A JOINT SOURCE/CHANNEL CODER WITH BLOCK CONSTRAINTS

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ABSTRACT

Joint source/channel coders obtained using MAP decoders tend to fail at low probability of error. In this paper we propose a modification of the standard approach which provides protection at low error rates as well.

1. INTRODUCTION

Implicit in the information transmission theorem [1], which provides justification for the separate design of the source coder and the channel coder, is the assumption that both the source encoder/decoder pair and the channel encoder/decoder pair are operating in an optimal fashion. In practical applications, due to the violation of the assumptions made in [1] and limits on complexity, this separation may not be possible [2], and the source encoder's output contains redundancy. Massey [3] showed that for distortionless transmission of the source under the constraint of linear source and channel coders, a significant reduction in complexity with equivalent performance can be achieved using a joint source/channel coder.

Research in the field of joint source/channel coding can be distinguished in three classes. The first class of the coders can be designated as joint source/channel coders because the source and channel coding operations are truly integrated. Another class of coders can be designated as concatenated source/channel coders. Coders that maximize the overall system performance by allocating the fixed bit rate between the known source coders and channel coders are placed in this class. The work of Modestino, Daut, and Vickers [4], and Comstock and Gibson [5] can be included in this second class.

The third class of coders can be denoted as constrained joint source/channel coders because the source coder and/or decoder are modified in order to consider the presence of a noisy channel. The work of Chang and Donaldson [6], Reinenger and Gibson [7], Sayood and Borkenhagen[8], Phamdo and Farvardin [11], and, the work of Sayood, Liu, and Gibson [9] belong to this class. The current work falls in the latter class. It is an extension of the system proposed in [8]. We therefore briefly review the approach proposed in [8] in the next section. We then describe the proposed encoder structure, and the corresponding decoder structure in Section 3, and Section 4 respectively. Finally the simulation results followed by a conclusion section is presented.

2. PREVIOUS WORK

In [8], Sayood and Borkenhagen made use of the residual redundancy in the source coder output for error protection. To show the validity of this approach they applied it to image coding using DPCM. Assuming that the source is an autoregressive process of order M generated according to (1), and the predictor is a linear predictor of order N, we can show that under even slightly nonideal conditions the transmitted sequence is correlated.Using this correlation, the decoder structure in [8] maximized the *a posteriori* probability

$$L(j,m,n) = P[\theta_i = \alpha_j, |\theta_{i-1} = \alpha_m, \hat{\theta}_i = \alpha_n]. \qquad (1)$$

where, α_i are elements of the channel input alphabet, θ_i is the transmitted symbol at time i, and $\hat{\theta}_i$ is the corrupted received value at time i. The previous symbol is also included in the quantity to be maximized in order to make use of the redundancy in the received sequence. The decoder structure was patterned after the Viterbi Decoder. Phamdo and Farvardin [11] used the same idea for VQ. Due to [8] and [11] maximizing (1) is equivalent to maximizing

$$P[\theta_i = \alpha_j, |\theta_{i-1} = \alpha_m] P[\hat{\theta}_i = \alpha_n, |\theta_i = \alpha_j] \qquad (2)$$

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The expression for L is in terms of the transition probabilities of the source encoder and the channel. In [11] it was shown that for a BSC, the decoder that implements (2) is useless for channel probability of errors satisfying

$$P \leq P_1 = \frac{(1-p)^2}{(N-1)^2 p^2 + (1-p)^2} \quad (3)$$

where, N is the alphabet size of the discrete Markov source, p is the probability of returning to the same state, P is the channel probability of error, and P_1 is the "critical bit error rate". In other words, they proved that the optimum sequence detection rule is taking the received symbols as the transmitted symbols, i.e., performing no decoding at all, if and only if the BSC's probability of error satisfies (3).

3. ENCODER STRUCTURE

The coders in [8] and [11] do not work well for channels with low probability of error. The distance between the correct and incorrect sequence is not sufficient for the decoder to discriminate between them. We can increase the distance by disallowing some of the sequences. In [9], this was done using a nonbinary convolutional code. However, this requires considerable overhead. If we could impose an explicit constraint on the source coder output without using a separate channel encoder, i.e., without using any additional overhead in order to have the constraint, we would be able to perform well for channels with low probability of error as well. To achieve this, the constraint has to be in some measure already inherent in the structure of the source coder output. We considered the DPCM system in particular. Due to residual redundancy, the DPCM output still has a low pass spectrum, and does not change "too much" in small blocks. Therefore, we considered the DPCM output in blocks of length M (M, even), $x_1, x_2, \ldots, x_M$, and modified this block such that $\Sigma_{i=1}^{M/2} x_{2i} = \Sigma_{i=1}^{M/2} x_{2i-1}$. However, once the DPCM output is obtained, if one changes some of the symbols in this sequence, this would result in great amounts of distortion due to the propagation of error, which is an intrinsic property of DPCM. In order to overcome this problem we implemented DPCM as a tree encoder [12]. For a block length M, a search depth of $M/2$ in the tree encoder guarantees that a block satisfying the constraint can always be found. The proposed "constrained tree encoder" finds the branch of length $M/2$ among the $Q^{M/2}$ branches which gives the MMSE. The next branch of length $M/2$ is the one with MMSE that enables the total branch of length M (together with the previous branch of length $M/2$) to satisfy the constraint.

4. DECODER STRUCTURE

A Viterbi Decoder that implements (2) would not make use of the constraint imposed in the encoder. Therefore, the proposed decoder structure is a List Viterbi Decoder (LVD)[10]. A LVD produces a rank ordered list of the L globally best candidates after a trellis search. Here, we use a parallel LVD that simultaneously produces the L best candidates. This algorithm requires maintaining a cost array of NL accumulated costs and a state array of NLxM which stores the path history. For each node at time t only the L branches with minimum costs (in a rank ordered fashion) survive.

Seshardi and Sundberg [10] have shown that the worst case asymptotic gain for the LVD with L outputs over the Viterbi Decoder is

$$10\log_{10}(G) = 10\log_{10}\left(\frac{2L}{L+1}\right) \quad (4)$$

The gains with L=4, 8, 16, are 2.04 dB, 2.50 dB, 2.75 dB respectively. For large values of L, the gain approaches 3 dB. However, they note that the gain presented in (4) is somewhat optimistic and the actual gain is often smaller. They have shown this result for decoding for the additive white Gaussian noise channel.

The overall propsed system consists of the constrained tree encoder acting as a joint source/channel coder at the transmitter, and a LVD acting as a joint source/channel decoder followed by a DPCM decoder at the receiver.

5. RESULTS

Three-bit DPCM, tree encoder, and constrained tree encoder are used as source encoders, in order to make a comparision of the proposed system with the existing systems. All have a one-tap predictor, and Lloyd-Max nonuniform quantizers. We used two different predictor coefficients given by

$$\rho_1 = \frac{R(1)}{R(0)} \quad (5)$$

$$a_1 = \frac{1-(1-\rho_1^2)^{1/2}}{\rho_1} \quad (6)$$

where R($\cdot$) is the autocorrelation function of the source, ρ_1 is the MSE optimized predictor coefficient, and a_1 is calculated using Chang and Donaldson's worst case result. The search depth of the tree encoders is chosen

to be 4 and the constraint length is 8. The channel was assumed to be BSC.
When the source encoder was the DPCM system or tree encoder, the Viterbi Decoder structure was used. The implementation of the Viterbi Decoder we chose incorporated a fixed delay of 35 time units to allow a regular symbol release. A symbol was released by looking at the root of the path with minimum cost. When the source encoder was the constrained tree encoder, LVD structure was used. The LVD we implemented releases a block of constraint length instead of releasing a single symbol. 16 best paths are produced for each node and after 64 time units, the decoder chooses the paths whose first eight elements satisfy our constraint. All other paths are pruned. The first eight elements of the path with minimum cost -among the remaining paths- is released. The simulation results for both of the decoder types show that the system performance was insensitive to increases in either the stack depth or the number of best candidates to be found.
The test image used for simulation was the USC GIRL image, for which a_1=0.778625 and ρ_1=0.96949. To obtain the set of source coder output transition probabilities, which both the decoders need to obtain the path metric defined in (2), we used the SENSIN image or the channel output as the training data. These two images are different enough to give us an idea about the robustness of the proposed system. The performance measure used to compare various systems is the signal-to-noise ratio (SNR) which is defined as

$$SNR = 10\log_{10}\frac{\sum {x_n}^2}{\sum(x_n-\hat{x}_n)^2} \qquad (7)$$

where x_n is the input to the source encoder and $\hat{x}_n$ is the output of the source decoder.

P	Pred. Coef=0.96949			Pred. Coef=0.778625		
	DPCM	tree	c.tree	DPCM	tree	c.tree
.1	2.63	2.76	2.90	8.47	8.57	8.49
.01	10.60	10.85	10.38	17.67	17.83	17.31
.001	19.35	19.54	18.99	21.43	21.94	21.13
0	27.66	28.14	25.31	25.06	25.74	23.25

Table 1: SNR values at the channel output before correction

As seen in Table 1, the tree encoder outperforms the DPCM about 0.5 dB and by using constrained tree encoder, we lose about 2.5 dB for the error free case, for both of the predictor coefficients. The loss is due to limiting the number of available sequences to be transmitted. The encoders that use Chang and Donaldson's worst case result to calculate the predictor coefficent

P	Pred. Coef=0.96949			Pred. Coef=0.778625		
	DPCM	tree	c.tree	DPCM	tree	c.tree
.1	5.83	5.70	5.50	11.39	11.40	10.58
.01	11.10	10.75	11.35	17.06	17.27	17.24
.001	15.10	14.51	16.74	20.19	20.42	20.16

Table 2: SNR values at the source decoder output, where the JSCD is a Viterbi Decoder using the SENSIN image as the training data

P	Pred. Coef=0.96949			Pred. Coef=0.778625		
	DPCM	tree	c.tree	DPCM	tree	c.tree
.1	2.63	2.76	2.90	8.96	8.94	8.74
.01	10.60	10.89	10.37	18.38	18.66	17.70
.001	19.33	19.52	18.98	21.40	21.94	21.06

Table 3: SNR values at the source decoder output, where the JSCD is a Viterbi Decoder using the channel output as the training data

(namely a predictor coefficient of 0.778625), which we will call the reoptimized systems, are less vulnerable to channel noise. Reoptimized systems outperform the classical systems (encoders in which the predictor has been obtained using only the source statistics, namely a prediction coefficent of 0.96949) by an amount of 6 dB for a BER of 0.1, 0.01, and 2 dB for a BER of 0.001, when no error correction is performed. Due to Table 1, and 2 the improvements for the reoptimized systems are about 3 dB for a BER of 0.1, -0.5 dB for a BER of 0.01, and -1 dB for a BER of 0.001. The improvements for reoptimized systems, according to Table 1, and 3 are about 0 dB for a BER of 0.1, 0.7 dB for a BER of 0.01, and 0 dB for a BER of 0.001. According to Table 4, the corresponding improvements with the proposed system are 2 dB for a BER of 0.1, 0.02 dB for a BER of 0.01, and 0.5 dB for a BER of 0.001, when the SENSIN image is used as the training data, and 0.5 dB for a BER of 0.1, 1.7 dB for a BER of 0.01, and 1.8 dB for a BER of 0.001, when the channel output is used as the training data. These results show that

T. D.	Pred. Coef=0.96949			Pred. Coef=0.778625		
	P=.1	P=.01	P=.001	P=.1	P=.01	P=.001
Sns.	5.55	12.25	18.88	10.57	17.52	21.64
Chn.	2.68	14.14	22.83	8.91	19.03	22.62

Table 4: SNR values at the source deocder output of the proposed system

Pred. Coef.	DPCM	tree	c.tree
0.96949	0.05	0.06	0.07
0.778625	0.04	0.03	0.04

Table 5: CBER for the simulated systems

one should use a training image,for high probability of errors, and the channel output for low probability of errors to obtain the transition probabilities. Note that the proposed system exhibits improvements even for low BER's, while other systems fail to do so. The proposed system also outperforms all the other systems for low BER's.

When we used the classical systems at the encoder, the improvements are almost the same as, but the final SNR values are lower than the reoptimized systems for high BER's. This is expected, because the reoptimized systems take into account the channel errors, when designing the predictor. In case of low BER's however, the proposed system (when using the channel output as the training data) outperformed the other sytems by an amount of 3.5-4 dB.

Also note that according to (6), having no improvements when using a Viterbi Decoder for a BER of 0.001, is expected since the CBER's shown in Table 5 are greater than 0.001. However, the propsed system shows an improvement of 2-3 dB even for this BER, because it is doing more than a VD that implements (5), hence results in improvements even for the BER's less than CBER. These results show that the proposed system using the channel output as the training data is a good solution for low BER's.

6. CONCLUSION

We presented an approach for obtaining improvements when image transmission is performed using joint sorce/channel coding over channels with low BER's. It incurs non-trivial operations at both the encoder and the decoder. The approach does not require any overhead for error protection and is reasonably robust since it suffices using the channel output as the training set.

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