 3], is limited by the channel distortions. A real life communication channel typically adds noise and other distortions like linear filtering, time varying fade and multi-path. In many wireless communication scenarios, we can assume slow time variation of the fades and the delays of the various paths. In this paper, we propose methods to deal with noise and time varying fading and multi-path which would be useful and easy to implement in the context of communicating with chaos. In section 2 of this paper, we describe a noise reduction method exploiting the phenomenon of chaos synchronization [4] and illustrate the results using a numerical experiment. There have been various methods proposed for noise reduction in chaotic time-series [5, 6, 7, 8]. The proposed method would have advantage with respect to ease of implementation and would approximate the ML estimator of the transmitted chaotic trajectory under AWGN.

An approach to channel equalization was considered in [9], where it was shown that one could construct an equalizer such that the equalized signal driving the synchronizing receiver results in a small synchronization error. Their strategy hence proposed a separate system before the synchronizing receiver. As we show in section 3, in the context of fading and multi-path, one can directly exploit synchronization to nullify these distortions without the need of a *separate* system before the synchronizing receiver. We find the conditions for operation of such a receiver and in section 4, we deal with the tracking of slowly time-varying parameters characterizing such channels.

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2. NOISE REDUCTION

Let the transmitted chaotic signal be given by $x(t)$ and the noise added by the channel be given by $n(t)$. Hence the received signal is given by $x(t) + n(t)$. We will assume that the noise $n(t)$ is ergodic, zero mean and un-correlated with any trajectory of the transmitter chaotic system. We would expect that thermal noise in a telephone line for example, is uncorrelated with the trajectories of the transmitter system. Let $\bar{n}(t)$ be the estimate of the noise generated at the receiver by some means such that $y(t) = x(t) + n(t) + \bar{n}(t)$, is a trajectory of the transmitter system, not necessarily same as $x(t)$. Hence

$$\begin{aligned} E\{\bar{n}^2(t)\} &= E\{(x(t) - y(t) + n(t))^2\}, \\ &= E\{(x(t) - y(t))^2\} + \sigma^2, \end{aligned} \tag{1}$$

where we use the fact that $n(t)$ is un-correlated with $x(t)$ and $y(t)$, and σ^2 denotes the variance of $n(t)$. It follows from Eq. (1) that $\bar{n}(t)$ has minimum norm only if $x(t) = y(t)$. Due to exponential divergence, we expect that $x(t)$ and $y(t)$ are typically far from each other and un-correlated, in which case $E\{(x(t) - y(t))^2\}$ $= 2\text{Variance}(x(t))$. An exception would be when $y(t) = x(t-\tau)$, in which case $E\{(x(t) - y(t))^2\} = 2(R_{xx}(0) - R_{xx}(\tau))$, where $R_{xx}(.)$ denotes the autocorrelation function of $x(t)$. In both the cases, $E\{(x(t) - y(t))^2\}$ is a positive quantity. The pathological case of $n(t) = y(t) - x(t)$ is ruled out because of the un-correlated assumption of noise with any trajectory of the transmitter dynamical system. Hence we want to find the nearest trajectory to the received noisy signal. If $n(t)$ is white gaussian noise, then the nearest trajectory rule or minimum distance decoding corresponds to maximum likelihood (ML) decoding. In what follows we suggest a method exploiting chaos synchronization to find the nearest trajectory.

We know that under no noise conditions, any trajectory passes through the synchronizing receiver undistorted. Hence we generate $\bar{n}(t)$ such that $x(t) + n(t) - \bar{n}(t)$ drives the synchronizing receiver and let $y(t)$ denote its output. If the input and output are same i.e. $y(t) = x(t) + n(t) - \bar{n}(t)$, then we may conclude that $y(t)$ is a trajectory of the transmitter chaotic system. The method is illustrated in Fig. 1. To find the nearest trajectory, it follows from Eq. (1) that $\bar{n}(t)$ must have minimum norm. Let the available noisy time series at the receiver be $\{x(i) + n(i)\}_{i=1}^{N}$, which is the appropriately sampled version of the continuous time received

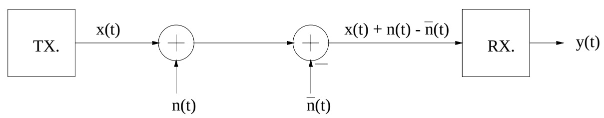

Figure 1: The estimate of the noise is subtracted from the received signal.

signal. Define the functions T_j $(j = 1, \cdots, N-M+1)$, as follows

$$T_j(\bar{n}(j), \bar{n}(j+1), \cdots, \bar{n}(j+M-1)) = \\ \sum_{i=j-K}^{j-1} e^{\alpha(i-j)}(y(i)+\bar{n}(i)-x(i)-n(i))^2 \\ + \sum_{i=j}^{j+M-1} ((y(i)+\bar{n}(i)-x(i)-n(i))^2 + \lambda\, \bar{n}^2(i)) \\ + \sum_{i=j+M}^{j+M+K-1} e^{\alpha(j-i)}(y(i)+\bar{n}(i)-x(i)-n(i))^2, \tag{2}$$

where λ is regularization parameter added to get the minimum norm solution. We use an iterative procedure consisting of successive minimizing "passes", where in each pass we minimize T_j incrementing j from 1 to $N-M+1$, and we use the estimate of $\bar{n}(t)$ from the previous pass as the initial condition for minimization on the current pass. In the expression for T_j, Eq. (2), the first and the last summation terms are added since the information about a sample is also contained in the preceding and succeeding terms and the exponential factor indicates the decay of this information. K limits the sum when the exponential term becomes small and varies $\sim 1/\alpha$. We use the previously generated estimates $\bar{n}(j-K), \cdots, \bar{n}(j-1)$ and $\bar{n}(j+M), \cdots, \bar{n}(j+M+K-1)$ for calculation and minimize over the variables $\bar{n}(j), \cdots, \bar{n}(j+M-1)$. The state vector of the synchronizing receiver at time instant $i = 1$ denoted by $\mathbf{w}(1)$, is unknown and is used to solve for the output of the synchronizing receiver at later instants. We estimate $\mathbf{w}(1)$ in each pass by minimizing the function

$$G(\mathbf{w}(1)) = \sum_{i=1}^{N} (y(i) - x(i) - n(i))^2. \tag{3}$$

It was observed in our numerical experiments that this combination of synchronization and adjustment of the initial state vector works well and does not impose severe precision constraints on $\mathbf{w}(1)$. (In particular, if we only adjust $\mathbf{w}(1)$ without using the synchronization method, extremely high precision of $\mathbf{w}(1)$ would be required due to the exponential sensitivity of chaotic systems.) Our typical strategy involves choosing λ as initially large and then gradually decreasing it after each pass by a scaling factor. For minimization, we use the downhill simplex method available in numerical recipes, requiring only function computations. The method was applied to the continuous time Lorenz system. The Lorenz equations (variables X, Y, Z) and the equations for the synchronizing receiver (variables $X_r(t)$, $Y_r(t)$, $Z_r(t)$) are as follows [2] :

$$\dot{X} = \sigma(Y - X) \; ; \; \dot{Y} = \rho X - Y - XZ \; ; \; \dot{Z} = XY - \beta Z \; ;$$

$$\dot{X_r} = \sigma(Y_r - X_r) \; ; \; \dot{Y_r} = \rho X - Y_r - XZ_r \; ; \; \dot{Z_r} = XY_r - \beta Z_r,$$

where $(\sigma, \rho, \beta) = (16.0, 45.0, 4.0)$. The time-series in this case consists of the received signal sampled at 0.01 units. The sampled signal was then interpolated and used for integration to calculate the output of the synchronizing receiver. Polynomial interpolation of order two (quadratic) was used, where the point $X(n+\alpha)$ at a distance $\alpha \in [0,1]$ from $X(n)$ is determined as : $X(n+\alpha) = 0.5\alpha(\alpha+1)X(n+1) + (1-\alpha^2)X(n) - 0.5\alpha(1-\alpha)X(n-1)$. We minimized the function T_j as defined by Eq. (2) when the signal to noise ratio (SNR) of the received signal was 10 dB. The parameters α, K, M, N were chosen as 0.25, 5, 6, 80 respectively. The parameter λ was decreased by a factor of 2 after each pass with an initial value of 5.0.

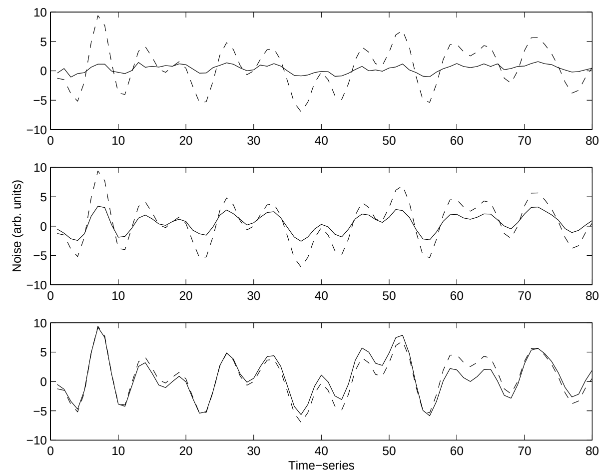

Figure 2: The estimated noise (solid line) as the number of passes over the time-series are increased : (a) 1 pass, (b) 5 passes, (c) 15 passes. Dashed line represents the additive gaussian noise with $\sigma = 3.69$, having same the bandwidth as the transmitted chaotic signal.

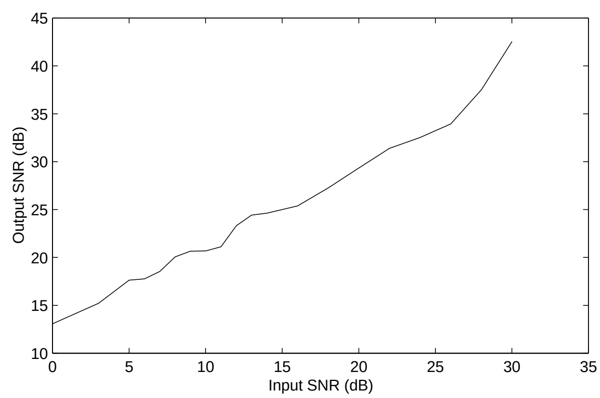

Figure 3: The output SNR as a function of the input SNR.

Figure 2 shows the actual and the estimated noise at various stages of the minimization. As shown in Fig. 2, the estimate of the noise $\bar{n}(t)$ approaches the actual noise $n(t)$ as the number of passes are increased and λ is decreased. The final value of SNR after 15 passes was 20.7 dB. Figure 3 plots the input versus the output SNR. Typical gain in SNR by this filtering operation is around 10 dB. The performance is limited by the interpolation inaccuracy and a higher order interpolation would be expected to give a larger gain in the final SNR.

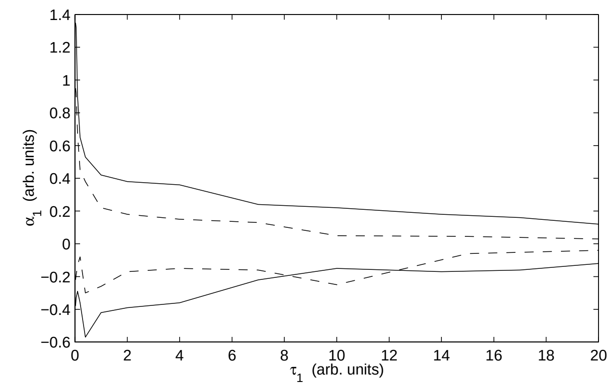

Figure 4: The allowable region for synchronization to occur in the α_1-τ_1 plane. The solid lines denote the boundary of the region for $L = 2$ and the dashed lines for $L = 3$ when $(\alpha_2, \tau_2) = (0.1, 10)$.

3. COMBATING FADING AND MULTI-PATH

Our strategy is to use synchronization for the recovery of a chaotic signal transmitted through a fading and multi-path channel. We illustrate this idea by transmitting the state variable $Y(t)$ of the Lorenz system. The model for fading and multi-path channels where the received signal $R(t)$ is a weighted sum of time delayed copies of the transmitted signal $Y(t)$, is given by

$$R(t) = \sum_{i=0}^{L-1} \alpha_i Y(t - \tau_i). \tag{4}$$

The synchronizing receiver for this system is constructed as

$$\begin{aligned} \dot{X}_r &= \sigma(Y_r - X_r) + \sigma[R(t) - \sum_{i=0}^{L-1} \alpha_i \, Y_r(t - \tau_i)], \\ \dot{Y}_r &= \rho X_r - Y_r - X_r Z_r, \\ \dot{Z}_r &= X_r Y_r - \beta Z_r, \end{aligned} \tag{5}$$

where the fades α_i's and delays τ_i's are assumed to be known at the receiver. Later we consider the case when α_i and τ_i are time varying and unknown at the receiver. Since a necessary condition for synchronization is that all the conditional Lyapunov exponents of the receiver should be negative, it is of interest to know how they behave as a function of α_i and τ_i. We assume $\tau_0 = 0$ which can be done by shifting the time axis of the receiver, and $\alpha_0 = 1$ since the overall gain term if known, can be compensated at the receiver, but this is not the case with the relative magnitudes. We consider first the case of $L = 2$ (i.e two paths). Figure 4 plots the allowable region in the α_1-τ_1 plane where the conditional Lyapunov exponents are sufficiently negative, such that good synchronization is achieved. If the least negative Lyapunov exponent is too near zero, then there may be bubbling in the attractor due to Lyapunov exponents of invariant measures embedded in the synchronized chaos being positive [10, 11]. The region between the solid lines is the allowable region (no bubbling) for the above case ($L = 2$). We also plotted the same region for $L = 3$ (in the dashed line) when (α_2, τ_2) are kept fixed at $(0.1, 10)$. As can be seen from the plot, the region is smaller than the previous case when $L = 2$. We comment here that the region is dependent on the choice of the synchronizing receiver, and it is possible that this region is larger for some other choice of the synchronizing receiver [12]. The bandwidth of the signal is ~ 5 (appropriate units). It is interesting to note that chaos synchronization becomes an effective tool for signal processing in this case, extracting the signal from a mixture of time delayed versions of the same signal. Since α_i's and τ_i's are typically modeled as random processes, one could find from Fig. 4, the probability that good synchronization is observed.

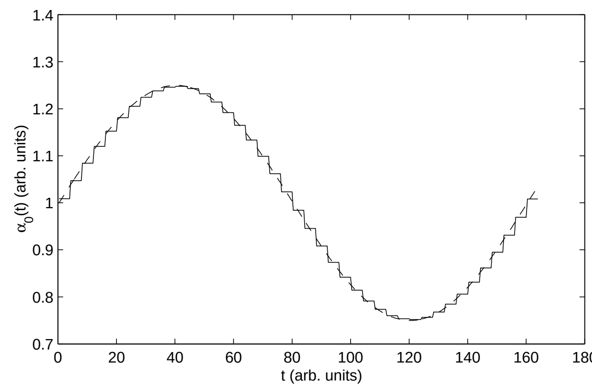

Figure 5: Tracking of the parameter $\alpha_0(t)$.

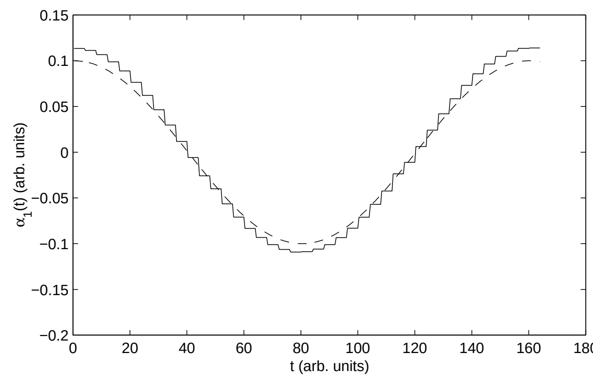

Figure 6: Tracking of the parameter $\alpha_1(t)$.

4. TRACKING OF PARAMETERS

Due to the typical occurrence of time variation of the parameters, it is of interest to know whether they can be tracked at the receiver. One can use simple methods to track the time varying parameters based on the minimization of synchronization error [13]. The synchronization error measurable at the receiver is given by

$$e(t) = \sum_{i=0}^{L-1} [\alpha_i(t) \, Y_r(t - \tau_i(t)) - \tilde{\alpha_i} \, Y_r(t - \tilde{\tau_i})].$$

where $\tilde{\alpha_i}$ and $\tilde{\tau_i}$ are the receiver estimates of α_i and τ_i, respectively. We generate the estimates at the receiver by minimizing the function

$$J(\tilde{\alpha_i}, \tilde{\tau_i}) = \int_t^{t+\bar{t}} e^2(t) dt$$

where $\bar{t}$ is chosen large enough such that the variance of J is small when initial conditions of the synchronizing receiver are varied

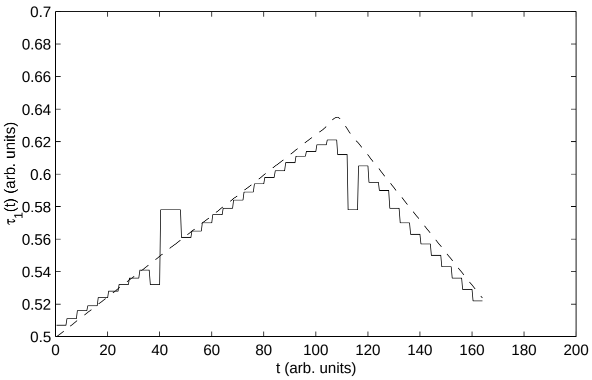

Figure 7: Tracking of the parameter $\tau_1(t)$.

[13]. We assume that the parameters are slowly time varying. This is a reasonable assumption in many practical cases of interest. We consider the case of $L = 2$ and vary the parameters α_0, α_1 in a sinusoidal fashion and τ_1 linearly. The first equation of the synchronizing receiver (Eq. (5)) is replaced by

$$\begin{aligned} \dot{X}_r &= \sigma(Y_r - X_r) \\ &+ \frac{\sigma}{\tilde{\alpha}_0(t)}(R(t) - \tilde{\alpha}_0(t)\,Y_r(t) - \tilde{\alpha}_1(t)\,Y(t - \tilde{\tau}_1(t))), \end{aligned}$$

Figures 5,6,7 plot their time variation and the result of the tracking algorithm. Since the tracking algorithm estimates the parameters by minimization of synchronization error over a time interval, the estimation is not exact due to continuous time variation of parameters. In the tracking of the time τ_1, the estimation goes bad twice. These are the points where $\alpha_1(t)$ is nearly zero and bad estimate of τ_1 does not have much bearing on the synchronizing error. Also note that, τ_1 was decreased at a larger rate than when it was increased. As the plot shows, the error in the estimation of τ_1 is larger when it is decreased, which results in a larger synchronization error.

5. CONCLUSIONS

In conclusion, we have shown that chaos synchronization can be used to nullify various distortions introduced by the channel. The method allows us to generate a minimum norm noise estimate without imposing precision constraints on the initial conditions. In the case of fading and multi-path, one can directly use the given signal to drive the receiver which does the inverse operation by synchronization. The region of operation for such a receiver is determined by the conditions under which synchronization holds. Slow time variation can be tracked at the receiver by minimization of measurable synchronization error.

6. REFERENCES

[1] S. Hayes, C. Grebogi, and E. Ott, “Communicating with Chaos”, Phys. Rev. Lett., vol.71, pp.65-68, 1993.

[2] K.M. Cuomo, and A.V. Oppenheim, “Chaotic signals and systems for communications”, Proc. 1993 IEEE ICASSP, vol.3., pp.137-140, 1993.

[3] L. Kocarev, and U. Parlitz, “General Approach for Chaotic Synchronization with Applications to Communication”, Phys. Rev. Lett. vol.74, pp.5028-5031, 1995.

[4] L.M. Pecora, and T.L. Caroll, “Synchronization in Chaotic Systems”, Phys. Rev. Lett. vol.64, pp.821-824, 1990.

[5] E.J. Kostelich and J.A. Yorke, “Noise Reduction in Dynamical Systems”, Phys. Rev. A. vol.38, pp.1649-1653, 1990.

[6] S.M. Hammel, “A noise reduction method for chaotic systems”, Phys. Lett. A vol.148, pp.421-428, 1990.

[7] P. Grassberger, R. Hegger, H. Kantz, C. Schaffrath and T. Schreiber, “On noise reduction method for chaotic data”, CHAOS, vol.3, pp127-135 , 1993.

[8] E. Rosa, S. Hayes and C. Grebogi, “Noise Filtering in Communication with Chaos”, Phys. Rev. Lett. vol.78, pp.1247-1250, 1997.

[9] K.M. Cuomo, A.V. Oppenheim, and R.J. Barron, “Channel Equalization for self-synchronizing Chaotic Systems”, Proc. 1996 IEEE ICASSP, pp.1605-1608, 1996.

[10] E. Ott, J.C. Sommerer, J.C. Alexander, I. Kan, and J.A. Yorke, “Scaling Behavior of Chaotic Systems with Riddled Basins”, Phys. Rev. Lett. vol.71, pp.4134-4137, 1993.

[11] S.C. Venkataramani, B.R. Hunt, and E. Ott, “Transitions to Bubbling of Chaotic Systems”, Phys. Rev. Lett. vol.77, pp.5361-5364, 1996.

[12] M. Ding, and E. Ott, “Enhancing synchronism of chaotic systems”, Phys. Rev. E., vol.49, pp.945-948, 1994.

[13] N. Sharma and P.G. Poonacha, “Tracking of Synchronized Chaotic Systems with Applications to Communications”, Phys. Rev. E., vol.56, pp.1242-1245, 1997.