LEAST-SQUARES CMA WITH DECORRELATION FOR FAST BLIND MULTIUSER SIGNAL SEPARATION

F.R.P. Cavalcanti, J.M.T. Romano, A.L. Brandão

Communications Department, Campinas State University
CP 6001, Campinas, 13083970, Brazil

ABSTRACT

A blind algorithm with implicit signal selectivity capability is proposed. The algorithm is an evolution of the original multiuser constant modulus algorithm of [1]. The new algorithm features a least-squares type updating rule for fast convergence rate and an adaptive control of the weight of the decorrelation term which improves the steady-state error variance. The expected improvements of the proposed algorithm are verified through simulations with smart antennas in a spatial-division multiple access system.

1. INTRODUCTION

The problem of multiuser signal separation has received attention due to its relevance for spatial and code division multiple access techniques (S/CDMA). In this context, it has been proposed recently in [1] a constant modulus algorithm with multiuser signal separation capability (MU-CMA). This capability was achieved by introducing in the optimization criterion a term which penalizes cross-correlations between multiuser output signals. Proposed MU-CMA has a LMS-like updating rule and therefore features a slow convergence rate which may preclude its use in fast time-varying environments. Furthermore, we point out in this paper that the extra decorrelation term increases the steady-state error variance, which degrades the corresponding bit error rate performance. Based on the above observations, a new algorithm is proposed which differs from the MU-CMA in the following aspects:

- a least-squares type updating rule is employed in order to increase the convergence rate and,
- an adaptive control of the decorrelation term weight is used in order to reduce steady-state error variance after multiuser signals are sufficiently separated.

The improvements obtained with the proposed algorithm are demonstrated through simulations with smart antennas in a SDMA application.

The rest of this paper is organized as follows. In section 2 we explain the application of interest in this paper. Section 3 reviews the original MU-CMA algorithm and analyzes its performance through simple simulations. Section 4 presents the proposed algorithm. Section 5 presents simulation results that validate the superior performance of the proposed algorithm. Finally, section 6 summarizes the present paper.

2. APPLICATION OF INTEREST

The application of interest is an AWGN symbol-synchronous SDMA system with possible power imbalances among users. An M-element uniform linear antenna array is placed in the receiver. A digital beamformer is provided for each active user. The output signal of the i-th user's beamformer is given by:

$$y_i[n]=\boldsymbol{w_i}^{T}\boldsymbol{x} \tag{1}$$

where: $\boldsymbol{w_i}=[w_{i1}\ w_{i2}\ \ldots\ w_{iM}]^{T}$ is i-th beamformer weight vector, $\boldsymbol{x}=[\ x_1\ \ldots\ x_M\ ]^{T}$ is beamfomer input vector and we have omitted the time index $[n]$ in the right hand side of eq.(1) for convenience. Performance among different algorithms will be accessed through the *constant modulus error*(*CME*): $CME(n)=(|y_i(n)|-1)^2$.

3. CONVENTIONAL MULTIUSER CMA

It is well known that the original CMA [2,3] does not have the signal selectivity capability. Therefore, when operating in multiuser signal environments, such as S/CDMA-based systems, additional procedures must be implemented in order to avoid user ambiguity. One possibility which is attractive because of its simplicity is the MU-CMA proposed in [1]. In this case a term which penalizes cross-correlations among multiuser output signals is added to the conventional constant modulus cost function. The cost function that must be minimized corresponding to the i-th user is given by:

$$\phi_i = E\left\{\left(|y_i[n]|^2-1\right)^2\right\}+\gamma\sum_{l=1}^{K}\sum_{\substack{j=1\\ j\neq l}}^{K}\left|r_{lj}\right|^2 \tag{2}$$

where: K is the number of users, $y_i[n]$ is i-th user's beamformer output, γ is the decorrelation term weight and $r_{lj} = E\{y_l[n]y_j^*[n]\}$ is the cross-correlation between l-th and j-th users. As we will point out in this section, the decorrelation weight γ will be important in meeting a compromise between the steady-state error variance (which increases with increasing γ) and the probability of user loss (which decreases with increasing γ). A user is lost when it is not included in the set of recovered users which means that another user has been recovered more than one time. A LMS-like algorithm can be obtained by the conventional stochastic gradient procedure. The gradient of the cost function ϕ_i with respect to $\boldsymbol{w_i}$ is given by:

$$\nabla_{\boldsymbol{w}_i}^{\phi_i} = \left(|y_i[n]|^2 - 1\right) y_i[n] \boldsymbol{x}^*[n] + \gamma \sum_{\substack{l=1 \\ l \neq i}}^{K} r_{li} E\left\{ y_l(n) \boldsymbol{x}_i^*(n) \right\} \quad (3)$$

where we have omitted some multiplicative constants that arise from the derivation. In order to implement this algorithm, the quantities r_{li} and $E\{y_l[n]x_i^*[n]\}$ must be estimated through temporal averages. This is implemented using a single pole filter as follows:

$$\boldsymbol{R}_{yy}(n+1)=\lambda \boldsymbol{R}_{yy}(n) + (1-\lambda)\boldsymbol{y}[n]\boldsymbol{y}^{\boldsymbol{H}}[n] \quad (4)$$

$$\boldsymbol{P}(n+1)=\lambda \boldsymbol{P}(n) + (1-\lambda)\boldsymbol{x}^{*}[n]\boldsymbol{y}^{\boldsymbol{T}}[n] \quad (5)$$

where $\boldsymbol{y}^{\boldsymbol{T}}[n]=[y_1[n] \; .... \; y_K[n]]$, superscripts ^{T} and ^{H} denotes ordinary and Hermitian tranpositions, and $\lambda<1$ is a smoothing factor. The estimates of the ensemble averages in (3) can be taken using eqs. (4) and (5). Thus, the MU-CMA is given by:

$$\begin{aligned} \boldsymbol{w_i}(n+1) &= \boldsymbol{w_i}(n) - \mu\left(|y_i[n]|^2 - 1\right) y_i[n] \boldsymbol{x}^*[n] - \\ &- \gamma \sum_{\substack{l=1 \\ l \neq i}}^{K} \hat{r}_{li}(n) \boldsymbol{p_l}(n) \end{aligned} \quad (6)$$

where $\hat{r}_{li}(n)$ is the (l,i) element of $\boldsymbol{R}_{yy}(n)$ and $\boldsymbol{p_l}(n)$ is the l-th column of matrix $\boldsymbol{P}(n)$. Two major disadvantages of the algorithm in (6) are its slow convergence rate and an increase in the steady-state error variance as a result of the additional decorrelation term. The latter occurs because the cross-correlations do not actually vanish due to imperfections on the estimation procedure. As a possible illustration of these disadvantages consider the following simulation. Table I shows the SDMA system setup. In the present case only the perfect power control scenario is considered. The additive noise power was set to zero. Fig. 1 shows the mean CME of the MU-CMA for 100 independent transmissions of 5000 QPSK symbols. Each curve is for a particular value of the decorrelation weight as indicated in the figure. Also indicated in the figure it is the percentage of lost users for each curve. Clearly, the steady-state error variance increases with increasing γ while the percentage of lost users decreases accordingly, and vice-versa. The penalty in steady-state error variance due to a successful decorrelation ($\gamma=10^{-3}$) can be significant as shown in fig.1. In practice, there will be a minimum value of γ for which no user is lost. However, this choice of γ will depend on the number of active users in the system. Furthermore, as seen in fig. 1, the algorithm takes up to 2000-3000 iterations to reach the minimum CME floor. These disadvantages led us to propose a new algorithm for fast and efficient blind multiuser signal separation.

Table I: SDMA System Configuration
(Uniform Linear Array of M=8 antennas)

User #	**DOA (degrees)**	**Power Control Scenario Relative Power (dB)**	
		Perfect	*Near-Far*
1	1	0	- 6
2	-52	0	- 3
3	29	0	+3
4	76	0	+6

4. LEAST-SQUARES WITH ADAPTIVE DECORRELATION MULTIUSER CMA

As discussed in last section, the slow convergence rate and the uncertainty about the choice of the decorrelating weight in the original MU-CMA, may preclude its use in practice. As a solution for the slow convergence rate, let us now propose a different constant modulus criterion for multiuser signal separation. Suppose that at the N-th time instant there are (N+1) data vectors $\boldsymbol{x}(0)$ ... $\boldsymbol{x}(N)$ as well as (N+1) array output signals per user $y_i(0)$ ... $y_i(N)$. Moreover, assume the availability of the cross-correlations statistics r_{lj} , $1\le l,j\le K$, $l\neq j$. Then, for the i-th user, we minimize a cost function given by:

$$\phi_i(N) = \frac{1}{N+1} \sum_{n=0}^{N} \varepsilon_i^2(n) + \zeta \quad (7)$$

where:

$$\varepsilon_i(n) = \left(|y_i(n)|^2 - 1\right)$$

$$\zeta = \gamma \sum_{l=1}^{K} \sum_{\substack{j=1 \\ j \neq l}}^{K} \left| r_{lj} \right|^2$$

Our aim is to choose the array weight vector $\boldsymbol{w_i}$ that minimizes $\phi_i(N)$. The gradient of $\phi_i(N)$ with respect to $\boldsymbol{w_i}$ is given by:

$$\begin{aligned} \nabla_{\boldsymbol{w}_i}^{\phi_i(N)} &= \frac{1}{N+1} \sum_{n=0}^{N} \left(|y_i(n)|^2 - 1\right) y_i(n) \boldsymbol{x}^*(n) + \\ &+ \gamma \sum_{\substack{l=1 \\ l \neq i}}^{K} r_{li} E\left\{ y_l(n) \boldsymbol{x}^*(n) \right\} \end{aligned} \quad (8)$$

where, again, we have omitted some multiplicative constants. By setting eq.(8) to zero we have:

$$\frac{1}{N+1}\sum_{n=0}^{N}\left|y_i(n)\right|^2 y_i\,(n)\boldsymbol{x}^*(n)=$$

$$=\frac{1}{N+1}\sum_{n=0}^{N}y_i(n)\boldsymbol{x}^*(n)-\gamma\sum_{\substack{l=1\\l\neq i}}^{K}r_{li}\,E\left\{y_l(n)\boldsymbol{x}^*(n)\right\} \qquad (9)$$

We can rewrite eq.(9) as:

$$\boldsymbol{R_i}(N)\boldsymbol{w_i}(N)=\boldsymbol{d_i}(N) \qquad (10)$$

where:

$$\boldsymbol{R_i}(N)=\frac{1}{N+1}\sum_{n=0}^{N}\left|y_i(n)\right|^2\boldsymbol{x^*}(n)\boldsymbol{x^T}(n)$$

$$\boldsymbol{d_i}(N)=\frac{1}{N+1}\sum_{n=0}^{N}y_i(n)\boldsymbol{x}^*(n)-\gamma\sum_{\substack{l=1\\l\neq i}}^{K}r_{li}\,E\left\{y_l(n)\boldsymbol{x}^*(n)\right\}$$

Finally:

$$\boldsymbol{w_i}(N)=\boldsymbol{R_i^{-1}}(N)\boldsymbol{d_i}(N) \qquad (11)$$

Note that this optimization procedure resembles the ones used to obtain the conventional recursive least-squares algorithm [4] and recursive CMA [5]. For real time implementation the required quantities in eq.(11) can be estimated using, again, a single pole filter. Hence, the algorithm can be summarized as follows:

$$\boldsymbol{w_i}(n)=\boldsymbol{R_i^{-1}}(n)\boldsymbol{d_i}(n) \qquad (12.a)$$

$$\boldsymbol{R_i}(n+1)=\lambda\boldsymbol{R_i}(n)+(1-\lambda)|y_i(n)|^2\boldsymbol{x^*}(n)\boldsymbol{x^T}(n) \qquad (12.b)$$

$$\boldsymbol{d_i}(n+1)=\lambda\boldsymbol{d_i}(n)+(1-\lambda)y_i(n)\mathbf{x}^*(n)-\gamma\sum_{\substack{l=1\\l\neq i}}^{K}\hat{r}_{li}(n)\boldsymbol{p_l}(n) \qquad (12.c)$$

where $\lambda<1$ is a smoothing factor, $\hat{r}_{li}(n)$ and $\boldsymbol{p_l}(n)$ are temporal estimates of the corresponding ensemble averages taken respectively from $\boldsymbol{R_{yy}}(n)$ and $\boldsymbol{P}(n)$ in eqs. (4-5), as explained before. The algorithm in eq.(12) will improve the convergence rate of MU-CMA but not its steady-state error variance. A further step into improving the performance of original MU-CMA is to control somehow the decorrelation weight γ. In fact, the necessity of the decorrelation term is less prominent when the weight vectors of the several users have provided the desired separation. Hence, the decorrelation weight could be made a function of the level of cross-correlation among users. For this sake we need to define a measure of the level of cross-correlation per user:

$$\bar{r}_i(n)=\frac{1}{K-1}\sum_{\substack{j=1\\j\neq i}}^{K}\left|\hat{r}_{ij}(n)\right|^2 \qquad (13)$$

This measure is an average over the number K of users in the system and therefore independent of it. Now a simple transformation on eq.(13) will enable us to control the decorrelation weight in a per-user basis. The value of γ on eq. (12.c) must be substituted by:

$$\gamma_i(n)=tanh\left[\bar{r}_i(n)\right] \qquad (14)$$

where tanh(•) is the hyperbolic tangent function. This function is an ad-hoc though suitable choice for the control of the decorrelation weight. The complete algorithm comprised of eqs.(12-14) will be called *least-squares with adaptive decorrelation* - multiuser CMA (LSAD-CMA).

5. SIMULATION RESULTS

In this section we present some comparative simulations with the proposed algorithm and the existing MU-CMA. The system configuration is given again by the parameters in Table I. Fig. 2 shows the CME performance averaged over all users and 200 independent transmissions of 400 QPSK data symbols. Signal-to-noise ratio was set to 20 dB. A total of 12 dB relative power imbalance is considered among users in the near-far scenario, as shown in table I. The CME performance of the continuously trained recursive least squares (RLS) [4] algorithm is included as a bound on performance. The smoothing factor for all temporal averages was set to λ=0.96. For every transmission, a verification of lost users was performed based on the measured bit error rate. Performance of LSAD-CMA reaches the error floor in as few as 400 symbols with no lost user throughout all independent transmissions in both power control scenarios. The performance of the original MU-CMA is poor as expected. Fig. 3 shows the temporal behavior of the decorrelation weight averaged over all users and repetitions for the near-far scenario. Fig. 4 shows an example of the set of antenna patterns provided by LSAD-CMA for all users after the last weight update and for the near-far scenario. Note the implicit power control performed by the antenna array: gains directed towards each user are inversely proportional to its received power level. The patterns provided by MU-CMA were meaningless after 400 hundred iterations.

6. CONCLUSIONS

We have verified some drawbacks of the original multiuser constant modulus algorithm of [1] and proposed an alternative technique which improves the convergence rate and the steady-state error variance. A least-squares version of the multiuser CMA was derived to enhance the convergence rate and an adaptive control of the decorrelation weight was introduced to improve the steady-state error variance. Simulation results confirm the expected improvements. The proposed technique, which has been named least-squares with adaptive decorrelation - multiuser CMA, is then an interesting approach for the task of blind multiuser signal separation.

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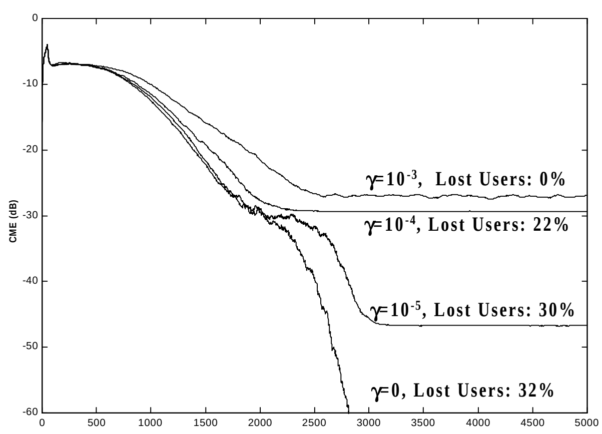

Fig. 1 - CME performance of MU-CMA for different values of the decorrelation weight

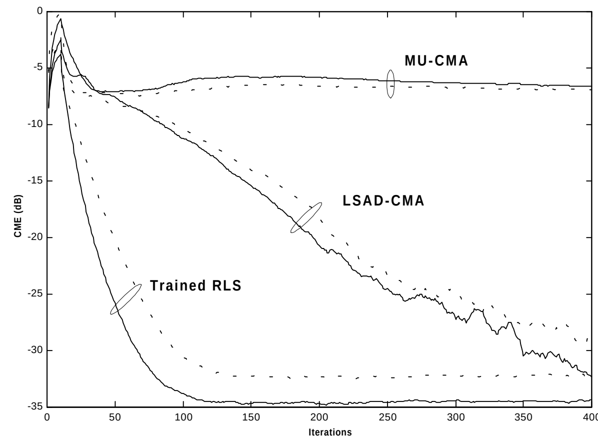

Fig. 2 - CME Comparative Performances of several algorithms for perfect (–) and near-far (- -) power control scenarios

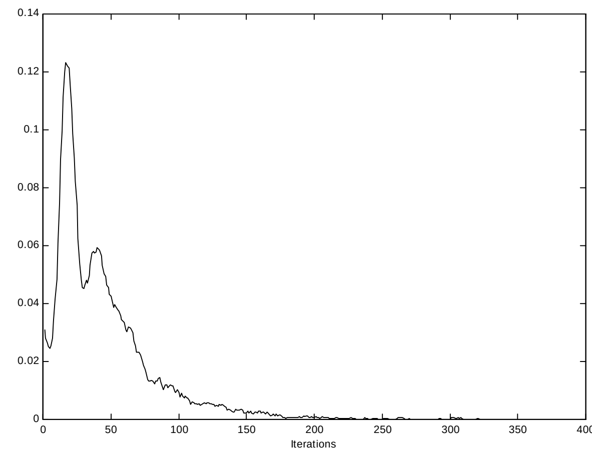

Fig.3 - Averaged temporal evolution of the adaptive decorrelation weight (near-far scenario)

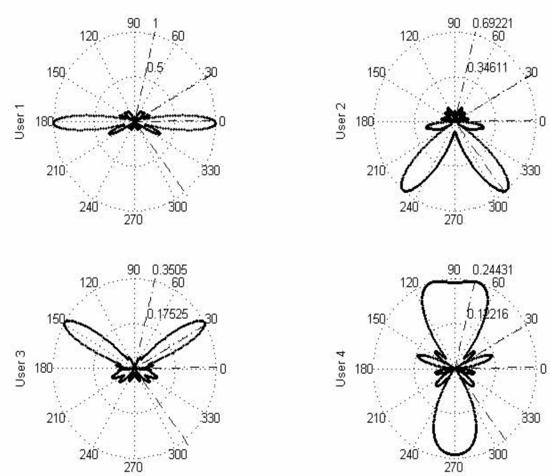

Fig. 4 - Antenna patterns for LSAD-CMA and near-far scenario after last weight update.