A NOVEL TIME-FREQUENCY EXCISER IN SPREAD SPECTRUM COMMUNICATIONS FOR CHIRP-LIKE INTERFERENCE

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ABSTRACT

A novel time-frequency exciser is developed for the removal of chirp-like interferences in direct sequence spread spectrum (DSSS) communications. The chirplet decomposition iteratively expands signals in terms of time-frequency localized waveforms. The interference signal components are highly correlated with chirplets, and are represented by a few of the highest energy components of the decomposition. These components are excised from the received signal, and an excised signal goes through a detector for a decision. The proposed time-frequency exciser outperforms the conventional Fourier transform based excisers for chirp-like interference classes.

1. INTRODUCTION

In spread spectrum communications, the transmitted signal occupies more bandwidth than the necessary information bandwidth of a signal. The increase in bandwidth results in a processing gain that increases the noise and interference immunity of the system. Because of this, the spread spectrum systems are used in CDMA and low probality of intercept communications.

In this paper, we consider pseudo-noise (PN) or direct sequence spread spectrum (DSSS) communication systems. The DSSS transmitter, shown in Fig. 1, spreads the incoming data bit stream $d_k (d_k \in \{-1, 1\},$
$\forall k)$, by multiplying it with a spreading sequence $c(n)$,
$(c(n) \in \{-1, 1\}$ for $n = 1, ...N)$. During the transmission, the channel adds a noise term $w_k(n)$ and other interferences $i_k(n)$. Therefore, the received signal $r_k(n)$ can be written as [1]

$$r_k(n) = d_k c(n) + i_k(n) + w_k(n). \quad (1)$$

Each data bit d_k has a duration of T_d sec. The PN spreading code has a chipping rate of T_c sec., where $T_d \gg T_c$. Therefore, the length of the transmitted PN code is $N = T_d/T_c$. Without any interfering signal $i_k(n)$, the transmitted DSSS signal has a flat spectrum. The receiver correlates the received signal with a properly synchronized version of the spreading sequence $c(n)$. A length-N PN spreading code has the energy $< c, c >= N$. The decision variable is therefore obtained as

$$\begin{aligned} U_k &= < r_k, c > = d_k < c, c > + < i_k, c > + < w_k, c > \\ U_k &= N d_k + < i_k, c > + < w_k, c > . \end{aligned} \quad (2)$$

Equation (2) indicates that the despreading operation recovers the desired signal while spreading the interference. The bandwidth expansion in a DSSS system is translated into a processing gain as a power improvement factor caused by signal mapping or spreading, and it can be expressed as $G_p \triangleq 10log(\frac{T_d}{T_c})$.

In fact, if the interference power is greater than the system's jamming margin, the DSSS receiver fails to operate. The interference immunity of a DSSS receiver can be further improved by excising the jammer before despreading. The commonly used interference excision techniques work under the assumption that either the jammer is a narrow-band or a time-localized signal [2], [3], [4]. However, for a chirp-like interference these assumptions are not valid anymore.

In this work, we use the recently developed chirplet decomposition [5], [6] to detect and excise any kind of interference that is localized in the joint time-frequency plane. Since chirp-like signals are localized in the joint time-frequency plane, the chirplet decomposition will detect these components with the closest chirplets.

2. CHIRPLET DECOMPOSITION

The chirplet decomposition expands any signal in terms of four-parameter time-frequency atoms, that are localized in the time-frequency plane. The Gaussian function is chosen as the basic atom because of its minimum area property in the time-frequency plane. Applying a series of area-preserving transformations to a Gaussian, a four-parameter chirplet atom is obtained.

The Gaussian $g(t) = \frac{1}{\pi^{1/4}} e^{-\frac{t^2}{2}}$ is scaled to obtain $g_s(t) \triangleq \frac{1}{\sqrt{s}} g(\frac{t}{s})$. Rotation in the time-frequency plane is achieved by using the fractional Fourier transform (FRFT), that is defined for any function $f(t)$ as [7]

$$\begin{aligned} f_{-\alpha}(t) &\triangleq (\Gamma_{-\alpha} f)(t) \\ &\triangleq \sqrt{\frac{1 - j\cot\alpha}{2\pi}} e^{j\frac{\cot\alpha}{2}t^2} \int_{-\infty}^{\infty} f(\tau) e^{j\frac{\cot\alpha}{2}\tau^2} \\ &\quad \times e^{-j\csc\alpha\tau t} d\tau, \end{aligned} \quad (3)$$

where Γ_α is the rotation operator corresponding to the counter-clockwise rotation of α radians. The four-parameter atom is obtained by successive applications of the scaling,

This work was partially supported by U.S. Air Force Grant No.: F30602-97-1-0290.

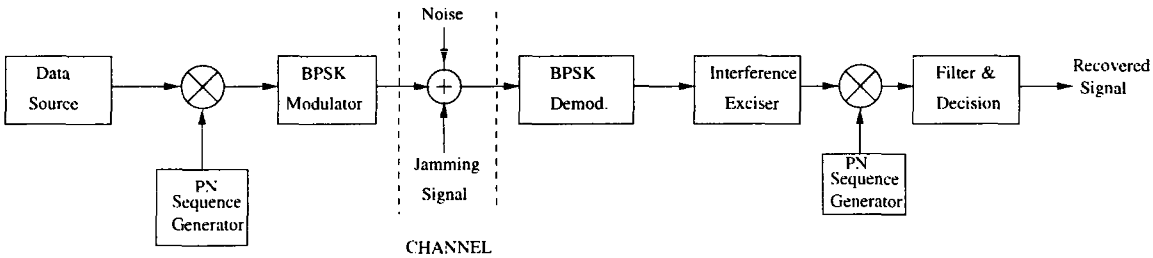

Figure 1: Block diagram of a DSSS communication system [1].

rotation, time and frequency-shift operators to the Gaussian function, giving

$$\begin{aligned} g_\beta(t) &\triangleq (\Gamma_\alpha g_s)(t-u)e^{j\nu t} \\ &\triangleq g_{s,\alpha}(t-u)e^{j\nu t}, \end{aligned} \tag{4}$$

where $\beta \triangleq (s, \alpha, u, \nu) \in \Upsilon$ is the index of the atom. The index set Υ is given by

$$\Upsilon \triangleq \begin{cases} (1,\infty) \times (-\pi/2, \pi/2) \times \mathbf{R}^2, & \alpha \neq 0 \\ \mathbf{R}^+ \times \mathbf{R}^2, & \alpha = 0. \end{cases} \tag{5}$$

The scaled and rotated atom $g_{s,\alpha}(t)$ is found as

$$g_{s,\alpha}(t) = \frac{\sqrt{s}e^{jc_1}}{\pi^{1/4}(\sin^2\alpha + s^4\cos^2\alpha)^{1/4}} \times \exp\{-\frac{s^2 - j(s^4-1)\cos\alpha\sin\alpha}{2(\sin^2\alpha + s^4\cos^2\alpha)}t^2\}, \tag{6}$$

where $c_1 = \frac{\pi}{4} - \frac{\alpha}{2} - \frac{\arctan(s^2\cot\alpha)}{2}$. Atoms are normalized, i.e., $||g_\beta(t)|| = 1$.

To obtain a decomposition in terms of these functions, the four-parameter space is discretized to obtain a discrete chirplet dictionary, $\mathcal{D}_\mu \triangleq \{g_\beta(t)\}_{\beta\in\Upsilon_\mu}$, that is complete in the Hilbert space [5]. The discrete parameters are selected such that the time-frequency centers of the atoms constitute lattices in the time-frequency plane for all the possible combinations of the parameters [5]. After parameter discretization, atoms are sampled, and we obtain

$$g_\beta(n) = g_{s,\alpha}(n-q)e^{jp\frac{2\pi}{N}n}. \tag{7}$$

Since the signals under consideration are real, we need the real form of the atoms. Real atoms are defined as

$$g_{\beta,\phi}(n) = K_{\beta,\phi}Real\{e^{j\phi}g_\beta(n)\}, \tag{8}$$

where $K_{\beta,\phi}$ is a normalization constant.

Matching pursuit is an iterative algorithm that selects an element at each iteration from a dictionary of atoms $\mathcal{D}_\mu$ to best match the inner structures of a signal, and finds its corresponding coefficient. The algorithm is initialized by computing the inner products $\{< f, g_\beta >\}_{\beta\in\Upsilon_\mu}$. Then the index β_0 of the best matching atom in the first iteration is found from

$$|< f, g_{\beta_0} >| = \sup_{\beta\in\Upsilon_\mu} |< f, g_\beta >|. \tag{9}$$

The phase ϕ_0 of the optimum atom $g_{\beta_0,\phi_0}(n)$ is set equal to the complex phase ϕ_β of $< f, g_\beta >$ [6]. The residual vector after approximating $f(n)$ in the direction of $g_{\beta_0,\phi_0}(n)$ is given by

$$Rf(n) = f(n) - < f, g_{(\beta_0,\phi_0)} > g_{(\beta_0,\phi_0)}(n). \tag{10}$$

The matching pursuit repeats the same operation for $Rf(n)$, i.e., it sub-decomposes the residue $Rf(n)$ by projecting it on an atom in $\mathcal{D}_\mu$ that best matches $Rf(n)$. Using induction, the flow of the matching pursuit algorithm can be explained easily. Let $R^0f(n) = f(n)$, it is supposed that the optimum match for the m^{th} order residue is searched by

$$|< R^m f, g_{\beta_m} >| = \sup_{\beta\in\Upsilon_\mu} |< R^m f, g_\beta >|. \tag{11}$$

Then the optimum atom is $g_{(\beta_m,\phi_m)}(n)$, where the phase $\phi_m = \arg\{< R^n f, g_\beta >\}$. Therefore, the search for the optimum index in the real decomposition is done by using the complex atoms, where the phase parameter is not searched. The residue can be written as

$$R^{m+1}f(n) = R^m f(n) - < R^m f, g_{(\beta_m,\phi_m)} > g_{(\beta_m,\phi_m)}(n). \tag{12}$$

The optimum index for $R^{m+1}f(n)$ is searched from the iterative formula

$$< R^{m+1}f, g_\beta > = < R^m f, g_\beta > - < R^m f, g_{(\beta_m,\phi_m)} > \times < g_{(\beta_m,\phi_m)}, g_\beta >. \tag{13}$$

Since $< R^m f, g_\beta >$ and $< R^m f, g_{(\beta_m,\phi_m)} >$ are stored in the previous iteration, only $< g_{(\beta_m,\phi_m)}, g_\beta >$ is computed in the current iteration. Since an analytical expression is used to calculate the inner product of two Gabor atoms $< g_{(\beta_m,\phi_m)}, g_\beta >$, a few complex multiplications and additions are necessary to calculate Equation (13).

When the residual signal energy becomes less than the required percentage of the signal energy, i.e.,

$$||f||^2 - \sum_{m=0}^{M-1} |< R^m f, g_{(\beta_m,\phi_m)} >|^2 \leq \epsilon^2 ||f||^2, \tag{14}$$

the algorithm terminates with the following decomposition

$$f(n) = \sum_{m=0}^{M-1} < R^m f, g_{(\beta_m,\phi_m)} > g_{(\beta_m,\phi_m)}(n). \tag{15}$$

The total number of the iterations M, is generally a small fraction of the number of signal samples [6].

3. INTERFERENCE EXCISION BY CHIRPLETS

In [8], the use of the matching pursuit algorithm as a denoising tool is shown. The main idea is to separate the white noise and signal components by looking at the correlations of the noisy signal residues at each iteration of the matching pursuit algorithm. The signal components that have high correlations with the dictionary elements are called coherent components. In this way, a noisy signal is divided into coherent and non-coherent parts. This idea can be applied to interference removal, as well.

The procedure can be outlined as follows. First, the PN sequence $c(n)$ used in the DSSS system is decomposed by the chirplet decomposition. Then at each iteration, the correlation ratios of the best matches with respect to the dictionary elements are found from

$$C(R^m c) = \frac{|< R^m c, g_{\beta_m} >|}{||R^m c||}. \quad (16)$$

At the receiver, the demodulated signal $r_k(n)$ is decomposed iteratively by the chirplet decomposition. At each iteration, the correlation ratios of the signal residues are found and compared with those of the PN sequence. The coherent structures of the signal $r_k(n)$ are the first K atoms $\{g_{(\beta_m,\phi_m)}\}_{0\leq m<K}$ that have higher correlation ratios than those of $R^m c(n)$. This means that, $r_k(n)$ has K coherent structures if and only if for $0 \leq m < K$

$$C(R^m r) > C(R^m c), \quad (17)$$

and

$$C(R^K r) \leq C(R^K c). \quad (18)$$

The correlation ratios can be computed from [8]

$$C(R^m r) = \sqrt{1 - \frac{||R^{m+1} r||^2}{||R^m r||^2}}. \quad (19)$$

A more coherent signal implies larger correlation ratios of signal residues. Since the chirplet dictionary consists of atoms that are well-localized in the time-frequency plane, the interference signal components are coherent with respect to the dictionary. However, the PN sequence that is spread in the joint time-frequency plane is not coherent with the dictionary. Therefore, the coherent parts of the received signal are excised and the residue is the desired signal part plus white noise.

4. PERFORMANCE ANALYSIS AND CONCLUSIONS

During the experiments, we have used the average number of erroneous chips per information bit as the performance measure of DSSS system. Since the decision device of the receiver has two outputs $-1, +1$, when the number of erroneous chips per data bit exceeds the half of the length of the PN code $N/2$, the corresponding data bit will be received in error.

To compare the performance of the chirplet-based interference exciser with a conventional technique, we use a transform domain exciser. First, the discrete Fourier transform (DFT) of the received signal is computed. The size of the DFT is taken as twice the PN code length to increase the frequency resolution. The coefficients of FFT that are above a fixed threshold level is set to zero and the signal is reconstructed with an inverse DFT.

For the two different interference levels, performance of the chirplet-based exciser and the DFT-based exciser are compared. Since the number of chips is 256, the processing gain of the system is 24 dB. The effect of the chirp interference on the PN-sequence and the removal of the interference are shown in Fig. 3. In Fig. 2, the average number of erroneous chips vs. signal-to-noise ratio (SNR) is given for three cases: No excision, DFT-based excision and chirplet-based excision, where the signal-to-interference ratio (SIR) is taken as -26 dB, and jammer chirp-rate is 0.5. As seen from the figure, the DSSS system without exciser or with FFT exciser cannot compensate for this case. However, for all the SNR values, the chirplet-based exciser works without error. In Fig. 4, this experiment is repeated for SIR $= -2$ dB.

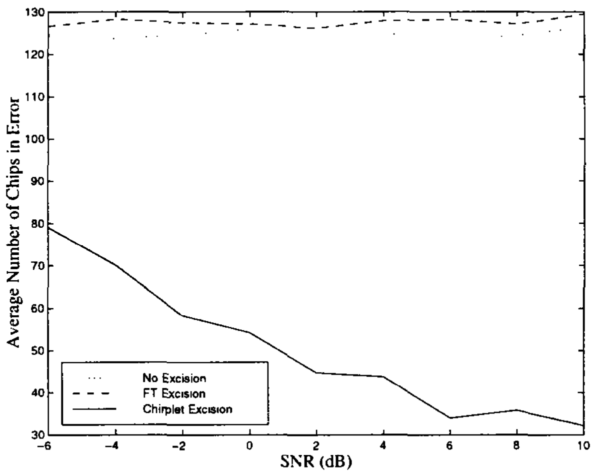

Figure 2: Average number of chips in error per information bit as a function of SNR. SIR$= -26$ dB, and chirp-rate of 0.5 are selected.

It is observed in this study that, the proposed scheme identifies chirp-like components of the received signal quite, satisfactorily. Therefore, the performance of DSSS communication system is significantly improved for this class of interference.

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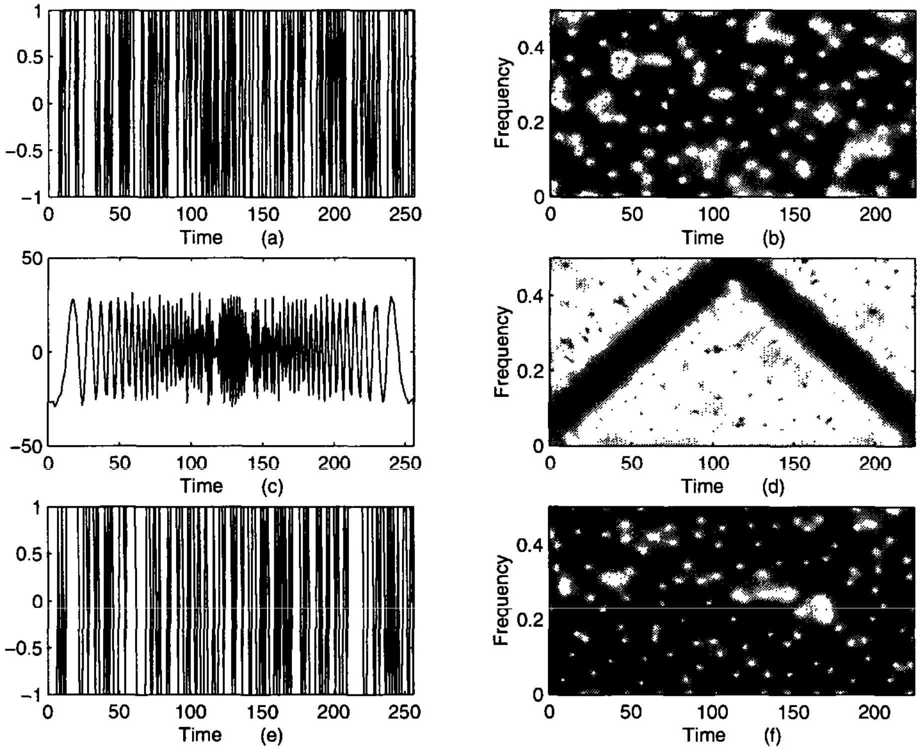

Figure 3: Chirp interference excision example for SNR= −6 dB, SIR= −26 dB, and chirp-rate= 0.5. (a) The PN sequence, (b) STFT of the PN sequence, (c) received signal that consists of the PN sequence, jammer, and white Gaussian noise, (d) STFT of the received signal, (e) interference excised and binary quantized signal, (f) STFT of the interference excised signal.

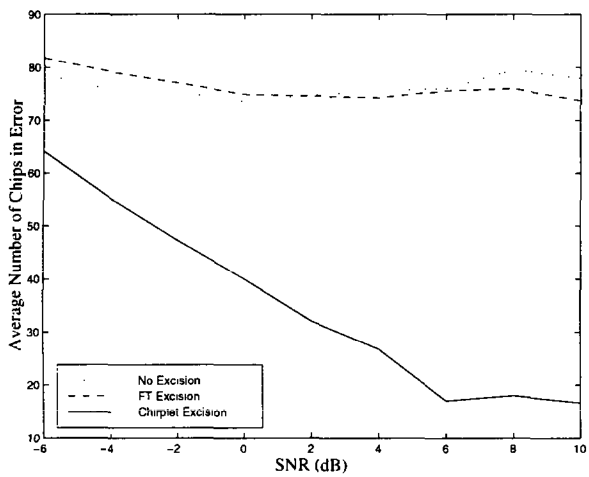

Figure 4: Average number of chips in error per information bit as a function of SNR. SIR= −2 dB, and chirp-rate of 0.5 are selected.

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