BLIND CLOSED-FORM ARRAY RESPONSE ESTIMATION IN WIRELESS COMMUNICATION

Liang Jin, Min-li Yao and Qin-ye Yin

Information Engineering Institute, School of Electronics and Information Engineering,
Xi’an Jiaotong University,
Xi’an, Shaanxi 710049, P. R. China

ABSTRACT

In this paper, a closed-form array response estimation (CARE) technique for blind source separation in wireless communication is developed. By exploiting the data structure of second-order statistics of the array output in the presence of multipath, we construct a *signature matrix* in such away that its eigenvectors corresponding to none-zero eigenvalues are just the array response vectors. Thus a closed-form solution of array response can be obtained by eigen-decomposition. The theoretical analysis and the simulations show that the proposed method achieves array response estimation with little constraint on signal property and propagation environment such as scatters or angular spread. Moreover, the array considered here can be of arbitrary geometry and even uncalibrated.

1. INTRODUCTION

Currently, the use of antenna arrays as a tool for canceling co-channel interference, increasing system capacity and improving cell coverage in wireless communications has been suggested [1][2]. Based on the different array response to each of the signals, the mobile users occupying the same frequency, same code and same time, but in different locations, will be separated via spatial filtering [3][4], even in the presence of multipath. In Time-Division-Duplex systems, since the receiving and transmitting channels are reversible, the array responses obtained from uplink can also be utilized to form sensor weights for downlink selective transmission [5][6], i.e., sending message towards one user and not at another in the same channel. Bearing the rich information about multi-users’ position, the array response reveals the feature of the channel between the user and the array in the short-delay multipath scenarios and is often referred to as spatial signature or spatial channel as well. Obviously, the estimation of array response plays an important role in the exploitation of spatial diversity.

The traditional algorithms combine high resolution direction-finding techniques such as MUSIC and ESPRIT with the estimation of array responses [3][7], relying on the fact the array response is a known function of DOA. In a coherent multipath environment, however, these algorithms either require that the number of signals including multipath reflections be less than the number of sensors, or involve a point source model for approximation, which restricts their applicability in real wireless settings. Talwar *et al.* [4] attacked this problem and introduced a theoretically simple and efficient approach which estimates the users’ array response vectors by exploiting the finite-alphabet property of a digital communication signal, but with the limitation that signals from all users must be perfectly synchronized at the bit-level. Recently, under the assumption of local scattering, which means the angular spread of coherent signals is confined to a relatively small region, a generalized array manifold model was proposed [8] approximating the array response to some vector from the conventional planewave manifold. Unfortunately, the drawback of the method is that the array needs to be well calibrated. Besides, it is pointed out [10] that, in the presence of distant scatters and base-station scatters, the local scattering model is not so preferable, especially for micro-cell and indoor wireless telephony.

In this work we focus on the uplink signal separation based on array response and propose a novel Closed-form Array Response Estimation (CARE) method by constructing and eigen-decomposing a *signature matrix*. The method imposes little constraint on signal property or propagation environment such as scatters or angular spread. Furthermore, the array considered here can be of arbitrary geometry and even uncalibrated, thus the method is particularly useful in practical wireless communications. Using M sensors, the array responses to M users can be estimated, which means M uplink signals can be blindly separated.

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2. PROBLEM FORMULATION

Consider d narrowband signals together with their respect multipath impinging at an array of M sensors with arbitrary characteristics. The scenario is assumed to be time-invariant during the observation period. The output of ith sensor is given by

$$x_i(t) = \sum_{k=1}^{d}\sum_{l=1}^{L_k} g_i a_i(\theta_{kl})\alpha_{kl} s_k(t-\tau_{kl}) + n_i(t) \qquad (1)$$

where L_k is the number of multipath for kth signal $s_k(t)$; α_{kl} and τ_{kl} are, respectively, the attenuation and time delay corresponding to lth multipath; g_i is a complex scalar standing for the gain of ith sensor, which is assumed to be unknown herein; $a_i(\theta_{kl})$ is the element of steering vector $\mathbf{a}(\theta_{kl}) = [a_1(\theta_{kl}),...,a_d(\theta_{kl})]^T$ corresponding to the DOA θ_{kl}, $n_i(t)$ represents additive Gaussian noise with covariance σ^2, which is assumed to be temporally and spatially white and independent of signals, i.e.,

$$R_{n_i n_j}(\tau) = E\{n_i(t+\tau/2)n_j^*(t-\tau/2)\} = \sigma^2 \cdot \delta(\tau)\delta(i-j)$$

$$R_{n_i s_k}(\tau) = E\{n_i(t+\tau/2)s_k^*(t-\tau/2)\} = 0 \qquad (2)$$

where $\delta(\bullet)$ is Dirac function.

We assume that delay spread caused by multipath propagation is much smaller than the inverse bandwidth of the signals, i.e.,

$$\max_l(\tau_{kl}) << 1/BW_k$$

Then the narrowband assumption applies and the data model becomes

$$x_i(t) = \sum_{k=1}^{d} g_i a_{ik} s_k(t) + n_i(t) \qquad (3)$$

where a_{ik} is ith element of spatial signature vector $\mathbf{a}_k$ for $s_k(t)$,

$$\mathbf{a}_k = [a_{1k},...,a_{ik},...,a_{Mk}]^T = \sum_{l=1}^{L_k}\mathbf{a}(\theta_{kl})\cdot\alpha_{kl}e^{-j\omega_c\tau_{kl}} \qquad (4)$$

and ω_c is carrier frequency. Collect the signals at the array outputs, we have a vector form of (3),

$$\mathbf{x}(t) = \sum_{k=1}^{d}\tilde{\mathbf{a}}_k s_k(t) + \mathbf{n}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \qquad (5)$$

where $\tilde{\mathbf{a}}_k$ is referred to as *array response* vector corresponding to $s_k(t)$,

$$\tilde{\mathbf{a}}_k = [\tilde{a}_{1k},...\tilde{a}_{ik},...,\tilde{a}_{Mk}]^T, \quad \tilde{a}_{ik} = a_{ik}\cdot g_i \qquad (6)$$

and $\mathbf{s}(t) = [s_1(t),...,s_d(t)]^T$, $\mathbf{n}(t) = [n_1(t),...,n_M(t)]^T$, $\mathbf{A} = [\tilde{\mathbf{a}}_1,...,\tilde{\mathbf{a}}_d]$. Given the observation of $\mathbf{x}(t)$, the array response matrix $\mathbf{A}$ is what we intend to estimate in the following section.

3. THE CARE METHOD

We select arbitrary two sensors in the array, namely, #1 and #2 sensor, as "guiding sensors". Since $\{s_k(t)\}$ are mutually not correlated with each other or with $n_i(t)$ and $n_i(t)$ is the additive Gauss white noise, according to (2) (3) and (6), we have the cross-correlation function between the normal sensor output $x_i(t)$ and the guiding sensor output $x_1(t)$ as follows,

$$R_{x_i x_1}(\tau) = E\{x_i(t+\tau/2)x_1^*(t-\tau/2)\} \qquad (i=1,2,...,M)$$

$$= E\{[\sum_{k=1}^{d}\tilde{a}_{ik}s_k(t+\frac{\tau}{2}) + n_i(t+\frac{\tau}{2})][\sum_{k'=1}^{d}\tilde{a}_{1k}^* s_{k'}^*(t-\frac{\tau}{2}) + n_1^*(t-\frac{\tau}{2})]\}$$

$$= \sum_{k=1}^{d}[R_{s_k s_k}(\tau)\tilde{a}_{1k}^*]\cdot\tilde{a}_{ik} + \sigma^2\delta(\tau)\delta(i-1)$$

$$= \sum_{k=1}^{d}[R_{s_k s_k}(\tau)\tilde{a}_{1k}^*]\cdot\tilde{a}_{ik} \qquad (\tau \neq 0, i=1,...,M) \qquad (7)$$

where $R_{s_k s_k}(\tau) = E\{s_k(t+\tau/2)s_k^*(t-\tau/2)\}$. Similarly, for the other guiding sensor we have

$$R_{x_i x_2}(\tau) = E\{x_i(t+\tau/2)x_2^*(t-\tau/2)\}$$

$$= \sum_{k=1}^{d}[R_{s_k s_k}(\tau)\tilde{a}_{2k}^*]\cdot\tilde{a}_{ik}$$

$$= \sum_{k=1}^{d}[R_{s_k s_k}(\tau)\tilde{a}_{1k}^*]\cdot\tilde{a}_{ik}\cdot(\tilde{a}_{2k}^*/\tilde{a}_{1k}^*) \qquad (\tau \neq 0, i=1,...,M) \qquad (8)$$

Alternative forms for (7)(8) are:

$$\mathbf{R}_1(\tau) = \mathbf{A}\mathbf{R}_s(\tau) \qquad (9)$$

$$\mathbf{R}_2(\tau) = \mathbf{A}\Phi\mathbf{R}_s(\tau) \qquad (10)$$

where $\mathbf{R}_1(\tau) = [R_{x_1x_1}(\tau), R_{x_2x_l}(\tau),...,R_{x_Mx_1}(\tau)]^T$

$$\mathbf{R}_2(\tau) = [R_{x_1x_2}(\tau), R_{x_2x_2}(\tau),...,R_{x_Mx_2}(\tau)]^T$$

$$\mathbf{R}_s(\tau) = [R_{s_1s_1}(\tau)\cdot a_{11}^*, R_{s_2s_{21}}(\tau)\cdot a_{21}^*,...,R_{s_Ds_D}(\tau)\cdot a_{D1}^*]^T$$

and Φ is a diagonal matrix,

$$\Phi = diag[a_{21}^*/a_{11}^*, a_{22}^*/a_{12}^*,...,a_{2d}^*/a_{1d}^*]$$

By sampling $\mathbf{R}_1(\tau)$, $\mathbf{R}_2(\tau)$ uniformly at N (N>M) lags $\tau_n(\tau_n = T_s, 2T_s,...,NT_s)$, the "pseudo snapshots" are collected as follows,

$$\mathbf{X}_1 = [\mathbf{R}_1(T_s), \mathbf{R}_1(2T_s), ..., \mathbf{R}_1(NT_s)] \quad (11)$$

$$\mathbf{X}_2 = [\mathbf{R}_2(T_s), \mathbf{R}_2(2T_s), ..., \mathbf{R}_2(NT_s)] \quad (12)$$

and also we have

$$\mathbf{X}_1 = \mathbf{AS}, \quad \mathbf{X}_2 = \mathbf{A}\Phi\mathbf{S} \quad (13)$$

where $\mathbf{S} = [\mathbf{R}_s(T_s), \mathbf{R}_s(2T_s), ..., \mathbf{R}_s(NT_s)]$. It can be seen that the pseudo snapshot matrix $\mathbf{X}_1$ and $\mathbf{X}_2$ are formally similar to conventional matrix pencil in the ESPRIT [11] or DOA-MATRIX [12] method, but the ESPRIT method can only obtain matrix Φ, not $\mathbf{A}$. To solve matrix $\mathbf{A}$, we define the *signature matrix* as

$$\mathbf{R} = \mathbf{X}_2[\mathbf{X}_1]^- \quad (14)$$

where $[\bullet]^-$ denote the pseudo-inverse, i.e., $\mathbf{X}_1[\mathbf{X}_1]^- = \mathbf{I}$.

Lemma: If the array response matrix $\mathbf{A}$ is full-rank, then the signature matrix $\mathbf{R}$ has its d non-zero eigenvalues equal to the d diagonal elements of Φ and corresponding eigenvectors equal to the d column vectors of matrix $\mathbf{A}$, i.e.,

$$\mathbf{RA} = \mathbf{A}\Phi, \quad (15)$$

Proof: From (13) we obtain

$$\mathbf{S} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{X}_1 \quad (16)$$

Substitute (16) back into (13)

$$\mathbf{X}_2 = \mathbf{A}\Phi(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{X}_1$$

Then,
$$\begin{aligned}\mathbf{RA} &= \mathbf{X}_2[\mathbf{X}_1]^-\mathbf{A}\\ &= \mathbf{A}\Phi(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{X}_1[\mathbf{X}_1]^-\mathbf{A}\\ &= \mathbf{A}\Phi(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{A}\\ &= \mathbf{A}\Phi \qquad \square\end{aligned}$$

Thus, we can estimate the array response matrix A from the eigenvectors of $\mathbf{R}$ in a closed form. We call this method the Closed-form Array Response Estimation (CARE) method. Obviously, with M sensors, the proposed CARE method is capable of estimating up to M array responses corresponding to M users.

There is a scalar ambiguity in the array response vector estimates, since

$$\mathbf{x}(t) = \sum_{k=1}^{d}\tilde{\mathbf{a}}_k s_k(t) + \mathbf{n}(t) = \sum_{k=1}^{d}(\tilde{\mathbf{a}}_k / c_k)[c_k s_k(t)] + \mathbf{n}(t)$$

$$= \sum_{k=1}^{d}\tilde{\mathbf{a}}_k^{'} s_k^{'}(t) + \mathbf{n}(t)$$

For any scalar c_k, both $\tilde{\mathbf{a}}_k$ and $\tilde{\mathbf{a}}_k^{'}$ are valid solutions. However, this ambiguity hardly matters to array response based source separation or downlink beamforming.

4. SIMULATION

In our simulations, we adopt a uniform linear array of M=7 sensors for simplicity, though the CARE method is applicable to any kind of arrays. Since we have assumed that the array is uncalibrated, so the complex gain of each sensor is randomly chosen. $N_t = 100$ independent Monte-Carlo trials are taken. In each trial 100 real snapshots and 30 pseudo snapshots are collected. The signals of each user are co-channel ones, i.e., share the same carrier frequency.

CASE 1: Consider $d = 2$ signals arriving along their corresponding $L_k = 4$ paths from [$40^0, 60^0, 35^0, 15^0$] and [$50^0, 75^0, 30^0, 12^0$], respectively. Note that the angular spread is very large here. To measure the accuracy of the estimated array responses, define Root Mean Square Error (RMSE) as

$$RMSE_k = \sqrt{\frac{1}{N_t}\sum_{n=1}^{N_t}[1 - \left\|\tilde{\mathbf{a}}_k^H \cdot \hat{\mathbf{a}}_k^n\right\|^2 / (\left\|\hat{\mathbf{a}}_k^n\right\|^2 \left\|\tilde{\mathbf{a}}_k\right\|^2)]}$$

where $\hat{\mathbf{a}}_k^n$ is the estimation of $\tilde{\mathbf{a}}_k$ in the *n*th trial. Fig.1 illustrates the results of RMSE versus SNR, which shows that CARE method performs quite well regardless of wide angular spread and low SNR.

CASE 2: In this case we intend to evaluate the ability of CARE to perform signal separation. With estimate of the array response matrix $\hat{\mathbf{A}}$, the estimated signals are given as

$$\hat{\mathbf{s}}(t) = (\hat{\mathbf{A}}^H\hat{\mathbf{A}})^{-1}\hat{\mathbf{A}}^H\mathbf{x}(t) .$$

Two independent signals with 20dB and 30 dB SNR are present. Each signals has two paths at the DOA of [$30^0, 10^0$] and [$30^0 + \Delta\theta, 10^0$], respectively. The corresponding path attenuation and delays of two signals are set to be identical, i.e., $\alpha_{1l} = \alpha_{2l}$, $\tau_{1l} = \tau_{2l}$ ($l = 1,2$). Thus, the smaller $\Delta\theta$ is, the closer two array response vectors are, and the more difficult it may be to separate two sources. The signal to interference and noise ratio (SINR) is defined as follows to evaluate the performance.

$$SINR_k = 20\log_{10}\frac{\left\|s_k(t)\right\|}{\left\|s_k(t) - \hat{s}_k(t)\right\|} \quad (k=1, 2)$$

where $\hat{s}_k(t)$ is the estimation of the *k*th user's signal $s_k(t)$。The average SINR of the estimated weaker signal for different angular interval $\Delta\theta$ is shown in Fig2. Clearly, the Care method can separate two co-channel signals successfully even if their array responses are very close to each other.

5. CONCLUSION

We have constructed a signature matrix and proved that its eigenvetors corresponds to the array response vectors, which leads to the proposed CARE methods. Using M sensors, the array responses for M users can be estimated, which means M uplink signals can be blindly separated. The method imposes little constraint on signal property and propagation environment such as scatters or angular spread. Furthermore, the array considered here can be of arbitrary geometry and even uncalibrated, thus the method is particularly useful in practical wireless communications. The simulation results show the effectiveness of the method.

6. REFERENCES

[1] J. Winters, J. Salz, and R. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Comm.*, vol.42, pp. 1740-1751, 1994.

[2] B. Ottersten, "Array processing for wireless communication," in *Proc. 8th IEEE Signal Processing Workshop on Statistics Signal and Array Processing*, pp. 466-473, July 1993.

[3] B. Ottersten, R. Roy, T. Kailath, "Signal waveform estimate in sensors array processing," in *Proc. 23rd Asilomar Conf. Signals, Syst., Comput.*, pp. 787-791, 1989.

[4] S. Talwar, M. Viberg, A. Paulraj, "Blind separation of synchronous co-channel digital signals using an antenna array – Part I: Algorithms," *IEEE Trans. Signal Processing*, vol.44, no. 5, pp.1184-1197, 1996.

[5] G. Xu, H. Liu, "An effective transmission beamforming scheme for frequency-division-duplex digital wireless communication systems", in *Proc. IEEE ICASSP'95*, vol .3, pp. 1729-1732, 1995.

[6] L. Jin, J. Li, Q. Yin, "A system scheme for downlink selective beamforming in smart antenna," accepted by *Proc. IEEE ISCAS'98*, 1998.

[7] S. Andersson, M. Millnet, M. Viberg, B. Wahlberg, "An adaptive array for mobile communication system," *IEEE Trans. Veh. Technol.*, vol. 40, no.1, pp. 230-236, 1991.

[8] D. Asztely, B. Ottersten, A. Swindlehurst, "A generalized array manifold model for local scattering in wireless communications," in *Proc. IEEE ICASSP'97*, vol. 5, pp. 4021-4024, 1997.

[9] B. Shynk, R. Gooch, "The constant modulus array for cochannel signal copy and direction finding," *IEEE Trans. Signal Processing*, vol. 44, no.3, pp. 652-660, 1996.

[10] J. Fuhl, A. Molisch, E. Bonek, " Unified channel model for mobile radio systems with smart antennas," *IEE Proc. Radar, Sonar, Navig.*, vol. 145, no.1, pp. 32-41, 1998.

[11] A. Paulraj, R. Roy, T. Kailath, "Estimation of signal parameters via rotational invariance techniques – ESPRIT," in *Proc. 19th Asilomar Conf. Circuits, Syst., Comput.*, Nov 1985.

[12] Q. Yin, R. Newcomb, L. Zou, "Estimating 2-D Angles of Arrival via two Parallel Linear Arrays," in *Proc. IEEE ICASSP'89*, vol. 4, pp. 2803-2806, 1989

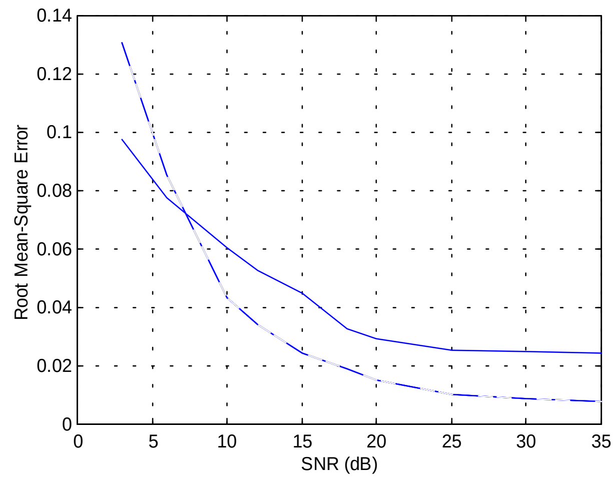

Fig.1 RMSE of two array responses versus SNR

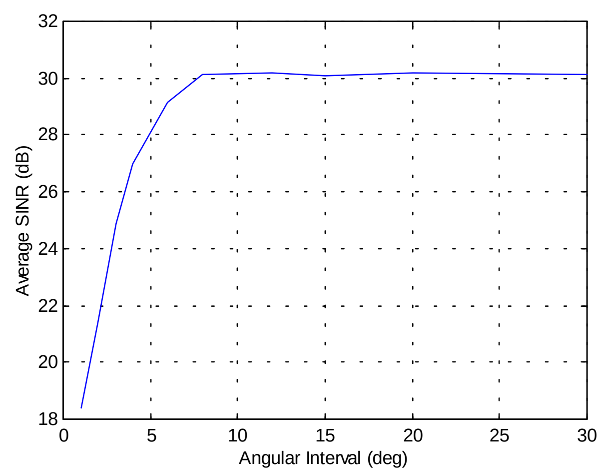

Fig.2 Performance of signal separation based on estimated array response.