BLIND IDENTIFICATION AND ORDER ESTIMATION OF FIR COMMUNICATIONS CHANNELS USING CYCLIC STATISTICS

Giampiero Panci, Gaetano Scarano and Giovanni Jacovitti

Dip. INFOCOM, Università di Roma "La Sapienza"
Via Eudossiana 18, I-00184 Roma, Italy
Fax: +39.6.4873.300, Tel: +39.6.4458.5500
E-Mail: gpanci, gaetano, gjacov@infocom.ing.uniroma1.it

ABSTRACT

In this contribution we address the problem of the *blind* joint identification and order estimation of a non–minimum phase FIR communication channel by exploiting the cyclostationarity of the received signal sampled at rate greater of the symbol rate. We show that the identification can be formulated as a "subspace fitting" problem; this allows for using the subspace distance as a test statistic to detect the correct channel length (among different hyphoteses). Moreover, mimicking estimation procedures proposed in the framework of DOA estimation, an asymptotically efficient procedure is proposed which obtains the channel estimate in two steps: the channel estimate obtained in the first step determines optimal weights which are then employed in the second step to refine the channel estimate. The accuracy of the identification is comparable with other methods described in the literarature (*e.g.* the subspace method [4]) while the test for order detection performs quite well also in presence of channel disparity outperforming commonly used tests based on the eigenvalues of the covariance matrix.

1. INTRODUCTION

Blind identification of single input/multi output (SIMO) FIR channels has received great attention in the very recent literature and estimation techniques based on geometric properties of suitable matrices of second–order statistics has been proposed and their accuracy analyzed [4, 10, 7, 5].

Moreover, fractionally sampling of communications signals reveals the ciclostationarity induced by the trasmitted symbols which can be exploited using multichannel representations so allowing for classical stationary analysis. The ciclostationarity of fractionally sampled communications signals is fully exploited in [1, 2, 3] to obtain solutions to both the identification and the equalization of the channel.

Despite the identifiability issue, *i.e.* common factors among subchannels cannot be identified from second order statistics, all the methods strongly rely on the knowledge of the FIR channel length since model mismatching dramatically affects the accuracy of the estimation. Usually, the order of the channel is estimated by means of statistical tests based on the hyphotesis of equal "noise" eigenvalues of suitable second order statistics matrices which are not particular robust for channel with lack of disparity (loosely speaking, channels close to non identifiability conditions).

Here, by further exploiting the property of matrices of cyclic statistics already presented in [3], we propose a "subspace fitting" (SF) approach to the blind identification of FIR channels which embeds the detection of the channel order as the measure of the distance between suitably defined subspaces of such matrices.

Moreover, estimation procedures based on SF already used for DOA estimation in array processing can be usefully applied to the case of interest and, in particular, asymptotically optimal techniques such as "Mode" technique [8] are here considered as well as other, non optimal but computationally less intensive, techniques such as the so–called "Reduced Order" technique [6].

Accuracy analysis is conducted by computer simulations to asses to applicability of the proposed techniques and results concerning the power of the order detection test are also reported.

2. BLIND IDENTIFICATION USING CYCLIC STATISTICS

We refer to the following observed discrete–time cyclostationary process

$$y_n = \sum_m w_m \cdot h[n - mP] + v_n \tag{1}$$

where the possibly coloured additive noise v_n is independent of the i.i.d. "symbols" w_n.

The time–varying (biargumental) correlation is periodic with period P (w.r.t. the index n):

$$\begin{aligned} R_y[n,k] &\stackrel{\text{def}}{=} \mathrm{E}\{y_{n+k}\overline{y}_n\} \\ &= \sigma_w^2 \sum_m h[k+n-mP] \cdot \overline{h}[n-mP] + R_v[k] \end{aligned} \quad (2)$$

In (2), σ_w^2 is the power of the symbols, $R_v[k]$ is the autocorrelation of the stationary noise v_n and the overbar denotes complex conjugation.

The cyclic correlations are the DFS coefficients of the periodic (for k fixed) sequence $R_y[n,k]$:

$$\begin{aligned} R_y^\alpha[k] &= \frac{1}{P} \sum_{n=0}^{P-1} R_y[n,k] e^{-j\frac{2\pi}{P}\alpha n} \\ &= \frac{\sigma_w^2}{P} \sum_l h[k+l] \cdot \overline{h}[l] e^{-j\frac{2\pi}{P}\alpha l} + R_v[k] \cdot \delta[\alpha] \\ \alpha &= 0, \cdots, P-1 \end{aligned} \quad (3)$$

Let us focus on the case $P = 2$, typical in digital communication scheme with fractional sampling; moreover, let us consider FIR channels of length L_h. We have, for $|k| \leq L_h - 1$,

$$R_y^0[k] = \frac{\sigma_w^2}{2} \sum_{l=0}^{L_h-1} h[k+l] \cdot \overline{h}[l] + R_v[k]$$

$$R_y^1[k] = \frac{\sigma_w^2}{2} \sum_{l=0}^{L_h-1} h[k+l](-1)^l \cdot \overline{h}[l]$$

and the relative cyclic spectra (Fourier transforms) are

$$P_y^0(e^{j\omega}) \stackrel{\text{def}}{=} \mathcal{F}\{R_y^0[k]\} = \frac{\sigma_w^2}{2} H(e^{j\omega})\overline{H}(e^{j\omega}) + P_v(e^{j\omega}) \quad (4)$$

$$P_y^1(e^{j\omega}) \stackrel{\text{def}}{=} \mathcal{F}\{R_y^1[k]\} = \frac{\sigma_w^2}{2} H(e^{j\omega}) \cdot \overline{H}(e^{j\omega+j\pi}) \quad (5)$$

The previous developements are well known in the literature and are reported for the sake of comprehension and for notational purposes only.

As shown in [3] using these statistics we can write the following identification system:

$$\mathbf{M}_y \cdot \overline{\mathbf{h}} = \sigma_v^2 \cdot \mathbf{M}_v \cdot \overline{\mathbf{h}} \quad (6)$$

where the channel coefficients are collected in the vector $\overline{\mathbf{h}} = \left[\overline{h}[0], \cdots, \overline{h}[L_h-1]\right]^T$ and the matrices $\mathbf{M}_y$ and $\mathbf{M}_v$ collects second order cyclic statistics as follows

$$\|\mathbf{M}_y^0\|_{kl} = R_y^0[k+l](-1)^l \quad ; \quad \|\mathbf{M}_y^1\|_{kl} = R_y^1[k+l]$$

$$\mathbf{M}_y = \mathbf{M}_y^0 - \mathbf{M}_y^1$$

$$\|\mathbf{M}_v\|_{kl} = \rho_v[k+l](-1)^l$$

having denoted the noise correlation $R_v[k] = \sigma_v^2 \cdot \rho_v[k]$.

We see that, apart a complex constant, the channel coefficient $\overline{\mathbf{h}}$ are given by the generalized eigenvector of the pencil $(\mathbf{M}_y, \mathbf{M}_v)$ corresponding to the generalized eigenvalue σ_v^2. When the channel order is not matched, there are multiple generalized eigenvalues equal to σ_v^2 and we can only individuate a subspace where the vector of the channel coefficients lies. This happen also when the order is known and there are non-identifiable components in the channel. For the sake of symplicity, in the following we consider only identifiable channels to exploit the properties of the kernel of the pencil $(\mathbf{M}_y, \mathbf{M}_v)$; these can be readily extended to channels with non identifiable components as it will appear in the following.

Let us consider the white noise case, to which the problem can be reconducted through whitening by a known square root of $\mathbf{M}_v$. In this case, the solution of (6) is given by the less significative eigenvector of the matrix $\mathbf{M}_y$ when the channel order is matched, *i.e.* the kernel of $\mathbf{M}_y - \sigma_v^2 \cdot \mathbf{I}$ has dimension equal to one.

Let is prove now the following

Theorem 1
Let $N \geq L_h$ be the number of columns of the matrix $\mathbf{M}_y$. Then, the kernel of the matrix $\mathbf{M}_y - \sigma_v^2 \cdot \mathbf{I}$ has dimension $\mathrm{int}\,[(N - L_h + 2)/2]$ and it admits a basis having the following structure:

$$\mathbf{V}(\mathbf{h}) \stackrel{\text{def}}{=} \begin{bmatrix} h_0 & h_1 & h_2 \cdots h_{L_h-1} & 0 & \cdots & 0 \\ 0 & 0 & h_0 \cdots h_{L_h-3} & h_{L_h-2} \cdots & & 0 \\ \vdots & \vdots & \vdots \cdots \quad \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 \cdots \quad 0 & 0 & \cdots & h_{L_h-1} \end{bmatrix}^H \quad (7)$$

Proof: First we determine the kernel dimension. The linear system (6) is the time domain expression of the homogeneous polynomial equation

$$\left(P_y^0(z) - P_v(z)\right)\overline{H(-z)} = P_y^1(z)\overline{H(z)}$$

in the unknown polynomial $H(z)$.

All the solutions must be in the form $H(z)H'(z)$ where $H'(z) = \prod_k (1 - a_k z^{-1})(1 + a_k z^{-1})$ has zeroes uniformly spaced on circles (due to the presence of both $H(z)$ and $H(-z)$) and $H(z)$ is the minimum degree polynomial without zeroes uniformly spaced on circles, *i.e.* the identifiable part of the channel since all other factors are embedded in $H'(z)$

Then, if $N = L_h$ the solution is unique and the dimension of the kernel is 1. For $N > L_h$, due to the nature of the polynomial $H'(z)$ which has an even number of roots, the dimension of the kernel increases as $1 + \mathrm{int}\,[(N - L_h)/2]$

As far as the structure of the kernel is concerned, the columns of $\mathbf{V}$ are linearly indipendent and all are solution of the form $H(z)z^{-2k}$ so constituting a basis of the kernel. □

The theorem 1 can be employed to phrase the joint order and channel estimation as a minimization of a suitably defined subspace distance criterion. In fact, given a generic basis, say $\mathbf{U}$, of the kernel of the matrix $\mathbf{M}_y - \sigma_v^2 \cdot \mathbf{I}$ (*e.g.* obtained through ordinary SVD) the correct channel order and coefficients zeroes the following functional

$$J(\mathbf{h}, L_h) = \|\mathbf{V}(\mathbf{h}) - \mathbf{U} \cdot \mathbf{T}\|_{\mathrm{F}}^2 \qquad (8)$$

where $\mathbf{T}$ is a non singular transformation between the bases $\mathbf{U}$ and $\mathbf{V}(\mathbf{h})$ and $\|\cdot\|_{\mathrm{F}}$ denotes Frobenius norm.

The functional (8) depends on both $\mathbf{h}$ and $\mathbf{T}$ which are unknowns. A similar functional is miminized in the so–called "Reduced Order" DOA estimation technique [6]. This technique basically exploits the fact that transformation $\mathbf{T}$ can be determined exploiting the special sparse structure of $\mathbf{V}(\mathbf{h})$ which allows to build a homogeneous linear systems in the unknowns elements of $\mathbf{T}$ using only those equations

$$\mathbf{U} \cdot \mathbf{T} = \mathbf{V}(\mathbf{h})$$

which corresponds to zeros in the right hand side. Solving for $\mathbf{T}$ the resulting overdetermined[1] homogeneous linear system, allows for the determination of the matrix $\mathbf{V} = \mathbf{U} \cdot \mathbf{T}$ which will admit the structure (7) iff the channel order is exact, *i.e.* the functional $J(\mathbf{h}, L_h)$ is zero.

Another procedure is obtained mimicking the so-called "MODE" technique described in [8] for DOA estimation. It stem out from the fact that the minimization of (8) can be separately done for $\mathbf{T}$ and $\mathbf{V} \stackrel{\mathrm{def}}{=} \mathbf{V}(\mathbf{h})$. Eliminating $\mathbf{T}$ and using the eigendecomposition of the matrix $\mathbf{M}_y = \Psi\Lambda\Psi^{\mathrm{H}}$, (8) is rewritten as follows

$$J(\mathbf{h}, L_h) = \mathrm{tr}\left[\mathbf{V} \cdot (\mathbf{V}^{\mathrm{H}}\mathbf{V})^{-1} \cdot \mathbf{V}^{\mathrm{H}}\mathbf{Q}\right] \qquad (9)$$

where $\mathbf{Q} \stackrel{\mathrm{def}}{=} \Psi\Delta\Psi^{\mathrm{H}}$, and $\Delta = \Lambda^{-1}(\Lambda - \sigma_v^2\mathbf{I})^2$.

Note that the functional (9) is highly nonlinear in the unknowns $\mathbf{h}$ and it should be minimized using numerical techniques. Order detection is carried out by minimizing over different hyphoteses.

The MODE technique [8] obtain a solution in two steps: in the first step the functional (9) is minimized dropping out the term $(\mathbf{V}^{\mathrm{H}}\mathbf{V})^{-1}$, *i.e.* minimizing the following **quadratic** functional for the channel coefficients $\mathbf{h}$:

$$J_1(\mathbf{h}_1, L_h) = \mathrm{tr}\left[\mathbf{V}\mathbf{V}^{\mathrm{H}} \cdot \mathbf{Q}\right]$$

under some suitable constraint such as $\|\mathbf{h}\| = 1$.

[1] In fact, said $x^2 = (1 + \mathrm{int}\,[(N - L_h)/2])^2$ the number of unknows, the available equations are $2x(x-1)$. This does not occur in the DOA case where the linear system is square.

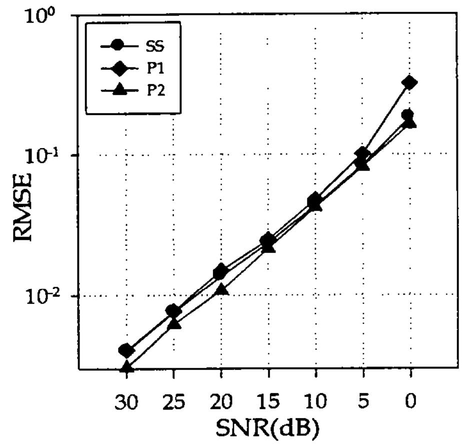

Figure 1: RMSE of various estimators vs. SNR for channel without lack of disparity.

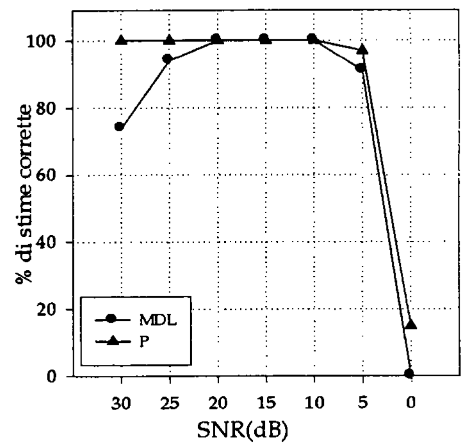

Figure 2: Percentage of succesfull order detection vs. SNR for channel without lack of disparity.

Then, substituting the obtained consistent estimate of $\widehat{(\mathbf{V}^{\mathrm{H}}\mathbf{V})}^{-1} = \left(\mathbf{V}^{\mathrm{H}}(\mathbf{h}_1)\mathbf{V}(\mathbf{h}_1)\right)^{-1}$ in (9), another **quadratic** functional is obtained

$$J(\mathbf{h}, L_h) = \mathrm{tr}\left[\mathbf{V} \cdot \left(\mathbf{V}^{\mathrm{H}}(\mathbf{h}_1)\mathbf{V}(\mathbf{h}_1)\right)^{-1} \cdot \mathbf{V}^{\mathrm{H}}\mathbf{Q}\right] \qquad (10)$$

The minimization of (10) yields an estimate of $\mathbf{h}$ whose accuracy is asymptotically the same of the estimation drawn from (9), as proved in [8].

3. SIMULATION RESULTS AND CONCLUSION

We have performed computer simulations to assess the accuracy of the proposed estimation procedures and for comparison purposes with existing techniques. In particular, we have considered the subspace (SS) technique described in [4] as a reference. In fig.1 the RMSE defined as RMSE$=\left(\sqrt{\frac{1}{N_r}\sum_{i=1}^{N_r}\|\tilde{\mathbf{h}}-\mathbf{h}\|^2}\right)$ (considering normalized unit norm vectors) is plotted *vs.* SNR for the channel $\mathbf{h}=[1.165, 0.6268, -1.0751, 0.3516, -0.6965, 2.6961]^T$ already tested in literature which does not shows lack of disparity and binary i.id. symbols. We have denoted by SS, $P1$ and $P2$ the results relative to the SS technique, the proposed "Reduced Order" technique and the proposed "MODE" technique, having drawn sample estimates of statistics from 1000 fractionally sampled observations. In fig.2 the percentage of succesfull order detection is reported for the same channel compared to application of MDL criterion to the estimation of channel order.

To test robustness under lack of disparity the channel $\mathbf{h}=[1, 1, -1.902113, -1.61803, 1, 1]^T$ has been considered in figs. 3 and 4. Note the improved performance is observed in the order detection, while channel estimation accuracy is comparable if not better than accuracy obtained by SS technique.

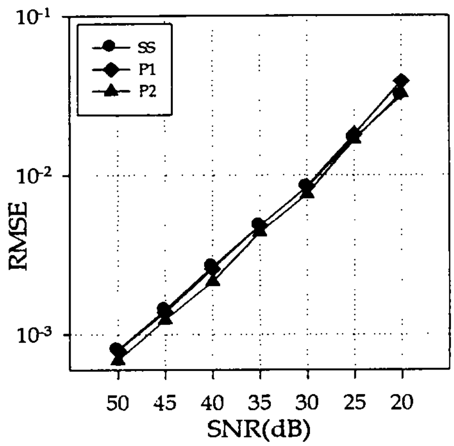

Figure 3: RMSE of various estimators vs. SNR for channel with lack of disparity.

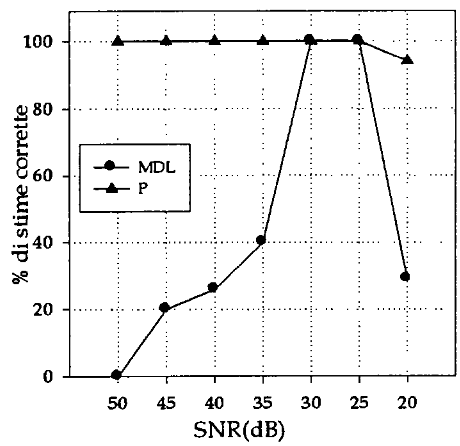

Figure 4: Percentage of succesfull order detection vs. SNR for channel with lack of disparity.

4. REFERENCES

[1] G.B. Giannakis, "Linear cyclic correlation approaches for blind identification of FIR channels", *Proc. 28th Asilomar Conf. Sig., Sys. and Comp.*, Pacific Grove (CA), Nov. 1994.

[2] G.B. Giannakis, S. D. Halford, "Blind fractionally-spaced equalization of noisy FIR channels: adaptive and optimal solutions", *Proc. IEEE Int. Conf. Acoust. Speech Sig. Proc.*, Detroit (MI), May 1995.

[3] G. Scarano, G. Panci, "On cyclic correlation approaches for blind identification of FIR communications channels", *First IEEE Workshop on Signal Processing Advances in Wireless Communications*, Paris, France, April 1997.

[4] E. Moulines, P. Duhamel, J.F. Cardoso, S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters", *IEEE Trans. SP*, vol. 43, no. 2, Feb. 1995.

[5] K. Abed-Meraim, P. Loubaton, E. Moulines, "A subspace algorithm for certain blind identification problem", *IEEE Trans. IT*, vol. 43, Mar. 1997.

[6] S.J. Orfanidis, **Optimum signal processing: An introduction**, McGraw Hill, 1990.

[7] W. Qiu, Y. Hua, "Performance analysis of the subspace method for blind channel identification", *IEEE Trans. IT*, vol. 41, Jan. 1995.

[8] P. Stoica, K. Sharman, "Maximum likelihood methods for direction-of-arrival estimation", *IEEE Trans. ASSP*, vol. 38, no.7, Jul. 1990.

[9] G. Xu, H. Liu, L. Tong, T. Kailath, "A deterministic approach to blind channel identification", *IEEE Trans. Sig. Proc.*, no. , Dec. 1995

[10] L. Tong, G. Xu,B. Hassibi, T. Kailath, "Blind channel identification and equalization based on second-order statistics: A frequency–domain approach", *IEEE Trans. IT*, vol. 41, Jan. 1995.