Poly-Phase Based Blind Deconvolution Technique Using Second-Order Statistics

Hiroshi Ochi & Mirai Oshiro
Department of Information Engineering, University of the Ryukyus
Okinawa, 903-01 Japan
ochi@ie.u-ryukyu.ac.jp

Abstract

A novel second-order statistics-based blind deconvolution and equalizer technique is proposed in this literature. This technique makes use of a two-channel perfect reconstruction filter bank derived from a two-component poly-phase decomposition of transmission channel in order to make exact system identifications possible. The proposed blind deconvolution algorithm is superior to conventional algorithms in view of simple structure and the uniqueness of solution. In order to verify the effectiveness of this method, several computer simulations including a 256 QAM signal equalizer and a blurred image recovery have been shown.

1. INTRODUCTION

The need for blind deconvolution arises in a number of important areas such as data transmission, reverberation cancellation, seismic deconvolution, and image restoration. Especially, a high-speed data transmission(e.g., ADSL using 64-256QAM signals) over a communication channel relies on the use of blind adaptive equalization. We may identify three broadly defined families of blind deconvolution and equalization algorithms, depending on the additional information that is used by the algorithm to make up for the unavailability of the channel input:

1. Bussgang algorithm; Sato[1], Godard[2]
2. Tricepstrum-based or cumulant-based algorithms; [3]
3. Cyclostationary statistics-based algorithm; [4]

The Bussgang and the tricepstrum-based algorithms rely on higher order statistics of the received signal in an implicit or explicit sense. This, in turn, requires the use of some form of nonlinearity. Therefore, the Bussgang algorithm suffers from the potential likelihood of being trapped in a local minimum w.r.t. a nonconvex cost function. The tricepstrum-based algorithm can be avoided the local minimum problem; however, it requires a high computational complexity. In addition, a limitation common to both of these approaches is a slow rate of convergence.

This slow rate convergence may be overcome by using the cyclostationary statistics-based algorithms. However, these methods face several difficulties: 1) Nonconvex optimization, 2) Channel order determination, 3) High computational load numerical arithmetic such as a singular decomposition.

In order to avoid these problems in conventional blind equalizers, we shall propose a novel blind deconvolution algorithm. The proposed blind equalizer makes use of a two-channel perfect reconstruction filter bank based on a two-component poly-phase decomposition of transmission channel. The proposed algorithm is derived from by solving a set of linear simultaneous equation so that we can obtain the unique impulse response of an unknown system. Furthermore, unlike conventional cyclostationary blind algorithms using a single-input multiple-output model(SIMO) of a channel, the proposed algorithm exploits a single-input double-output(SIDO) model only and can attain an exact blind deconvolution. This results in simple structure, i.e., less computational complexity, compared with the conventional methods.

The organization of this paper is as follows. In Section II, the poly-phase based architecture for a blind deconvolution and equalization is derived. We then present a new blind deconvolution algorithm based on the poly-phase based architecture in Section III. We also discuss on the blind equalization technique using second-order statistics. In Section IV, several computer simulations including a 256 QAM signal equalizer and a blurred image recovery have been shown in order to verify the effectiveness of this method.

2. ARCHITECTURE OF POLY-PHASE BLIND ADAPTIVE FILTERS

2.1. Poly-Phase Decomposition of a Channel

The channel output of a communication system, e.g., quadratic-amplitude-modulation (QAM), can be described using the baseband representation as

$$x(t) = \sum_{k=0}^{\infty} a(k)h(t-kT) + w(t) \qquad (1)$$

In this formulation, a sequence of data $a(n)$, which is possibly complex, with symbol rate T is sent by the transmitter through a band-limited linear time invariant(LTI) channel with impulse response $h(t)$ or transfer function $H(s)$. The channel output may be corrupted by channel noise $w(t)$; however, we assume throughout this paper that $w(t)$ is negligible.

Fig.1(a) shows a transmission channel model with a D-A converter in the transmitter and an A-D converter in the receiver. In here, we assume that the received signal $y(n)$ is up-sampled with factor 2. Then $H(s)$ can be modeled by the digital transfer function $H(z)$ as shown in Fig.1(b). In this configuration, $y(n)$ is divided into even numbered samples $y_0(n)$ and odd numbered samples $y_1(n)$, respectively.

Furthermore, since the transmitted signal are up-sampled, Fig.1(b) can be redrawn as shown in Fig.1(c) by using a two-component poly-phase decomposition w.r.t. $H(z)$ and Noble identity[5]. $H_0(z)$ and $H_1(z)$ represent poly-phase components in terms of $H(z)$, respectively. As a result, the sequence of sampled channel outputs becomes a SIDO model as follows.

$$\begin{aligned} x_0(nT) &= \sum_{k=0}^{p-1} a(k)h_0(nT-kT) \quad (2-a) \\ x_1(nT) &= \sum_{k=0}^{q-1} a(k)h_1(nT-kT) \quad (2-b) \end{aligned}$$

where we assume $H(z)$ is a LTI-FIR system with $p+q$ taps.

2.2. Poly-Phase Based Blind Equalizer

The poly-phase decomposition of a transmission channel provides a new architecture of a blind deconvolution and equalizer based on SIDO model as depicted in Fig.2. The blind equalizers $F_0(z)$ and $F_1(z)$ should attempt to recover the input data sequence $a(n)$ from the measurable channel output $y_0(n)$ and $y_1(n)$. In order to reach this our goal, the following theorem should be satisfied.

Theorem1 : In Fig.2, $a(n)$ can be recovered from $x(n)$, when $H_0(z)$ and $H_1(z)$ have no common zeros, if and only if

$$H_0(z)F_0(z) + H_1(z)F_1(z) = 1 \quad (3)$$

This is nothing but the perfect reconstruction condition of two-channel filter banks without rate conversions.

This scheme can be also applicable in case that two distinct blurred signals $y_0(n)$ and $y_1(n)$ with same inputs are simply available as shown in Fig.2.

3. BLIND DECONVOLUTION ALGORITHM

3.1. Blind System Identification

Before applying Theorem 1 for a blind deconvolution, the poly-phase components both $H_0(z)$ and $H_1(z)$ should be identified *in priori*. In this chapter, we present an algorithm for blind system identifications by using second-order statistics in order to identify $H_0(z)$ and $H_1(z)$.

In Fig.2, the cross-correlation function between $y_0(n)$ and $y_1(n)$ can be given as follows.

$$\begin{aligned} r_{y_0y_1}(\tau) &= E[y_0(n)y_1(n+\tau)] \\ &= E[\sum_{k=0}^{p-1} h_0(k)a(n-k)\sum_{l=0}^{q-1} h_1(l)a(n+\tau-l)] \\ &= \sum_{k=0}^{p-1}\sum_{l=0}^{q-1} h_0(k)h_1(l)E[a(n-k)a(n+\tau-l)] \\ &= h_0(-\tau) * h_1(\tau) * R_{aa}(\tau) \quad (4) \end{aligned}$$

where * denotes a convolution.

We assume in here that $x(n)$ is wide sense stationary (WSS) signal. Likewise, the auto-correlation function in terms of $y_0(n)$ is given by

$$\begin{aligned} r_{y_0y_0}(\tau) &= E[y_0(n)y_0(n+\tau)] \\ &= h_0(-\tau) * h_0(\tau) * R_{aa}(\tau) \quad (5) \end{aligned}$$

By cancelling out $R_{aa}(\tau)$ from both Eq.(4) and (5), we get

$$h_0(n) * r_{y_0y_1}(n) = h_1(n) * r_{y_0y_0}(n) \quad (6)$$

We can rewrite Eq.(6) as

$$r_{y_0y_1}(n) = -\sum_{k=1}^{p-1} h_0(k)r_{y_0y_1}(n-k) + \sum_{l=0}^{q-1} h_1(l)r_{y_0y_0}(n-l) \quad (7)$$

where we assume that $h_0(0) = 1$.

By expressing Eq.(7) in the matrix form, we get

$$\boldsymbol{P} = \boldsymbol{R}\boldsymbol{h} \quad (8)$$

We then finally obtain $\boldsymbol{h}$ by taking a matrix inversion in Eq.(8) as

$$\boldsymbol{h} = \boldsymbol{R}^{-1}\boldsymbol{P} \quad (9)$$

where

$$\begin{aligned} \boldsymbol{P} &= [r_{y_0y_1}(0)\, r_{y_0y_1}(1) \cdots r_{y_0y_1}(p+q-2)]^T \\ \boldsymbol{R} &= [\boldsymbol{R}_{y_0y_1}\ \boldsymbol{R}_{y_0y_0}] \end{aligned}$$

$$\boldsymbol{R}_{y_0y_1} = \begin{bmatrix} -r_{y_0y_1}(-1) & -r_{y_0y_1}(-2) & \cdots & r_{y_0y_1}(-p+2) & r_{y_0y_1}(-p+1) \\ -r_{y_0y_1}(0) & -r_{y_0y_1}(-1) & \cdots & r_{y_0y_1}(-p+3) & r_{y_0y_1}(-p+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -r_{y_0y_1}(p+q-3) & -r_{y_0y_1}(p+q-4) & \cdots & r_{y_0y_1}(q-1) & r_{y_0y_1}(q-2) \end{bmatrix}$$

$$\boldsymbol{R}_{y_0y_0} = \begin{bmatrix} r_{y_0y_0}(0) & r_{y_0y_0}(-1) & \cdots & r_{y_0y_0}(-q+2) & r_{y_0y_0}(-q+1) \\ r_{y_0y_0}(1) & r_{y_0y_0}(0) & \cdots & r_{y_0y_0}(-q+3) & r_{y_0y_0}(-q+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{y_0y_0}(p+q-2) & r_{y_0y_0}(p+q-3) & \cdots & r_{y_0y_0}(p) & r_{y_0y_0}(p-1) \end{bmatrix}$$

It should be noted that we do not need any statistical information in terms of input signal $a(n)$ to identify the unknown impulse response $\boldsymbol{h}$. This implies that we can achieve blind system identification by using Eq.(9). If it is necessary to reduce computational complexity in Eq.(9), we can apply the well-known *matrix inversion lemma* or *recursive least square (RLS)* algorithm.

3.2. Poly-Phased Based Blind Deconvolution Algorithm

We now summarize the proposed blind deconvolution algorithm.

Step 1 : Identify $\boldsymbol{h}$ by Eq.(9)

Step 2 : Substitute $\boldsymbol{h}$ into Eq.(3) and obtain $F_0(z)$ and $F_1(z)$

Step 3 : Deconvolve $x(n)$ through the circuit depicted in Fig.2 to recover $a(n)$

4. SIMULATION RESULTS

In order to verify the effectiveness of the proposed blind deconvolution algorithm, several computer simulations shall be shown in this chapter.

Experiment 1 - 256QAM Signal Equalization:

We first consider the performance of the proposed equalizer in the case of 256QAM signal inputs $a(n)$. The impulse response of channel is described by

$$h(n) = [1\ 2\ 3\ 2\ 1] \qquad (10)$$

The original binary data is band-limited by a raised-cosine filter with roll-off factor $\alpha = 0.4$.

Fig.3 shows the received constellation and the equalized constellation for 1000 symbols. Clearly, the blind equalizer has opened the eye almost completely.

We have also verified that even 256QAM signals can be equalized by using the proposed blind deconvolution algorithm.

Experiment 2 - Blurred Image Recovery:

In this experiment, we extend the proposed deconvolution algorithm for 2-dimensional version and investigate its performance by blurred image recovery. Suppose two distinct blurred images with a same original image are obtained as y_0 and y_1 as depicted in Fig.2. The transfer function of two channels are described by

$$H_0(z_1, z_2) = 1 + 2z_1^{-1} + 2z_1^{-2} + 2z_1^{-3} + z_1^{-4} \qquad (11)$$

$$H_1(z_1, z_2) = 1 + z_1^{-1} + z_1^{-2} + z_1^{-3} + z_1^{-4} \qquad (12)$$

Fig.4 shows two blurred images and a deconvoluted(equalized) image. As expected, the proposed equalizer has worked well even for image recovery.

5. CONCLUDING REMARKS AND FUTURE WORK

In this paper, a novel blind deconvolution algorithm based on second-order statistics has been proposed. The structure used in this algorithm has been derived from two-component poly-phase decomposition of channel. We have claimed here that it is possible to carry out blind deconvolution only by up-sampling with factor 2 of received signals unlike conventional cyclostationary statistics-based or SIMO structure-based algorithms. Two computer simulations have been shown in order to verify the effectiveness of the proposed algorithm.

The future work is to develop adaptive algorithms based on the proposed blind deconvolution scheme.

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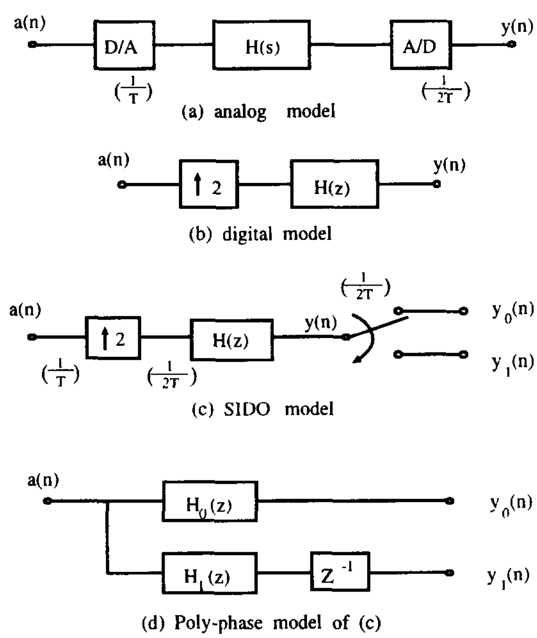


Figure 1. Ploy-phase decomposition of channel

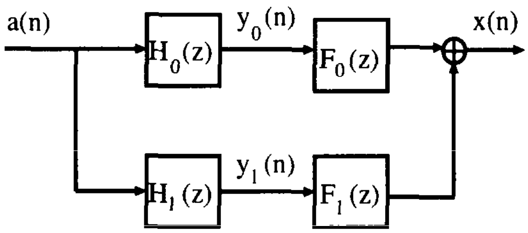
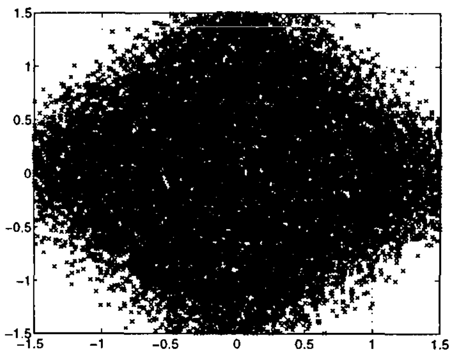
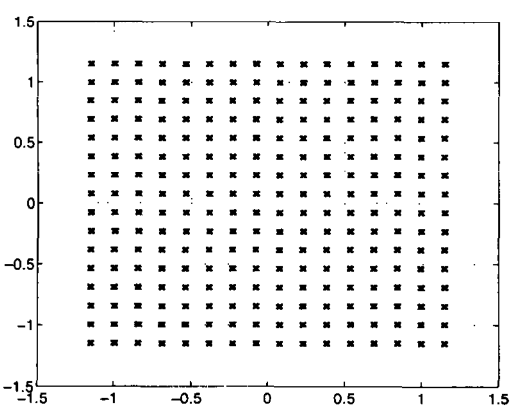


Figure 2. Poly-Phase based blind equalizer



(a) received y(n)



(b) equalized x(n)

Figure 3. 256QAM constellation

(a) $y_0(n_1, n_2)$

(b) $y_1(n_1, n_2)$

(c) Recovered image $x(n_1, n_2)$

Figure 4. Image recovery