ANALYSIS AND DESIGN OF NARROWBAND ACTIVE NOISE CONTROL SYSTEMS

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ABSTRACT

This paper presents an analysis and optimization of narrowband active noise control (ANC) systems using the filtered-X least mean-square (LMS) algorithm. First, we derive an upper bound for the eigenvalue spread of the filtered reference signal's covariance matrix, which provides insights into algorithm convergence speed. Amplitude of internally generated sinusoidal reference signal is optimized as the inverse of the secondary path's magnitude response at the corresponding frequency to improve the convergence speed. Second, we analyze the characteristic of asymmetric out-of-band overshoot. Based on the analysis result, the phase of sinusoidal reference signal is optimized to compensate for the phase shift of the secondary path. This phase optimization leads to the minimization of the out-of-band overshoot.

1. INTRODUCTION

Active noise control [1, 2] is based on the principle of superposition, where an unwanted noise is canceled by a secondary noise of equal amplitude and opposite phase. In many practical ANC applications, the primary noise is produced by rotating machine and is periodic. In this case, a reference sensor such as a tachometer or an accelerometer provides frequency information for a signal generator to synthesize an internally generated reference signal that contains the fundamental frequency and all the harmonics of the primary noise. The reference signal is then processed by an adaptive filter to generate a canceling signal that is fed to a secondary source. An error sensor measures the residual noise and uses it to update the coefficients of the adaptive filter by the filtered-X LMS (FXLMS) algorithm [3].

The adaptation of the FXLMS algorithm is slow because of the delay associated with the secondary path from the output of the adaptive filter to the output of the error sensor. If the reference signal consists of multiple sinusoids, another problem arises because the modulus of the secondary path at these sinusoidal frequencies will be very different. A step size (or convergence factor) must be chosen to guarantee that the system is stable for the frequency at which the response of the secondary path is largest. This will considerably slow down the convergence of the algorithm at frequencies where the response of the secondary path is small. In this paper, each sinusoidal reference signal's amplitude will be optimized as the inverse of the secondary path's magnitude response at the corresponding frequency in order to improve the convergence speed.

The stability and transient response of the adaptive notch filter using the FXLMS algorithm is analyzed in the complex weight domain [4]. The analysis shows a large out-of-band overshoot can lead to instability. Furthermore, when a periodic signal embedded in broadband noise is the subject of cancellation, the out-of-band overshoot on each side of the canceling notches will introduce significant undesired amplification of the background noise [4-6]. One solution is to equalize the secondary path over the entire band. However, there is an inherent tradeoff here because an additional filter will also introduce an extra delay, which will further slow down the convergence rate. In this paper, phase of internally generated sinusoidal reference signal will be optimized based on the normalized frequency and the secondary path's phase response in order to reduce the out-of-band overshoot.

2. NARROWBAND ANC SYSTEMS

A block diagram of a narrowband ANC system with the FXLMS algorithm is illustrated in Fig. 1. The reference signal $x(n)$ is the sum of K sinusoids, i.e.,

$$x(n) = \sum_{k=1}^{K} A_k \sin(k\omega_0 n), \tag{1}$$

where A_k is the amplitude of the k-th harmonic at frequency $k\omega_0$, with ω_0 being the fundamental frequency. The secondary signal $y(n)$ is generated as $y(n) = \mathbf{w}^T(n)\mathbf{x}(n)$, where $\mathbf{w}(n) = [w_0(n)\ \ w_1(n)\ \cdots\ w_{L-1}(n)]^T$ is the weight vector of the adaptive filter $W(z)$ with order L, T denotes transpose of a vector, and $\mathbf{x}(n) = [x(n)\ \ x(n-1)\ \cdots\ x(n-L+1)]^T$ is the sinusoidal reference signal vector. The weight vector is updated by the FXLMS algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}'(n), \tag{2}$$

where μ is the step size, $e(n)$ is the residual noise sensed by the error sensor, and $\mathbf{x}'(n) = [x'(n)\ \ x'(n-1)\ \cdots\ x'(n-L+1)]^T$ is the filtered reference signal vector with components $x'(n) = x(n) * \hat{s}(n)$. Here, $\hat{s}(n)$ is the impulse response of the secondary-path estimate $\hat{S}(z)$ and * denotes convolution.

Consider the case in which the control filter, $W(z)$, is changing slowly, the order of $W(z)$ and $S(z)$ in Fig. 1 can be commuted [3, 7]. We further assume that $\hat{S}(z) = S(z)$, Fig. 1 can be simplified to Fig. 2. Since the output of the adaptive filter now carries through directly to the error signal, the traditional LMS algorithm analysis method can be used, although the relevant reference signal is now $x'(n)$, which is produced by filtering $x(n)$ through $S(z)$.

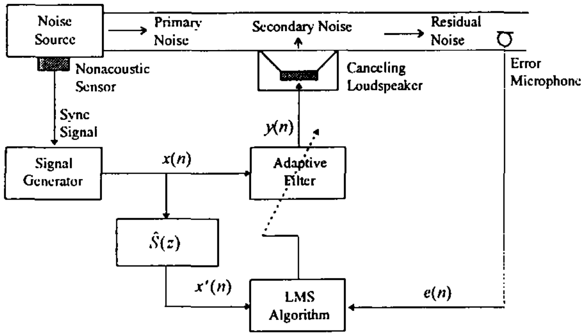

Fig. 1 Narrowband ANC system with the FXLMS algorithm

The convergence rate is determined by the eigenvalue spread of the covariance matrix of the filtered reference signal $x'(n)$, which is a function of the internally generated reference signal $x(n)$ and the secondary path $S(z)$. The characteristics of $S(z)$ are determined by the physical ANC system setup. The internally generated reference signal, however, can be optimized to improve the performance of narrowband ANC system.

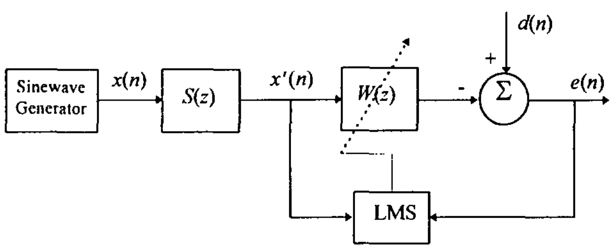

Fig. 2 Analysis model for the narrowband ANC system

3. CONVERGENCE ANALYSIS AND IMPROVEMENT

A convergence analysis of the narrowband ANC system is presented in this section using an eigen-decomposition approach.

3.1. Convergence Analysis

As shown in Fig. 2 and Eq. (1), based on the steady-state sinusoidal response, the filtered signal can be expressed as

$$x'(n) = \sum_{k=1}^{K} A'_k \sin(k\omega_0 n + \phi_k), \tag{3}$$

where ϕ_k is the phase of $S(z)$ at frequency $k\omega_0$ and

$$A'_k = A_k S_k. \tag{4}$$

Here S_k is the magnitude response of $S(z)$ at frequency $k\omega_0$. The covariance matrix of the filtered signal $x'(n)$, $\mathbf{R} = E[\mathbf{x}'(n)\mathbf{x}'^T(n)]$, can be decomposed as the sum of K simpler matrices with each being the covariance matrix of the filtered sinewave [8]

$$\mathbf{R} = \frac{1}{2}\sum_{k=1}^{K} A_k'^2 \mathbf{R}_k, \tag{5}$$

where

$$\mathbf{R}_k = \begin{bmatrix} 1 & \cos(k\omega_0) & \cdots & \cos[(L-1)k\omega_0] \\ \cos(k\omega_0) & 1 & \cdots & \cos[(L-2)k\omega_0] \\ \vdots & \vdots & \ddots & \vdots \\ \cos[(L-1)k\omega_0] & \cos[(L-2)k\omega_0] & \cdots & 1 \end{bmatrix}. \tag{6}$$

For the multiple sinusoidal reference signal $x(n)$ defined in Eq. (1), each matrix $\mathbf{R}_k$ defined in Eq. (6) has two nonzero eigenvalues. Therefore, only $2K$ eigenvalues of the matrix $\mathbf{R}$ are non-zero. It can be shown that

$$\frac{\lambda_{\max}}{\lambda_{2K}} \le C \frac{\max_{k=1}^{K} A_k'^2}{\min_{k=1}^{K} A_k'^2}, \tag{7}$$

where λ_{2K} is the smallest non-zero eigenvalues of the matrix $\mathbf{R}$ and C is a constant determined by the parameters L and ω_0.

Equation (7) indicates that if the upper bound is large, the convergence of the adaptive algorithms may be very slow for the mode corresponding to the small non-zero eigenvalues. Note that $A_k = 1$ is used in previous works, results in $A'_k = A_k S_k = S_k$, as shown in Eq. (4). Therefore, the error signal components converge slowly at the frequencies corresponding to the valleys of the magnitude response of $S(z)$. A simple method to speed up convergence is to use a larger step size μ. Unfortunately, a step size must be chosen to guarantee stability for the frequency component at which the secondary-path response is largest. The step size μ so chosen will considerably slow down the convergence of the algorithm at frequencies where the magnitude response of secondary path is small.

3.2. Amplitude Optimization

To speed up the convergence of the algorithm, we can choose the values of A_k such that the upper bound for the eigenvalue spread given in Eq. (7) is small and independent of the secondary path $S(z)$. As shown in Eq. (4), $A'_k = 1$ if we choose $A_k = 1/S_k$. Therefore, to improve the convergence speed of the narrowband ANC system, the amplitudes of internally-generated sinusoids are optimized as follows:

$$A_k = \frac{1}{\hat{S}_k} = \frac{1}{\left|\hat{S}(z)\right|_{z=e^{jk\omega_0}}}, \quad k = 1, 2, \cdots, K, \tag{8}$$

i.e., the amplitude of the k-th reference sinusoid is chosen to be the inverse of the magnitude response of the secondary-path estimate $\hat{S}(z)$ at the corresponding frequency $k\omega_0$.

From Eq. (7) and assuming that $\hat{S}(z) = S(z)$, we have $A'_k = A_k S_k = 1$ and

$$\frac{\lambda_{\max}}{\lambda_{2K}} \le C \frac{\max\limits_{k=1}^{K} A_k'^2}{\min\limits_{k=1}^{K} A_k'^2} = C. \tag{9}$$

Therefore, the modulation effect of the secondary path $S(z)$ is compensated by the optimized sinewave amplitudes given in Eq. (8). Extensive computer simulations were conducted to show that $\lambda_{\max}/\lambda_{2K}$ is close to one for the optimized algorithm [8]. It is important to note that the optimization of sinewave amplitudes defined in Eq. (8) can be done off-line after the completion of off-line secondary-path modeling. Thus, the optimized algorithm does not increase computational burden in real-time on-line ANC operation.

The transfer function of the secondary path $S(z)$ for computer simulations was measured from an experimental duct setup for acoustic ANC. The primary noise $d(n)$ used in computer simulations consists of white noise and 16 harmonics. For the optimized narrowband ANC algorithm, we used A_k defined in Eq. (8). For the original algorithm, $A_k = 1$ was used. The simulation results show that the optimized algorithm converges much faster than the original algorithm. The spectra of residual noises after 8,000 iterations are plotted in Fig. 3a for the optimized algorithm and in Fig. 3b for the original ANC algorithm. The results show that the optimized algorithm achieved much better performance than the original algorithm at normalized frequencies corresponding to the valleys of $S(z)$.

(a)

(b)

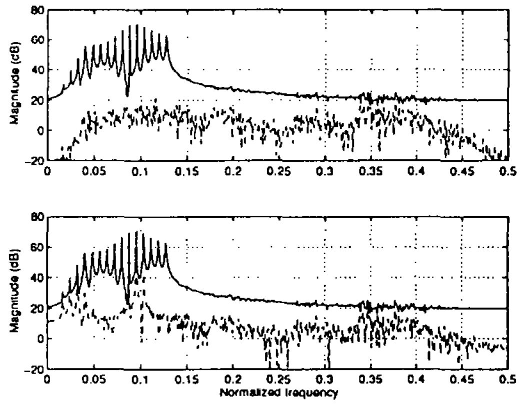

Fig. 3 Residual noise spectral after 8000 iterations: (a) optimized algorithm, (b) original algorithm. Solid line: before cancellation, dashed line: after cancellation.

4. OUT-OF-BAND OVERSHOOT REDUCTION

In this section, we consider a single-frequency narrowband ANC system which uses a second-order transversal filter [2]. The reference input $x(n)$ is a sinusoidal signal with frequency ω_0 and phase ϕ. As shown in Fig. 2, the output of $S(z)$ can be expressed as $x'(n) = A_s \sin(\omega_0 n + \phi + \phi_s)$, where A_s and ϕ_s are the magnitude gain and the phase shift, respectively, of the secondary path $S(z)$ at frequency ω_0.

4.1 Analysis of Narrowband ANC

The closed-loop transfer function from $d(n)$ to $e(n)$ can be derived as [9]

$$H(z) = \frac{z^2 - 2z\cos\omega_0 + 1}{z^2 + \left(k\alpha\cos\omega_0 + k\beta\sin\omega_0 - 2\cos\omega_0\right)z + 1 - k\alpha}, \tag{10}$$

where $k = -\mu A_s^2$, $\alpha = 2\left[\cos\left(2\phi + 2\phi_s - \omega_0\right)\cos\omega_0 - 1\right]$, and $\beta = -2\sin\left(2\phi + 2\phi_s - \omega_0\right)\cos\omega_0$. The roots of the numerator of Eq. (10) are the zeros $z_z = e^{\pm j\omega_0}$ of the narrowband ANC system. They are located precisely on the unit circle in the z-plane at angles of $\pm\omega_0$. The poles of the system can be obtained by setting the denominator of Eq. (10) equal to zero. The complex-conjugate poles are at $z_p = r_p e^{\pm j\theta_p}$ where [9]

$$\begin{aligned} r_p &= \sqrt{1 + 2\mu A_s^2[\cos(2\phi + 2\phi_s - \omega_0)\cos\omega_0 - 1]} \\ &= 1 - \eta\mu + o(\mu^2) \end{aligned} \tag{11}$$

is the radius of the poles and

$$\theta_p = \pm\cos^{-1}\left(\frac{2\cos\omega_0 - k(\beta\sin\omega_0 + \alpha\cos\omega_0)}{2\sqrt{1 - k\alpha}}\right) \tag{12}$$

is the angle of the poles.

The magnitude response of the narrowband ANC system can be obtained as [9]:

$$\begin{aligned} |H(\omega)| &= |H(z)|_{z=e^{j\omega}} \\ &= \sqrt{\frac{4(\cos\omega - \cos\omega_0)^2}{\left(r_p^2 - 1\right)^2 + 4\left(r_p^2 - 1\right)\cos\omega\left(\cos\omega - r_p\cos\theta_p\right) + 4\left(\cos\omega - r_p\cos\theta_p\right)^2}} \end{aligned} \tag{13}$$

where r_p and θ_p are the radius and angle of poles, respectively. Let $|H_+(\omega)| = |H(\omega)|_{\omega=\omega_0+\Delta\omega}$ where $\Delta\omega$ is an absolute offset value from ω_0, and let $|H_-(\omega)| = |H(\omega)|_{\omega=\omega_0-\Delta\omega}$, we can easily show that [9]

$$|H_+(\omega)| \ne |H_-(\omega)|, \tag{14}$$

which indicates that the magnitude response is asymmetric, thus the out-of-band overshoot is asymmetric with respect to ω_0 in most cases.

The maximum value of overshoot is [9]

$$|H(\omega)|_{\max} \approx \frac{2}{\eta\mu}\left|\frac{\cos\theta_p - \cos\omega_0}{\sin\theta_p}\right|. \tag{15}$$

Ignoring all terms which contain μ^2 or higher order, we have

$$\theta_p \approx \omega_0. \tag{16}$$

This equation shows the following important results:

1. For the narrowband ANC system, the maximum out-of-band overshoot occurs approximately at frequency $\omega = \theta_p$, where θ_p is the angle of the poles.

2. When $\theta_p = \omega_0$, i.e., the angle of poles aligned with the angle of zeros, the out-of-band overshoot is minimized.

4.2 Phase Compensation

The undesired out-of-band overshoot can be minimized when $\theta_p = \omega_0$, the angles of poles are aligned with the angles of zeros. From Eq. (16), we have

$$\cos\omega_0 = \cos\theta_p = \frac{2\cos\omega_0 - k(\beta\sin\omega_0 + \alpha\cos\omega_0)}{2\sqrt{1-k\alpha}}. \quad (17)$$

Using the facts that $k = -\mu A_s^2 \neq 0$, k is a very small value and some trigonometry formula, the optimal initial phase can be expressed as [9]

$$\phi_{\text{optimal}} = \frac{1}{2}(m\pi + \omega_0) - \phi_s, \quad (18)$$

where m is an integer number.

The optimal phase to minimize an undesired out-of-band overshoot is the function of the reference signal frequency ω_0 and the secondary path's phase ϕ_s. For a given physical narrowband ANC setup, the secondary-path transfer function $S(z)$ can be estimated off-line, thus the phase of the secondary path ϕ_s is known for a given frequency. Therefore, the optimal phase ϕ_{optimal} can be calculated after off-line modeling for a given frequency ω_0 using Eq. (18). By using the optimal phase in the internally generated sinusoidal signal, the out-of-band overshoot can be significantly reduced. It is important to note that the optimization of sinewave's phase can be done off-line after the completion of off-line secondary-path modeling. Thus, the optimized algorithm do not increase computational burden in real-time on-line ANC operation.

Computer simulations were conducted to verify the theoretical optimal phase expressed in Eq. (18). The parameters used in the simulation shown in Fig. 4 are $k = -0.05$, $\omega_0 = 0.2\pi$, and $\phi_s = 60^\circ$. By setting $m = 1$ in Eq. (18), the theoretical optimal phase is given by $\phi_{\text{optimal}} = 48^\circ$ and it is verified by the results shown in Fig. 4 that the 48° initial phase resulted in the best magnitude response; while other initial phases resulted in undesired out-of-band overshoot.

5. CONCLUSION

The effects of the secondary-path transfer function on the convergence of the FXLMS algorithm for narrowband ANC was studied based on the eigenvalue spread of the covariance matrix of the filtered reference signal. To speed up the convergence corresponding to valleys of the magnitude response of the $S(z)$, the reference signal was generated with the amplitude at each frequency being equal to the inverse of the magnitude response of $S(z)$ at that frequency. The undesired out-of-band overshoot was reduced by optimizing the phase of the internally generated sinusoidal reference signal to align the angles of poles with the angles of zeros.

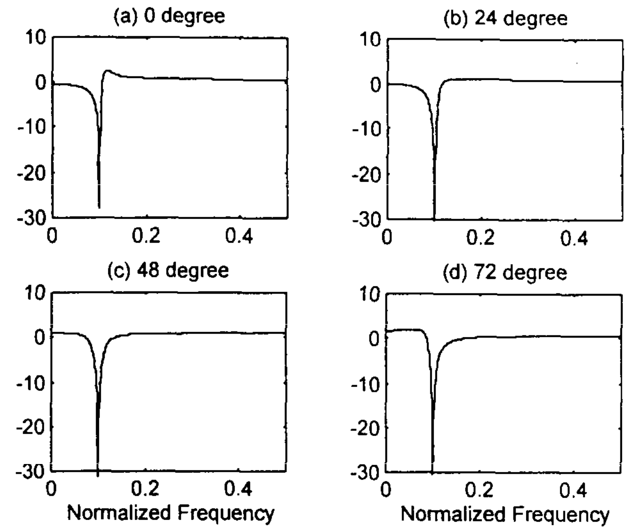

Fig. 4 Magnitude (in dB scale) responses of the narrowband ANC defined in (13) with $k = -0.05$, $\omega_0 = 0.2\pi$, $\phi_s = 60^\circ$ and four different ϕ : (a) 0°; (b) 24°; (c) 48°; (d) 72°.

6. REFERENCES

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