

SPATIAL SIGNATURE ESTIMATION IN CYCLIC CUMULANT DOMAIN AND MULTIUSER SIGNAL SEPARATION IN UPLINK SDMA

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ABSTRACT

In smart antenna system (SAS), the most compelling research work is the space-division multiple-access (SDMA) which includes uplink source separation and downlink selective transmission. In this paper, we propose a subspace-based spatial signature estimation algorithm of cochannel multiuser signals by jointly exploiting cumulant and cyclostationarity, and apply it to multiuser signal copy in uplink SDMA. Our method constructs a new matrix called spatial signature matrix (SS Matrix), and estimates multiuser spatial signature through the eigen-decomposition of the SS Matrix. Based on estimated spatial signature, a spatial filter bank is designed for cochannel multiuser waveform estimation. Computer simulations show that the spatial response of the filter bank can be used to find direction-of-arrival (DOA) of the correlated multiuser signals.

1. INTRODUCTION

The use of antenna arrays at base station in wireless communication system has gained much interest. Smart antenna systems (SAS) [1-5] proposed in the early 90's, can be used to extend the range of the base station, reduce the cost of the base station, mitigate fading, and increase the system capacity and performance. A smart antenna system consists of two fundamental operations: the uplink source separation and downlink selective transmission.

In flat fading channel [6], the time dispersion introduced by the multipath propagation is small in comparison with the reciprocal of the signals' width (IS-54 always be the case). In such case, signals from different paths of the same user are correlated, the channel between the users and the arrays may be modeled as a single vector referred to as spatial signature which reveals the spatial differences between users. For TDD system, the spatial signature captured during the uplink can be used for downlink weight design. Since the spatial signature, which is the linear combination of steering vector of multipath signals, contains sufficient information of DOA, and can be used directly in DOA estimation for downlink selective transmission in FDD system [3].

Recently, many researchers have studied and presented spatial signature estimation algorithms [4,5,7,8]. These algorithms are based on signal structure such as finite-alphabet [4] or constant-

modulus [7], and the approximation of propagation models such as point source [5] or local scattering model [8].

In this paper, we investigate the spatial signature estimation problem and present a subspace-based spatial signature estimation algorithm by exploitation cyclic cumulant. Due to its signal selectivity, the algorithm is insensitive to Gaussian noise and interference not having the same characteristic with the SOI (signal of interest), it also imposes few constraints on signal structure or propagation environment. Based on estimated spatial signature, we design a spatial filter bank for uplink multiuser signal separation. Computer simulations show that spatial response of the filter bank can be used to find direction-of-arrival (DOA) of the correlated multiuser signals. We also present a novel SDMA implementing scheme for multichannel multiuser case.

2. PROBLEM STATEMENT

Consider L cochannel narrow-band user signals $\{s_i(t)\}_{i=1}^L$. Suppose these signals undergo flat fading channel, producing $\{p_i\}_{i=1}^L$ delayed and scaled multipath signals impinging on an M -sensor d spacing uniform linear array from direction $\{\theta_{i,1}, \theta_{i,2}, \dots, \theta_{i,p_i}\}_{i=1}^L$. Assume that (AS1): $s_i(t)$ is a zero mean non-Gaussian process with non-zero fourth-order cyclic cumulant. (AS2): User signals are not fourth-order mutually cyclically correlated. And (AS3):

$$\tau_{mp} \ll BW^{-1}, \tau_{arr} \ll BW^{-1} \quad (1)$$

where τ_{arr} is the maximum transit time of a wavefront across the antenna array, τ_{mp} is the maximum differential delay due to multipath in the propagation medium, and BW is the signal bandwidth. Under above assumptions, the outputs of the array are modeled as

$$\mathbf{X}(t) = \sum_{i=1}^L \tilde{\mathbf{a}}_i s_i(t) + \mathbf{N}(t) = \tilde{\mathbf{A}} \mathbf{S}(t) + \mathbf{N}(t) \quad (2)$$

where $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}_1, \tilde{\mathbf{a}}_2, \dots, \tilde{\mathbf{a}}_L]$, $\mathbf{S}(t) = [s_1(t), s_2(t), \dots, s_L(t)]^T$, the i th column of $\tilde{\mathbf{A}}$, denotes the spatial signature of the signal $s_i(t)$. $\tilde{\mathbf{A}}$ is modeled as constant during the observation period. The spatial signature $\tilde{\mathbf{a}}_i$ is modeled as

$$\tilde{\mathbf{a}}_i = \sum_{l=1}^{p_i} \mathbf{a}(\theta_{i,l}) \beta_{i,l} \quad (3)$$

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where $\beta_{i,l}$ is the phase and amplitude differences between the l th path and direct path signal of the i th user. $a(\theta_{i,l})$ is the steering vector associating with the l th multipath, which has the following form

$$\mathbf{a}(\theta_{i,l}) = [1, \exp(j2\pi f_0 \omega \tau_{i,l}), \dots, \exp(j2\pi f_0 \tau_{i,l}(M-1))]^T \quad (4)$$

where $\tau_{i,l} = (d/c) \sin \theta_{i,l}$, f_0 and c are carrier frequency and wave velocity respectively.

Higher order cyclic cumulant can distinguish between stationary / cyclostationary and Gaussian / non-Gaussian processes. To handle symmetric probability density function from the sources of interest, we shall use fourth-order cyclic cumulant of array output [9]. Fourth-order (zero-log) spatial cyclic cumulant of an M -sensor array can be defined as [9,10]

$$\begin{aligned} C_{4,x}^\alpha(i, j, k, l) &= ccum\{x_i(t)x_j^{[*]}(t)x_k(t)x_l^{[*]}(t)\} \\ &= \lim_{T \rightarrow \infty} \sum_{t=0}^{T-1} E\{\{x_i(t), x_j^{[*]}(t), x_k(t), x_l^{[*]}(t)\} \\ &\quad - E\{x_i^{[*]}(t)x_j(t)\}E\{x_k^{[*]}(t)x_l(t)\} \\ &\quad - E\{x_i^{[*]}(t)x_k^{[*]}(t)\}E\{x_j(t)x_l(t)\} \\ &\quad - E\{x_i^{[*]}(t)x_l(t)\}E\{x_j(t)x_k^{[*]}(t)\}\} \exp(-j2\pi\alpha t) \end{aligned} \quad (5)$$

where $(i, j, k, l) \in \{1, 2, \dots, M\}$, $\{x_k(t)\}_{k=1}^M$ is M -sensor array output, $ccum$ is cyclic cumulant operator, selectable $[*]$ determines the selection of the cyclic frequency, If Eq. (9) contains conjugates, baud rate is selected as cyclic frequency. On the contrary, four times carrier frequency is selected as cyclic frequency [11]. we adopt the latter one in this paper.

3. SPATIAL SIGNATURE ESTIMATION IN CYCLIC CUMULANT DOMAIN

From Eq. (2), the algorithm estimates two $M \times M$ fourth-order spatial cyclic cumulant matrix \mathbf{C}_1^α and \mathbf{C}_2^α [9,10] with elements

$$\begin{aligned} C_1^\alpha(i, j) &= ccum\{x_1(t), x_1(t), x_i(t), x_j(t)\} \\ &= \sum_{k=1}^L C_{4,s_k}^\alpha a_k(i) a_k(j) \end{aligned} \quad (6)$$

$$\begin{aligned} C_2^\alpha(i, j) &= ccum\{x_2(t), x_1(t), x_i(t), x_j(t)\} \\ &= \sum_{k=1}^L C_{4,s_k}^\alpha a_k(i) a_k(j) [a_k(2)/a_k(1)] \end{aligned} \quad (7)$$

where $\mathbf{a}_i(k)$ represents the k th element of $\tilde{\mathbf{a}}_i$. $\mathbf{C}_{4,s_i}^\alpha$ is the fourth-order cyclic cumulant for the i th user. In above derivation, cumulant properties [CP1], [CP3], [CP5], and [CP6] in [11] were used. We also use the fact that fourth-order cyclic cumulant of a Gaussian noise are zero and the assumption (AS1). Collect Eq. (6) and (7) for $1 \leq i, j \leq M$ in matrix form ($L < M$)

$$\mathbf{C}_1^\alpha = ccum\{x_1(t), x_1(t), \mathbf{x}(t), \mathbf{x}^T(t)\} = \tilde{\mathbf{A}} \mathbf{D} \tilde{\mathbf{A}}^T \quad (8)$$

$$\mathbf{C}_2^\alpha = ccum\{x_2(t), x_1(t), \mathbf{x}(t), \mathbf{x}^T(t)\} = \tilde{\mathbf{A}} \boldsymbol{\phi} \tilde{\mathbf{A}}^T \quad (9)$$

where

$$\mathbf{D} = \text{diag}[C_{4,s_1}^\alpha, \dots, C_{4,s_L}^\alpha]$$

$$\boldsymbol{\phi} = \text{diag}[a_1(2)/a_1(1), a_2(2)/a_2(1), \dots, a_L(2)/a_L(1)]$$

Obviously, the rank of spatial fourth-order cumulant matrix \mathbf{C}_1^α is equal to L , if $M > L$. let $\{\mu_1 \geq \mu_2 \geq \dots \geq \mu_L\}$ and $\{v_1, v_2, \dots, v_L\}$ be the eigenvalues and corresponding eigenvectors of the matrix \mathbf{C}_1^α , respectively, and the pseudoinverse matrix of \mathbf{C}_1^α can be expressed as

$$\hat{\mathbf{C}}_1^\alpha = \sum_{i=1}^L \mu_i^{-1} v_i v_i^H \quad (10)$$

Now we define an $M \times M$ matrix referred to as the spatial signature matrix (SS Matrix)

$$\mathbf{C}^\alpha = \mathbf{C}_2^\alpha \hat{\mathbf{C}}_1^\alpha \quad (11)$$

Theorem: If $\tilde{\mathbf{A}}$ and \mathbf{D} have full rank L , the diagonal element $\boldsymbol{\phi}$ are not identical, then the SS matrix has its L non-zero eigenvalues equal to the L diagonal elements of $\boldsymbol{\phi}$, and the corresponding eigenvectors equal to the SS of L cochannel signals, i.e.

$$\mathbf{C}^\alpha \tilde{\mathbf{A}} = \tilde{\mathbf{A}} \boldsymbol{\phi} \quad (12)$$

Proof: The proof of the Theorem consists of the following steps. First, From Eq. (8), we know that

$$\tilde{\mathbf{D}} \tilde{\mathbf{A}}^T = (\tilde{\mathbf{A}}^T \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^T \mathbf{C}_1^\alpha \quad (13)$$

then substitute Eq. (13) into Eq. (9) yields

$$\mathbf{C}_2^\alpha = \tilde{\mathbf{A}} \boldsymbol{\phi} (\tilde{\mathbf{A}}^T \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^T \mathbf{C}_1^\alpha \quad (14)$$

Combined Eq. (14) with Eq. (11), we can obtain that

$$\begin{aligned} \mathbf{C}_2^\alpha \hat{\mathbf{C}}_1^\alpha \tilde{\mathbf{A}} &= \tilde{\mathbf{A}} \boldsymbol{\phi} (\tilde{\mathbf{A}}^T \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^T \mathbf{C}_1^\alpha \hat{\mathbf{C}}_1^\alpha \tilde{\mathbf{A}} \\ &= \tilde{\mathbf{A}} \boldsymbol{\phi} (\tilde{\mathbf{A}}^T \tilde{\mathbf{A}})^{-1} \tilde{\mathbf{A}}^T \tilde{\mathbf{A}} = \tilde{\mathbf{A}} \boldsymbol{\phi} \end{aligned} \quad (15)$$

The Theorem is proven.

Based on the above theorem, we can get SS estimation of the L cochannel user signals through the eigen-decomposition of SS Matrix \mathbf{C}^α for each given cyclic frequency α . The theorem is an extension of DOA matrix method [12]. In a smart antenna system, using M -sensor uniform linear array, the algorithm can estimate M SSs for M cochannel signals. With the estimated spatial signature $\hat{\mathbf{A}} = \tilde{\mathbf{A}}$, the array output of the L cochannel multiuser signals can be represented as

$$\hat{\mathbf{X}}(t) = \hat{\mathbf{A}} \mathbf{S}(t) \quad (16)$$

Under ideal condition, $\hat{\mathbf{X}}(t)$ only contains information of SOI with cyclic frequency α .

4. UPLINK SIGNAL SEPARATION

The goal of smart uplink is to separate multiple cochannel signals from array output with minimum crosstalk. Fig.1 shows signal

copy method in smart uplink. We regard the system as MIMO system. In order to implement smart uplink, we design a spatial filter bank \mathbf{W} to recover cochannel multiuser signals, let

$$\mathbf{W}=[\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_L] \quad (17)$$

where $\mathbf{w}_i=[w_{i1}, w_{i2}, \dots, w_{iM}]^T$ is the spatial filter for the i th user. The output of the designed filter bank can be expressed in matrix form as

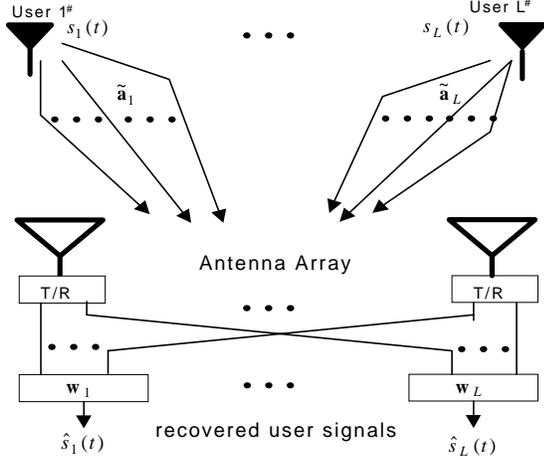


Fig. 1 The block diagram of uplink source separation

$$\hat{\mathbf{S}}(t)=\mathbf{W}^H \hat{\mathbf{X}}=\mathbf{W}^H \hat{\mathbf{A}} \mathbf{S}(t)$$

$$= \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1L} \\ h_{21} & h_{22} & \dots & h_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ h_{L1} & h_{L2} & \dots & h_{LL} \end{bmatrix} * \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_L(t) \end{bmatrix} = \mathbf{H} \mathbf{S}(t) \quad (18)$$

where \mathbf{H} is the response matrix of the spatial filter, and $h_{ij}=\mathbf{w}_i^H \hat{\mathbf{a}}_j$ denotes the i th spatial filter \mathbf{w}_i response to the j th user signal $s_j(t)$. To ensure correct signal separation without crosstalk \mathbf{H} must satisfy following equation

$$h_{ij} = \begin{cases} 1, & i=j; \\ 0, & i \neq j; \end{cases} \quad (19)$$

From Eq. (18) and Eq. (19), the relation between spatial filter bank and the estimated spatial signature can be represented as

$$\mathbf{W}^H \hat{\mathbf{A}} = \mathbf{I} \quad (20)$$

From Eq. (20), we can obtain

$$\mathbf{W}=\hat{\mathbf{A}}^+=[(\hat{\mathbf{A}}^H \hat{\mathbf{A}})^{-1} \hat{\mathbf{A}}^H]^H \quad (21)$$

where $\hat{\mathbf{A}}^+$ represents pseudoinverse of $\hat{\mathbf{A}}$, H denotes transpose. With designed spatial filter bank \mathbf{W} for SOIs, we can recover cochannel signals $\mathbf{S}(t)$ from array output $\mathbf{X}(t)$, i.e.,

$$\hat{\mathbf{S}}(t)=[(\hat{\mathbf{A}}^H \hat{\mathbf{A}})^{-1} \hat{\mathbf{A}}^H] \mathbf{X}(t) \quad (22)$$

where $\hat{\mathbf{S}}(t)$ is the recovered signal vector.

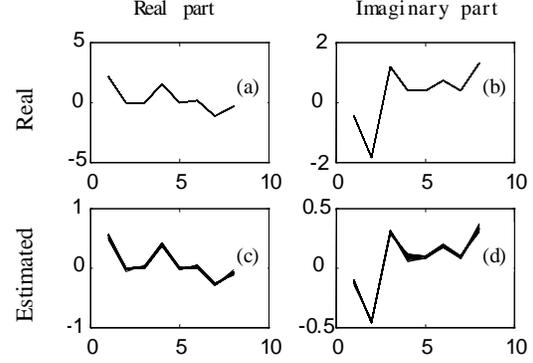


Fig. 2 Spatial signature estimation results for user 1[#]

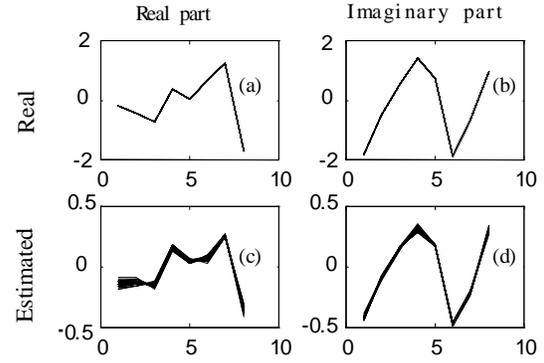


Fig. 3 Spatial signature estimation results for user 2[#]

5. SIMULATION AND ANALYSIS

In the experiment, an 8-sensor linear equispaced aperture whose spacing is half wavelength is considered. The array output is converted from center frequency f_0 (900 MHz) to frequency f_c , and quadrature sample with a rate f_s . Two cochannel BPSK user signals with baud rate f_B impinge on the array, $\{55^\circ, 35^\circ, 10^\circ\}$ and $\{1, 0.8+0.2j, 0.4-0.66j\}$ are DOAs and propagation fading for user 1[#], SNR=10dB. $\{20^\circ, 25^\circ, 45^\circ\}$ and $\{1, 0.34-0.7j, 0.5+0.5j\}$ are for user 2[#], SNR=10 dB. Another BPSK inference signal with baud rate f_B impinges from direction $\{-5^\circ\}$ with carrier frequency $(900M+200K)$ Hz. SNR =10dB. The number of snapshots is 2000, $f_c=0.1f_s$, $f_B=0.2f_s$, $\alpha=0.4f_s$. Fig. 2 and 3 are 30 independent spatial signature estimation results for user 1[#] and user 2[#], respectively. From which we can conclude that our method can estimate spatial signature of cochannel users up to a complex constant. Fig.4 shows 5 independent spatial response estimates of the spatial filters for user 1[#] and user 2[#], from Fig.4 (a) and (b), we can conclude that the peaks of the responses correspond the DOAs of users, so DOAs of SOIs can be found through spatial response of the filter bank. The responses at direction $\{-5^\circ\}$ are weaker, this is the result of signal selectivity of the spatial signature estimation algorithm. It is well known that DOA may be the only reference that can be used for downlink selective transmission in FDD system, Fig.5 present a

novel scheme to implement SDMA based on estimated spatial signature. For a thorough discussion of downlink selective transmission, the reader is referred to [13]. The two user spatial filters whose spatial response shown in fig.4 properly combined array output to separated two user signals. Fig 6 shows the signal separation results in which (a) and (b) are the constellations of the recovered signals for user 1[#] and user 2[#], respectively.

6. CONCLUSION

We present a new subspace-based estimation algorithm of spatial signature and apply it to waveform estimation in a smart antenna system. The theoretical analysis shows that multiuser spatial signature can be estimated through decomposition of constructed SS Matrix in cyclic cumulant domain. We also design spatial filter bank to recover cochannel multiuser signals. DOAs of correlated cochannel multiuser signals can be found through the spatial response of spatial filter bank. Some computer simulations demonstrate the performance of uplink signal separation.

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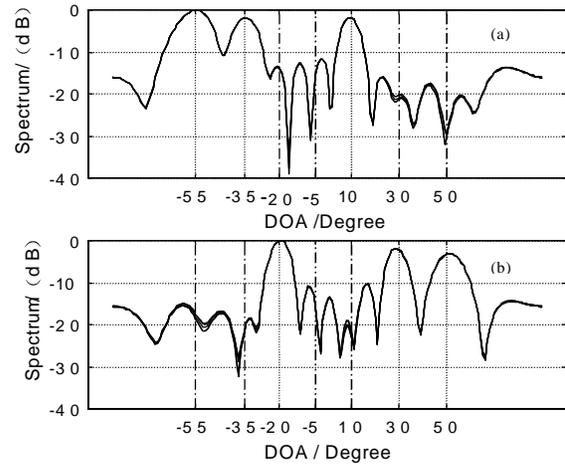


Fig. 4 Spatial frequency responses for user 1[#] ((a)) and user 2[#] ((b))

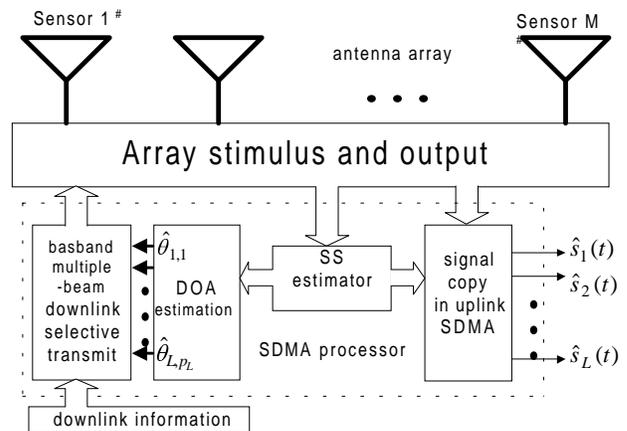


Fig. 5 A novel scheme for implementing SDMA

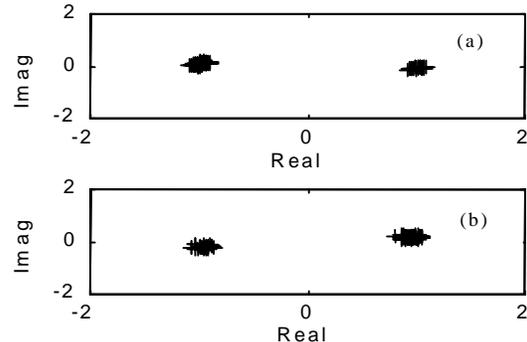


Fig. 6 The constellation of recovered two cochannel signals for user 1[#] ((a)) and user 2[#] ((b)).