Spatial and Temporal Stability of Vision Chips Including Parasitic Inductances and Capacitances

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Abstract

There are two dynamics issues in vision chips: (i) The *temporal* dynamics issue due to the parasitic capacitors in a CMOS chip, and (ii) the *spatial* dynamics issue due to the regular array of processing elements in a chip. These issues are discussed in [1, 2, 3] for the resistor network with only associated parasitic capacitances. However, in this paper we consider also parasitic *inductances* as well as parasitic capacitances for a more precise network dynamics model. We show that in some cases the temporal stability condition for the network with parasitic inductances and capacitances is equivalent to that for the network with only parasitic capacitances, but in general they are not equivalent. We also show that the spatial stability conditions are equivalent in both cases.

Key Words : Network Stability, Vision Chip, Neuro Chip, Neural Network

I. Introduction This study has been motivated by spatial versus temporal stability issues of analog image-processing neuro chips (vision chips). The image-smoothing vision chip in [4], for instance, consists of a regular array of photo-sensors with conductances $g_0 > 0$, $g_1 > 0$, $g_2 < 0$ (Fig.1). We refer the reader to [4] for the chip details. Since the chip involves negative conductances g_2, both spatial and temporal stability issues naturally arise. There are two intriguing elements. First, our earlier numerical experiments suggested that generally a vision chip is temporally stable if and only if it is spatially stable, where spatial stability means that the node voltage distribution behaves "properly." Second, spatial dynamics naturally induces a discrete linear dynamical system so that its stability should be checked by its eigenvalues. " A discrete linear dynamical system is stable if and only if all the eigenvalues lie inside the unit circle of the complex plane." This statement turned out to be false. Namely, due to the noncausal nature of the dynamics, if λ is an eigenvalue, so is $1/\lambda$, and hence the stability condition for causal linear systems is never satisfied.

Most of the fundamental issues involving these two elements have been settled in [1, 2] for 1D and 2D array cases. For instance, a network is temporally stable if and only if it is spatially stable, except for a set of Lebesgue measure zero in the parameter space. Another fundamental result was that a network is spatially stable if and only if the eigenvalues of the dynamics are off the unit circle, even though they can be outside the unit circle. These results are far from trivial. One of reasons that makes these results crucial is the boundary conditions associated with the finiteness of a network. Even if the eigenvalue conditions are satisfied, solutions can oscillate or explode if the boundary conditions are inappropriate.

Although the previous results in [1, 2] are completely rigorous, the results are for the resistor network with only associated parasitic capacitances; associated parasitic inductances are neglected as a first-order approximation. However, in this paper we consider parasitic inductances as well as parasitic capacitances for a more precise network dynamics model. We show in some cases that the temporal stability condition for the network with parasitic inductances and capacitances is equivalent to that for the network with only parasitic capacitances, but in general they are not equivalent. We also show that the spatial stability conditions are equivalent in both cases.

Our approach in this paper is a systematic exploitation of the circulant network structure for 1D cases; speaking roughly, a circulant network has a "ring" structure as shown in Fig.2 (a). The validity of such an approach has already been discussed in [2], and we also remark that our results here can be extended to 2D cases in a similar manner to [2].

II. Formulation Now let us consider a 1D network with N nodes numbered 0 through N-1, where each node k is excited by a current source u_k and has an admittance y_0 to ground, and an admittance y_p to nodes $(k+p)$ for $p = \pm 1, \pm 2, \ldots \pm m$. Note that $y_p = y_{-p}$ because node k connects to node $k+p$ with y_p whereas node $(k+p)$ connects to node $((k+p)-p)$, i.e., node k with y_{-p} and hence $y_p = y_{-p}$. The network is said to be circulant if the rightmost and leftmost nodes are connected together, and thus the network is of a ring structure. Fig.2 (a) shows a circulant network where $m = 2$ and the admittance y_p is composed of a conductance g_p and a capacitance c_p in parallel ($p = 0, 1, 2$, Fig.2 (b)). Then we obtain the following equation from Kirchoff Current Law at node k:

$$-(y_0 + 2\sum_{p=1}^{m} y_p)v_k + \sum_{p=1}^{m} y_p(v_{k-p} + v_{k+p}) + u_k = 0.$$

Then letting

$$\mathbf{v} := (v_0, v_1, ..., v_{N_1\text{-}1})^T, \quad \mathbf{u} := (u_0, u_1, ..., u_{N_1\text{-}1})^T,$$

$$\alpha_0 := -(y_0 + 2\sum_{p=1}^{m} y_p), \quad \alpha_p = y_p, \quad p = 1, 2, ..., m,$$

the state equation is given by

$$Y\mathbf{v} + \mathbf{u} = \mathbf{0} \tag{1}$$

where

$$Y := circl(\alpha_0, \alpha_1, ..., \alpha_m, 0, ..., 0, \alpha_m, ..., \alpha_1)$$

and $circl()$ denotes a circulant matrix [5]. Let F be a Fourier matrix with size $N \times N$ and note that $F^*F = I$, then Eq.(1) leads to

$$F^*YFF^*\mathbf{v} + F^*\mathbf{u} = \mathbf{0}. \tag{2}$$

It follows from [5] that F^*YF is diagonalized as follows:

$$F^*YF := \Lambda = diag(\lambda_0, \lambda_1, ..., \lambda_{N\text{-}1}),$$

where

$$\lambda_k := \alpha_k + 2\sum_{p=1}^{m} \alpha_p \cos(2\pi pk/N) \qquad k = 1, 2, ..., N\text{-}1.$$

Letting

$$-F^*\mathbf{v} := \mathbf{o} = (o_0, o_1, ..., o_m)^T,$$

$$F^*\mathbf{u} := \mathbf{i} = (i_0, i_1, ..., i_m)^T,$$

then Eq.(2) reads

$$\Lambda\mathbf{o} = \mathbf{i}.$$

Thus if Λ is nonsingular, the followings are obtained:

$$\frac{o_0}{i_0} = \frac{1}{\lambda_0}, \ \frac{o_1}{i_1} = \frac{1}{\lambda_1}, \ \cdots, \frac{o_{N\text{-}1}}{i_{N\text{-}1}} = \frac{1}{\lambda_{N\text{-}1}}. \tag{3}$$

We see that the network is temporally stable if and only if all the transfer functions of $\lambda_0^{-1}, \lambda_1^{-1}, ..., \lambda_{N\text{-}1}^{-1}$ are stable, i.e., all of their poles are located in the left-half of the s-plane.

This statement is very general for the temporal stability of the network and is consistent to the previous results [1, 2, 3].

III. RCL Network Now consider the case that the admittance y_p consists of a conductance g_p, a capacitance c_p and an inductance l_p as shown in Fig.3, where g_p, c_p and l_p can be *negative*. Then the admittance y_p is given by

$$y_p = \frac{1}{1/g_p + sl_p} + sc_p = \frac{g_p + sc_p + s^2 l_p g_p c_p}{1 + sl_p g_p}. \tag{4}$$

The reader may wonder why g_p, c_p and l_p can be negative. For the image processing purpose, some of g_p have to be negative [4] and this negative conductance $g_p < 0$ can be implemented with a positive conductance $g'_p(= -g_p) > 0$ and two admittance inverters as shown in Fig.4. Let c'_p and l'_p be parasitic capacitance and inductance associated with g'_p. Even if c'_p and l'_p are positive, these can be effectively negative between the nodes A and B due to the two admittance inverters; the effective admittance y_p between the nodes A and B is equal to $-y'_p$. Note also that in the previous cases [1, 2, 3], the parasitic inductances were neglected as shown in Fig.2 (b) where $l_p = 0$.

Proposition 1 Consider the RCL network where y_p is given in Fig.3 and also the following restriction is satisfied:

$$l_p g_p = d > 0 \qquad \text{for } p = 0, 1, 2, ..., m, \tag{5}$$

where d is a positive constant. In this case the temporal stability condition of the RCL network is equivalent to that of the RC network (where L is neglected as shown in Fig.2 (b).)

Proof : It follows from Eqs.(4) and (5) that α_p's are given by

$$\alpha_0 := \frac{(g_0 + 2\sum_{p=1}^{m} g_p) + (s + s^2 d)(c_0 + 2\sum_{p=1}^{m} c_p)}{1 + sd},$$

$$\alpha_p := y_p = \frac{g_p + sc_p + s^2 dc_p}{1 + sd} \qquad \text{for } p = 1, 2, ..., m.$$

Then the transfer functions of λ_k^{-1} described in Eq.(3) are given by

$$\frac{1}{\lambda_k} = \frac{-(1 + sd)}{-\mu_k + s\nu_k + s^2 d\nu_k} \quad \text{for } k = 0, 1, 2, ..., N\text{-}1,$$

where

$$\mu_k := -(g_0 + 2\sum_{p=1}^{m} g_p) + 2\sum_{p=1}^{m} g_p \cos(2\pi pk/N)$$

$$\nu_k := -(c_0 + 2\sum_{p=1}^{m} c_p) + 2\sum_{p=1}^{m} c_p \cos(2\pi pk/N).$$

From the Routh-Hurwitz stability criteria, we obtain the following temporal stability condition:

$$-\mu_k > 0,\ \nu_k > 0,\ d\nu_k > 0,\ d\nu_k^2 > 0 \text{ for } k = 0, 1, 2, ..., N\text{-}1.$$

Then the above conditions yield to the following:

$$\mu_k < 0,\ \nu_k > 0,\ d > 0 \qquad \text{for } k = 0, 1, 2, ..., N\text{-}1. \quad (6)$$

Let us compare this result (Eq.(6)) to the RC network case in [1, 2]. "$\mu_k < 0$ for all $k = 0, 1, 2, ..., N$-1" means that the system matrix A [1, 2] is negative definite, and "$\nu_k > 0$ for all $k = 0, 1, 2, ..., N$-1" is equivalent to that the capacitance matrix B [1, 2] is positive definite. We see that the temporal stability condition of the RCL network which satisfies Eq.(5) is equivalent to that of the RC network. (Q. E. D.)

Proposition 2 If $l_p g_p \neq l_q g_q$ for some $0 \leq p, q \leq m$, then the temporal stability condition of the RCL network is *not* necessarily equivalent to that of the RC network.

Proof : Consider the case $m = 1$ and $l_0 g_0 \neq l_1 g_1$. Then it follows from Eq.(4) that

$$y_0 = (g_0 + sc_0 + s^2 d_0 c_0)/(1 + sd_0),$$

$$y_1 = (g_1 + sc_1 + s^2 d_1 c_1)/(1 + sd_1),$$

where $d_0 := l_0 g_0$, $d_1 := l_1 g_1$ and $d_0 \neq d_1$. Then the transfer functions described in Eq.(3) are given by

$$\frac{1}{\lambda_k} = \frac{1}{\alpha_0 + 2\alpha_1 \cos(2\pi k/N)}$$

$$= \frac{1}{-(y_0 + 2y_1) + 2y_1 \cos(2\pi k/N)} = -\frac{(1 + sd_0)(1 + sd_1)}{n_k(s)},$$

where

$$n_k(s) := e_0 + se_1 + s^2 e_2 + s^3 e_3,$$

$$e_0 := g_0 \beta_k g_1,\ \ e_1 := d_1 g_0 + \beta_k d_0 g_1 + c_0 + \beta_k c_1,$$

$$e_2 := (d_0 + d_1)(c_0 + \beta_k c_1),\ \ e_3 := d_0 d_1 (c_0 + \beta_k c_1),$$

$$\beta_k := 2(1 - \cos(2\pi k/N)).$$

It follows from the Routh-Hurwitz stability criteria that the network temporal stability condition is given by

$$e_0 > 0,\ e_1 > 0,\ e_2 > 0,\ e_3 > 0,\ e_1 e_2 - e_0 e_3 > 0.$$

This result means that in addition to the negative definiteness of the system matrix A, the positive definiteness of the capacitance matrix B and $d_0 > 0, d_1 > 0$, we need to satisfy the following conditions:

$$d_1^2 g_0 + \beta_k d_0^2 g_1 + c_0 + \beta_k c_1 > 0, \quad (7)$$

$$d_1 g_0 + \beta_k d_0 g_1 + c_0 + \beta_k c_1 > 0. \quad (8)$$

If $d_0 = d_1 > 0$ (i.e. Eq.(5) is satisfied), the negative definiteness of A and the positive definiteness B automatically lead to the above conditions (7), (8), however, if $d_0 \neq d_1$, they do not. Hence in this case the temporal stability condition is more strict than that in the RC network. (Q.E.D.)

Proposition 3 The spatial stability condition of the RCL network is equivalent to that of the RC network.

Proof : We need to consider the equilibrium point for the spatial stability, and at the equilibrium, the admittance y_p is given by

$$y_p|_{s=0} = \frac{1}{1/g_p + sl_p} + sc_p|_{s=0} = g_p.$$

This is the same as y_p at the equilibrium in the RC network, and hence the spatial stability conditions are equivalent in both cases. (Q.E.D)

Lemma In the RCL network where y_p consists of g_p, c_p and l_p as shown in Fig.4 and Eq.(5) is satisfied, the spatial and temporal stability conditions are virtually equivalent.

Proof : Note that the spatial and temporal stability conditions of the RC network are virtually equivalent [1, 2]. Then we see that according to this fact and Propositions 1, 3, the above statement is valid. (Q.E.D.)

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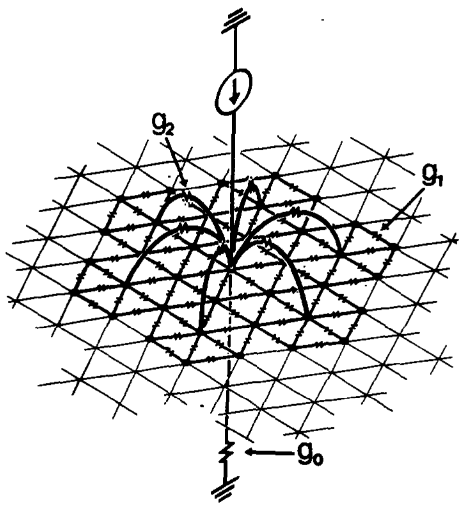


Fig.1 The image-smoothing neuro chip. Only one unit is shown.

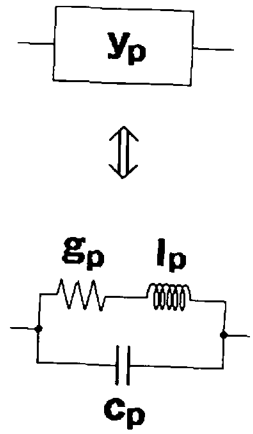
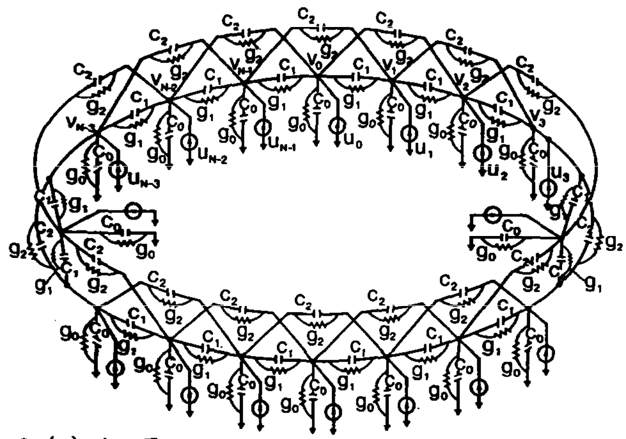


Fig.3 : The admittance y_p consists of a conductance g_p, a capacitance c_p and an inductance l_p.



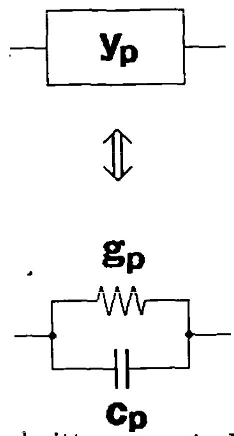


Fig.2 (a) A 1D circulant network with $m = 2$. (b) The admittance y_p in Fig.2 (a) consists of a conductance g_p and a capacitance c_p in parallel ($p = 0, 1, 2.$)

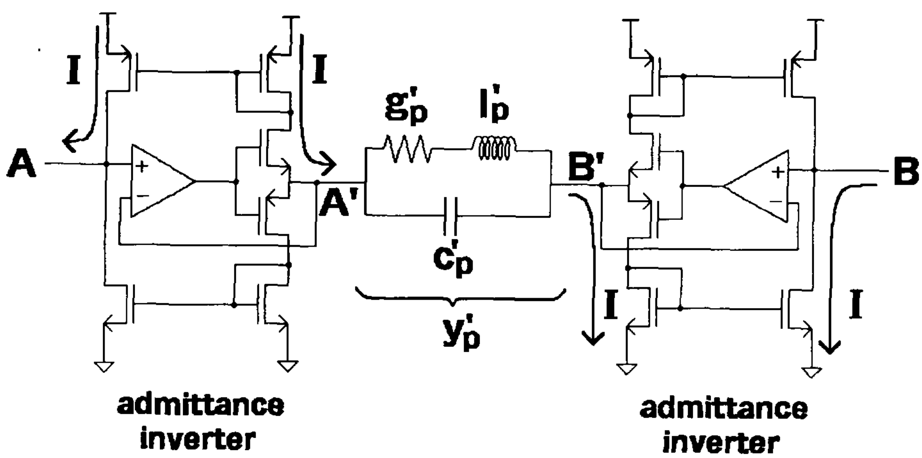


Fig.4 : Admittance inverters can realize *negative* conductances, capacitances and inductances. Suppose that V_A, V_B, V'_A and V'_B are node voltages of A, B, A' and B', and also $V_A > V_B$. Then, due to the voltage followers, $V_A \approx V'_A$ and $V_B \approx V'_B$, and the current I flows from node A' to B' with $I = y'_p(V'_A - V'_B)$. We see that the current I effectively flows from node B to node A with $I = y'_p(V_A - V_B)$ and the admittance y_p between node A and B is effectively equal to $-y'_p$.