ADAPTIVE H^{∞} DELAY TRACKING IN ASYNCHRONOUS DS-CDMA SYSTEMS

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ABSTRACT

In this paper a new H^{∞}-type delay acquisition/tracking approach is proposed which is based on partitioning of the PN-code matrix of the CDMA users into submatrices which are then used to form a 'state-space' H^{∞}-type linear combiner. The proposed novel formulation and approach is robust to modelling errors such as over- and underestimation of the number of signals present and provides a powerful *near-far* resistant delay-tracking solution in a multiuser DS-CDMA signal environment.

1. INTRODUCTION

It is well known that the major problem of conventional DS-CDMA systems is their performance degradation due to the *near-far* effect in a multi-user environment. This problem is not just restricted to the demodulator, but also affects the performance of delay time acquisition and tracking for synchronization between the transmitter and receiver.

Attempts to solve this problem can be seen in [1, 2, 3, 4, 5, 6] and the references therein. The most popular *near-far* resistant delay time estimators are based on subspace techniques as in [3, 5, 6]. The basic principle of subspace techniques is to estimate the signal/noise subspace, normally through Eigendecomposition of the covariance matrix of a received vector input. Assuming that the desired user code is known, the delay associated with the desired transmission can be obtained. Since the performance of these techniques is insensitive to signal power, they are naturally immune to the *near-far* problem.

However, these algorithms assume that the propagation delays are fixed during the observation interval (stationary environment). This is clearly not the case in many applications, particularly in mobile radio systems, where the propagation delay varies due to the movement of mobile stations (non-stationary environment). Moreover in such practical situations, there are bound to be signals appearing and disappearing during transient periods and weak signals which are not accounted for by the system. In addition, subspace methods require knowledge of the number of users and are sensitive to modelling errors. Current detection methods such as AIC and MDL [7] tend to overestimate the number of users. All these abnormalities in the signal environment perturb the assumed signal model, potentially degrading the performance of the acquisition/tracking process.

In this paper, a new delay tracking algorithm based on adaptive H^{∞} estimation theory [8] and the so called "propagator" method [9] is proposed. The "propagator" method is a linear subspace method which eliminates the need to perform Eigendecomposition of the received signal covariance matrix. This is achieved by determining a linear operator which may be used to obtain the noise subspace. H^{∞} estimation techniques not only provide the proposed algorithm with "adaptivity" but also reduce sensitivity to modelling errors. The two techniques are combined via reformulation of the linear operator in the form of a state-space model. An adaptive H^{∞} estimator is then obtained to estimate a state matrix which may be used as a linear operator. A cost function similar to MuSIC [3] is then used to estimate the propagation delay using the noise subspace derived from the state matrix. The combination of these techniques not only provides a computationally simple tracking solution but also an estimator which is robust to modelling errors in the signal environment as shown in the simulations.

2. MODELLING AND PROBLEM FORMULATION

In an M-user asynchronous DS-CDMA system with BPSK modulation, each user sends an information signal, $m_i(t)$, of the form

$$m_i(t) = \sum_{n} a_i[n] c_1(t - nT_{cs}) \tag{1}$$

where $\{a_i[n], \forall n \in \mathcal{N}\}$ is the i^{th} user's data sequence of $\pm$ 1s and $c_1(t)$ is the unit amplitude rectangular pulse of width T_{cs} shown in Figure (1). Note that in Equation (1), for a given n (i.e. for the n^{th} bit of the data sequence), the time t satisfies

$$nT_{cs} \leq t < (n+1)T_{cs}$$

The i^{th} user data signal $m_i(t)$ is modulated by a periodic PN-code signal modelled by

$$b_i(t) = \sum_{k} \alpha_i[k] c_2(t - kT_c) \tag{2}$$

where $\{\alpha_i[k], \forall k \in \mathcal{N}\}$ is the i^{th} user's PN-code sequence of $\pm$ 1s and is of period $\mathcal{N}_c$ and c_2 is a unit amplitude rectangular pulse of width T_c (i.e. T_c is the chip period), shown in Figure

(1). Each data bit consists of one period of the code sequence, of length $\mathcal{N}_c \in \mathcal{Z}$ chips which implies that $\mathcal{N}_c = T_{cs}/T_c$.

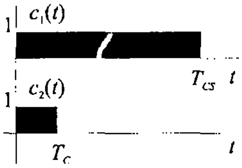

Figure 1: Rectangular Pulses $c_1(t)$ and $c_2(t)$.

Using a CDMA system with a 'data privacy feature' i.e. each user has coincident 'data' and 'PN code' clocks (though transmission is still asynchronous) then the baseband DS-CDMA signal of the i^{th} user becomes

$$\begin{aligned} s_i(t) &= m_i(t)b_i(t) \\ &= \sum a_i \left[\left\lfloor \frac{k}{\mathcal{N}_c} \right\rfloor\right] \alpha_i[k] c_2(t - kT_c) \end{aligned} \tag{3}$$

with $\lfloor \cdot \rfloor$ denoting the "round down to integer" operator and where, for a given k, the time t satisfies

$$\left\lfloor \frac{k}{\mathcal{N}_c} \right\rfloor T_{cs} + (k \bmod \mathcal{N}_c)T_c \le t < \left\lfloor \frac{k}{\mathcal{N}_c} \right\rfloor T_{cs} + ((k \bmod \mathcal{N}_c) + 1)T_c$$

The i^{th} user's transmitted signal is then formed by multiplying $s_i(t)$ with a carrier $\sqrt{2P_i}\cos(2\pi f_c t + \zeta_i)$, where P_i is the received power, ζ_i is a random carrier phase, uniformly distributed in $[0, 2\pi)$, and f_c is the carrier frequency. The received signal in an additive noise environment can be written

$$x(t) = \sum_{i=1}^{M} \mathrm{Re}\left\{ \sqrt{P_i} s_i(t - \tau_i) e^{j(2\pi f_c t + \psi_i)} \right\} + n(t) \tag{4}$$

where τ_i is the propagation delay of the i^{th} user received signal, $\psi_i = \zeta_i + 2\pi f_c \tau_i$, and $n(t)$ is an additive white noise waveform with two-sided power spectral density $N_o/2$.

By sampling the received signal $x(t)$ with an oversampling factor q (i.e. having a sampling period $T_s = T_c/q$) the ℓ^{th} sample can be written as follows

$$x[\ell] = \tag{5}$$
$$\sum_{i=1}^{M} \sqrt{P_i} e^{j\psi_i} a_i \left[\left\lfloor \frac{\ell - (l_i + d_i)}{qN_c} \right\rfloor\right] \alpha_i \left[\left\lfloor \frac{\ell - (l_i + d_i)}{q} \right\rfloor\right] + n[\ell]$$

where $\tau_i = (l_i + d_i)T_s$ and l_i and d_i respectively denote integer and fractional multiples of T_s. The additive noise sequence $n[\ell]$ is assumed to be circular Gaussian, such that

$$\mathcal{E}\{n[\ell_1] n^*[\ell_2]\} = \frac{2N_o}{T_s} \delta_{\ell_1, \ell_2} \tag{6}$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation operator, and δ denotes the delta function. Considering that $n = \left\lfloor \frac{\ell}{q\mathcal{N}_c} \right\rfloor$ represents the n^{th} data bit (symbol) period and rearranging the sampled received signal $\{x(\ell), \forall \ell\}$ into vector form yields

$$\underline{x}[n] = \mathbb{H}[n] \mathbb{G} \underline{a}[n] + \underline{n}[n] \tag{7}$$

where

$$\begin{aligned} \mathbb{H}[n] &= [\underline{h}_1[n] \;\; \underline{h}_1[n-1] \;\; \ldots \;\; \underline{h}_M[n] \;\; \underline{h}_M[n-1]] \\ \underline{a}[n] &= [a_1[n] \;\; a_1[n-1] \;\; \ldots \;\; a_M[n] \;\; a_M[n-1]] \\ \mathbb{G} &= \mathrm{diag}\Big([\sqrt{P_1} e^{j\psi_1}, \sqrt{P_1} e^{j\psi_1}, \ldots, \\ &\qquad \sqrt{P_M} e^{j\psi_M}, \sqrt{P_M} e^{j\psi_M}] \Big) \\ \underline{n}[n] &= [n_1[n] \;\; n_2[n] \;\; \ldots \;\; n_M[n]] \end{aligned}$$

with

$\underline{h}_i[n] =$
$$[\underline{0}_{l_i}^T, \alpha_i[\lfloor (nq\mathcal{N}_c + 1) - d_i \rfloor], \ldots, \alpha_i[\lfloor (n+1)q\mathcal{N}_c - (l_i + d_i) \rfloor]]^T$$
$\underline{h}_i[n-1] =$
$$[\alpha_i[\lfloor (nq\mathcal{N}_c + 1) - (l_i + d_i) \rfloor], \ldots, \alpha_i[\lfloor nq\mathcal{N}_c - d_i \rfloor], \underline{0}_{q\mathcal{N}_c - l_i}^T]^T$$

where $\underline{0}_p$ denotes the $(p \times 1)$ zero vector. The covariance of $\underline{n}[n]$ is

$$\mathcal{E}\left\{ \underline{n}(n_1) \underline{n}(n_2)^H \right\} = \frac{2N_o}{T_s} \delta_{n_1, n_2} \mathbb{I}_{q\mathcal{N}_c} \tag{8}$$

3. PROPOSED ADAPTIVE H^∞ DELAY-TRACKING

In the previous section we have seen that the matrix $\mathbb{H}$ has, as columns, delayed versions of the PN sequences of the M users corresponding to two successive data symbols. By adapting for CDMA delay estimation the definition of the "propagator" proposed for antenna arrays in [9], the matrix $\mathbb{H}[n]$ may be partitioned into two submatrices, $\mathbb{H}_1$ and $\mathbb{H}_2$ as follows

$$\mathbb{H}[n] = \overbrace{\begin{bmatrix} \mathbb{H}_1[n] \\ \mathbb{H}_2[n] \end{bmatrix}}^{2M} \begin{matrix} \}2M \\ \}q\mathcal{N}_c - 2M \end{matrix} \tag{9}$$

Assuming that $\mathbb{H}_1[n]$ has linearly independent column vectors, there is a unique linear operator $\mathbb{L}$ such that

$$\mathbb{L}^H[n]\mathbb{H}_1[n] = \mathbb{H}_2[n] \tag{10}$$

where $\mathbb{L}[n]$ is a $2M \times (q\mathcal{N}_c - 2M)$ matrix. Let

$$\mathbb{Q}[n] = \begin{bmatrix} \mathbb{L}[n] \\ \mathbb{I}_{q\mathcal{N}_c - 2M} \end{bmatrix} \in \Re^{q\mathcal{N}_c \times (q\mathcal{N}_c - 2M)} \tag{11}$$

where $\mathbb{I}_p$ denotes the $p \times p$ identity matrix. It can be shown that

$$\mathbb{Q}^H[n]\mathbb{H}[n] = \mathbb{O}_{(q\mathcal{N}_c - 2M) \times 2M} \tag{12}$$

where $\mathbb{O}_{p \times q}$ denotes the $p \times q$ zero matrix. This means that the subspace spanned by the columns of $\mathbb{Q}$ is orthogonal to the subspace spanned by the columns of $\mathbb{H}$.

The assumption that the first $2M$ rows of $\mathbb{H}$ have linearly independent column vectors may not be true in the case of this delay estimator. However, we can always find $2M$ rows that are linearly independent using the method described below.

In [11], a method is given for determining the rank of a matrix in the presence of roundoff error, using QR factorization with column pivoting ($A\Pi = QR$). Given a priori knowledge of the rank of the matrix ($\text{rank}(\mathbb{H}) = 2M$), this method may be adapted to carry out a partial $A\Pi = QR$ factorization on the rows of $\mathbb{H}$, using only the first $2M$ steps to find the rows which best form an orthogonal basis for $\mathbb{H}$. This method was found to be insensitive to high noise levels, working consistently at SNRs below $10dB$, as well as in the presence of moving sources.

After determining the independent rows and partitioning accordingly, the received signal-vector $\underline{x}[n]$ can also be partitioned into two vectors $\underline{x}_1[n]$ and $\underline{x}_2[n]$ as follows

$$\begin{bmatrix} \underline{x}_1[n] \\ \underline{x}_2[n] \end{bmatrix} = \begin{bmatrix} \mathbb{H}_1[n] \\ \mathbb{H}_2[n] \end{bmatrix} \mathbb{G}\underline{a} + \begin{bmatrix} \underline{n}_1[n] \\ \underline{n}_2[n] \end{bmatrix} \tag{13}$$

Using Equation (10), the relationship between $\underline{x}_1[n]$ and $\underline{x}_2[n]$ can be expressed as

$$\underline{x}_2[n] = \mathbb{L}^H[n]\underline{x}_1[n] + \left(\underline{n}_2[n] - \mathbb{L}^H[n]\underline{n}_1[n]\right) \tag{14}$$

It can be shown that $\underline{x}_2[n]$ can be described by the following state-space model

$$\begin{cases} \mathbb{Q}[n] & = & \begin{bmatrix} \mathbb{L}[n] \\ \mathbb{I}_{qN_c-2M} \end{bmatrix} \\ \underline{x}_2^H[n] & = & \underline{x}_1^H[n]\mathbb{L}[n] + \underline{v}[n] \\ \mathbb{L}[n+1] & = & \mathbb{L}[n] + \mathbb{B}\left(\triangle\mathbb{L}[n]\right) \end{cases} \tag{15}$$

$\mathbb{L}[n]$ is the state matrix or the propagator at the n^{th} symbol, $\underline{v}[n] = (\underline{n}_2[n] - \mathbb{L}^H[n]\underline{n}_1[n])$ is the unknown noise vector, $\mathbb{B}$ is a user-defined bound using prior knowledge of rate of change of $\mathbb{L}[n]$ from the n^{th} to the $(n+1)^{\text{th}}$ symbol and $\triangle\mathbb{L}[n]$ denotes the unknown variation in the state-matrix $\mathbb{L}[n]$ that will occur between the n^{th} and the $(n+1)^{\text{th}}$ symbol (i.e. disturbance).

The H^{∞} estimation methods can be seen as a powerful and robust solution to handle parameter variations, modelling uncertainties and noise effects with limited statistical information. Therefore, by choosing the *a priori* H^{∞} estimator for the state-space model defined in Equation (15) the following result can be derived

$$\widehat{\mathbb{L}}[n+1] = \tag{16}$$
$$\widehat{\mathbb{L}}[n] + \mathbb{P}[n]\underline{x}_1[n]\left(1 + \underline{x}_1^H[n]\mathbb{P}[n]\underline{x}_1[n]\right)^{-1}\left(\underline{x}_2^H[n] - \underline{x}_1^H[n]\widehat{\mathbb{L}}[n]\right)$$

Where $\widehat{\mathbb{L}}[n]$ denotes the estimate of $\mathbb{L}$ at the n^{th} data symbol. In Equation (15) the matrix $\mathbb{P}[n]$ is defined as follows

$$\mathbb{P}[n] = \widetilde{\mathbb{P}}[n] - \gamma^{-2}\underline{x}_1[n]\underline{x}_1^H[n] \tag{17}$$

where $\widetilde{\mathbb{P}}[n]$ satisfies the recursive equation

$$\widetilde{\mathbb{P}}[n+1] = \left[\widetilde{\mathbb{P}}[n] + (1-\gamma^{-2})\underline{x}_1[n]\underline{x}_1^H[n]\right]^{-1} + \mathbb{B} \tag{18}$$

initialized with $\widetilde{\mathbb{P}}[0] = \mu\mathbb{I}$ and $\mathbb{B} = \beta\mathbb{I}_M$. The parameters γ, μ and β are user-defined and may be optimized for a particular environment.

A stable solution of the estimator exists as long as

$$\gamma^2 \leq \sup_n \left(\beta + 1/\underline{x}_1^H[n]\underline{x}_1[n]\right)\underline{x}_1^H[n+1]\underline{x}_1[n+1] \tag{19}$$

At each iteration of the recursive function in equation (16), the orthogonal transformation $\widehat{\mathbb{Q}}[n] = \left[\widehat{\mathbb{L}}^T[n]\ \mathbb{I}_{qN_c-2M}\right]^T$ is determined. We may then use any signal subspace method (e.g. [3]) to obtain the delay of the desired signal.

4. SIMULATIONS

In the subsequent simulations, all users are assigned Gold sequences of length $N_c = 15$, generated by the polynomials $g_1(\mathrm{x}) = \mathrm{x}^4 + \mathrm{x} + 1$ and $g_2(\mathrm{x}) = \mathrm{x}^4 + \mathrm{x}^3 + \mathrm{x}^2 + \mathrm{x} + 1$. The signal is scaled so that the received power of the desired user is 1. The chip period T_c is taken equal to 0.8μsec (as in IS-95) and the rest of the signal parameters such as carrier phase and time offset are set to zero for simplicity. The data bits are binary random variables with ± 1 having equal probability. As a basis for comparison a simple exact Eigendecomposition (ED) algorithm is used [10].

Unless otherwise stated, the environment for the subsequent simulations is as follows. The number of users is $M = 2$, with propagation delays starting at $\underline{\tau} = [13.71\ 5.77] \times T_c$ and changing in a nonlinear fashion over the simulation interval. The parameters for the H^{∞} estimator are $\mu = 0.9$, $\lambda = 1.6$ and $\rho = 0.008$. The noise power is set $10dB$ below the desired signal power.

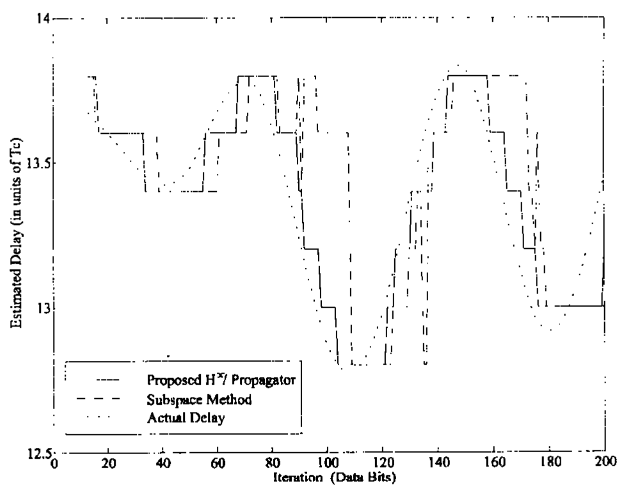

Figure 2: Tracking Performance and Adaptation of Proposed Algorithm in Power Controlled Environment.

We first investigate the tracking performance of the proposed algorithm under perfect power control, i.e. $P_i = 1$, for $i = [1, \ldots, M]$. Figure (2) shows the tracking capabilities of the two algorithms. The proposed algorithm performs substantially better than Eigendecomposition in this example, adapting to a change in the propagation delay in the space of a few bit periods. Next we examine the proposed algorithm in a near-far environment with the received signal power for the desired user set at $20dB$ below the interference. The performance of the two algorithms in this environment was found to be identical to their performance under perfect power control. Again, the proposed algorithm shows a significant improvement.

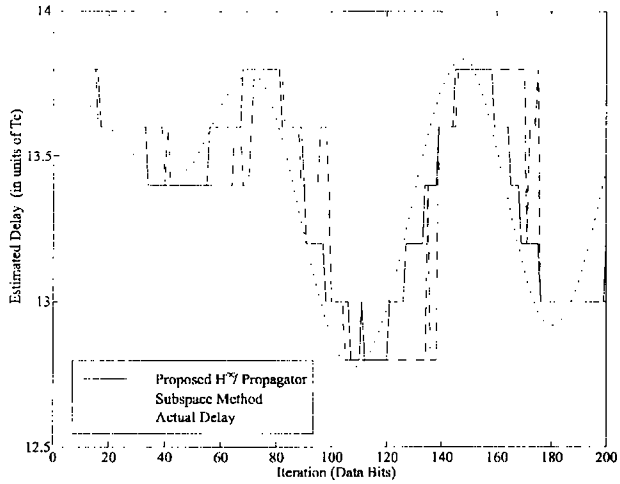

Figure 3: Tracking Performance and Adaptation of Proposed Algorithm in Overestimation Environment.

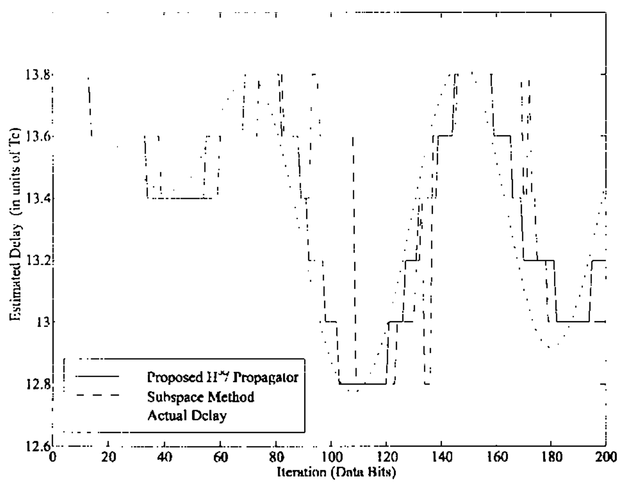

Figure 4: Tracking Performance and Adaptation of Proposed Algorithm with Intermittent Low-Powered Interference.

We next study the performance of the proposed algorithm when the number of sources is overestimated. Two equipowered sources are present, as above, but the algorithms are initialised to search for three sources. The performance of the subspace algorithm is substantially worse while the proposed algorithm shows only a slight degradation. Figure (3) illustrates the effect of overestimation of the number of sources on the tracking performance of the algorithms.

Finally we compare performance when a third source with delay $\tau_3 = 9.04 \times T_c$ and power $20dB$ below the desired signal is added to the environment intermittently over the simulation interval. The algorithms are initialised to search for only two sources. The results are plotted in Figure (4). Again, our algorithm is seen to be robust to this disturbance.

5. CONCLUSION

In this paper, a novel H^∞-type delay-tracking approach has been proposed, based on the reformulation of the PN-code matrix of the CDMA users into a state-space linear combiner. The approach has been demonstrated to track the delay of a desired user consistently better than a signal subspace-type algorithm over a range of signal environments. Simulations have shown it to perform well both when the number of signals present is overestimated and also when an additional intermittent interference source appears periodically through the simulation interval. The proposed approach has been shown to provide a powerful *near-far* resistant solution to the problem of delay tracking in a multi-user environment, robust to any modelling errors.

6. REFERENCES

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