# NEAR FIELD SUPERDIRECTIVITY (NFSD)

*W. Täger*
France Telecom, CNET
2 av. Pierre Marzin, F 22300 Lannion
wolfgang.tager@cnet.francetelecom.fr

## ABSTRACT

In some array applications, the source of interest is close to the array, so that we have to use a near field model. Almost always the near field is considered as an additional difficulty [1]. We contradict this point of view and show that if the desired source is in the near field and the other sources are in the far field, then even a small array can be at the same time highly directive and comparatively robust. Instead of relying on small phase differences for low frequencies, we fully exploit the fact that the amplitude vector of the source of interest is different from that of any other source. The array geometry should be chosen to enhance this effect. Unlike far field superdirectivity, we can steer the main lobe to arbitrary directions without prohibitive loss of performance. We applied our method to microphone array sound pick up for workstations. Simulation results and measurements of a real time implementation on a fixed point DSP are provided.

## 1. INTRODUCTION

The main problem for microphone array sound pick up is the large bandwidth of speech and music signals :

| | | |
|---|---|---|
| Analog telephony | 300-3400 Hz | $\lambda_{max} = 1.1\text{m}$ |
| Wide band telephony | 50-7000 Hz | $\lambda_{max} = 6.8\text{m}$ |
| HIFI | 20-20000 Hz | $\lambda_{max} = 17\text{m}$ |

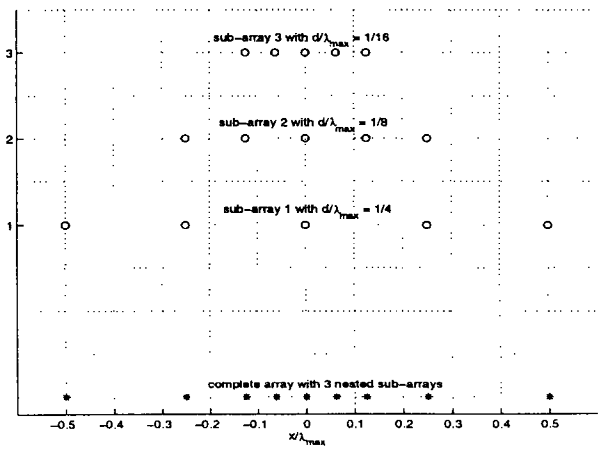
Several methods have been proposed to deal with this problem. We compare our new method (NFSD) with 3 of them : delay-weight-sum (DWS), far field superdirectivity (FFSD) and adaptive beamforming (AB).

In this paper we first present these 3 methods with their advantages and drawbacks. Then the definition of the directivity index is adapted to the near field. The optimization of this index under linear constraints and a robustness constraint is addressed next. Finally two numerical examples are provided.

## 2. PREVIOUS WORK

### 2.1 Delay-weight-sum (DWS)

Conventional delay-weight-sum beamforming with a uniformly spaced linear array is limited to reasonable values of $d/\lambda$ , where $d$ is the sensor spacing and $\lambda$ the wavelength. We can roughly cover the octave band $0.25 < d/\lambda \le 0.5$ with DWS without prohibitive loss of directivity. Let $M_s$ be the number of sensors for each sub-array and $O = \log_2(f_{max}/f_{min})$ the bandwidth in octaves, e.g. $M_s = 5$ and $O = 7$ for the wide band case. Then $M_s O = 35$ sensors are necessary to cover the whole frequency band. The number of sensors can be reduced to $M = M_s + (O-1)(M_s - 1)/2 = 17$ if $M_s$ is odd by nesting sub-arrays as shown in the figure below (only 3 sub-arrays are shown), but the total length of the array is $L = (M_s - 1)d_{max} = 6.8\text{m}$ . Many applications require a very much smaller array, e.g. if we want to put the array on a 17 inch CRT screen, then it should not be longer than 40cm. Poor performance at low frequencies is very annoying, since most undesired sources are basically in the low frequency band (cocktail speech noise, echo, background noise, reverberation).




### 2.2 Far field superdirectivity (FFSD)

Naturally the question arises if it is possible to design a small sized but nevertheless directive array. Under the far field assumption, the propagation vector of the source of interest is very close to the propagation vector of any other source for low frequencies. If we add the signal of interest in phase, then we also add all other signals nearly in phase, and we obtain poor performance. The only way to obtain a significant difference is to be close to zero. This means that at the output, the signal of interest is strongly attenuated, but all other propagating signals are even more attenuated.

The superdirective method can be derived by the optimization of the directivity index under linear and non linear constraints [3], which can be seen as a special case of our proposed method. It turns out that it is possible to achieve the mentioned objective, but that in the far field case the solution becomes highly sensitive to errors and amplifies spatially incoherent noise.

### 2.3 Adaptive beamforming (AB)

Delay-weight-sum and FFSD are fixed beamformers. Their directivity patterns do not depend on the impinging signals. Adaptive beamformers have time varying beam patterns and usually try to maximize the array gain. This is intrinsically the better approach, since finally we are more interested in high SNR than in high directivity. Adaptive arrays can achieve significant SNR enhancements if the noise field is very anisotropic [4,5].

The problem is that the solution depends on unknown parameters which have to be estimated. In a non stationary environment, this estimation cannot be perfect and the algorithm has to be robust against estimation errors, otherwise a cancellation of the signal of interest can occur. Adaptive beamforming can also fail in the presence of correlated sources. The adaptation speed must be chosen as a compromise between tracking speed and misadjustment. For low frequencies we have the already mentioned problem that all propagation vectors are very close, so the problem is inherently ill-conditioned, therefore convergence can be slow. Adaptive algorithms are always more complicated and require more calculations than fixed beamformers, and they often crucially depend on voice activity detection and speaker tracking. It is finally more difficult to combine them with other signal processing methods like acoustic echo cancellers or noise reduction algorithms.

## 3. NEAR FIELD EQUATIONS

Let c be the propagation speed, $s_p(t)$ the signal of source p, $d_{p,m}$ the distance between source p and sensor m, $m_{ref}$ the reference sensor and $x_m(t)$ and $b_m(t)$ the observation and the noise on sensor m respectively. If we want to use unidirectional sensors, then we have to multiply the signal amplitudes by $u_{p,m}$ which depends on the direction under which the sensor m sees the source p. This leads to the following near field model (reflections are treated as additional sources) :

$$x_m(t) = \sum_p \alpha_{p,m} s_p(t - \tau_{p,m}) + b_m(t)$$

with $\alpha_{p,m} = \frac{u_{p,m}}{d_{p,m}} d_{p,m_{ref}}$ and $\tau_{p,m} = \frac{d_{p,m} - d_{p,m_{ref}}}{c}$

In the frequency domain we obtain :

$$X_m(f) = \sum_p S_p(f) \alpha_{p,m} e^{-j2\pi f \tau_{p,m}} + B_m(f)$$

We want to compare the performance of the array with that of a single omnidirectional microphone. To be fair, we should use the microphone which is the closest one to the source of interest as the reference sensor. Let us suppose that we have two sources, the source of interest p= 1 located in the near field and another source p= 2 situated in the far field in the direction $\varphi,\theta$. The normalized attenuation and phase shift of the second source do not depend on the distance, so we replace $\alpha_{2,m}$ by $\alpha(\varphi,\theta,m)$ and $\tau_{2,m}$ by $\tau(\varphi,\theta,m)$. At the output of the filter-sum-beamformer (denoting the filter behind the sensor m $G_m(f)^*$ ), we get (using the superscripts $^{T}, ^{*}, ^{H}$ for transpose, complex conjugate and transpose complex conjugate) :

$$Y(f) = \sum_m G_m(f)^* X_m(f) = \tilde{S}_1(f) + \tilde{S}_2(f,\varphi,\theta) + \tilde{B}(f)$$

$$\tilde{S}_1(f) = S_1(f) \sum_m G_m(f)^* \alpha_{1,m} e^{-j2\pi f \tau_{1,m}}$$

$$\tilde{S}_2(f,\varphi,\theta) = S_2(f) \sum_m G_m(f)^* \alpha(\varphi,\theta,m) e^{-j2\pi f \tau(\varphi,\theta,m)}$$

$$\tilde{B}(f) = \sum_m G_m(f)^* B_m(f)$$

Now we can define the complex gain for the signal of interest as

$$A_1(f) = \sum_m G_m(f)^* \alpha_{1,m} e^{-j2\pi f \tau_{1,m}}$$

and the complex gain for a signal from direction $\varphi,\theta$ as

$$A_2(f,\varphi,\theta) = \sum_m G_m(f)^* \alpha(\varphi,\theta,m) e^{-j2\pi f \tau(\varphi,\theta,m)}$$

We assume that the noise process $B_m(f)$ is spatially white and with equal power on all sensors. We get the directivity index as :

$$F_D(f) = \frac{|A_1(f)|^2}{\frac{1}{4\pi} \int_\varphi \int_\theta |A_2(f,\varphi,\theta)|^2 \sin\theta \, d\varphi \, d\theta}$$

and the incoherent noise reduction as :

$$R_I(f) = \frac{E\left[\frac{1}{M} \sum_m |B_m(f)|^2\right]}{E\left[|\tilde{B}(f)|^2\right]} = \frac{1}{\sum_m |G_m(f)|^2}$$

where E[..] denotes the estimation operator.

## 4. NEAR FIELD SUPERDIRECTIVITY

In this section we address the problem of the maximization of the directivity index under linear constraints and a constraint on the incoherent noise reduction.

Using vector notation $\mathbf{G}(f) = (G_1(f), \cdots, G_M(f))^T$

we search :

$$\mathbf{G}_{opt}(f) = \arg\max(F_D(f)) = \arg\min\left(F_D(f)^{-1}\right)$$

under the linear constraints $\mathbf{C}(f)^H \mathbf{G}(f) = \mathbf{e}(f)$

and the robustness constraint $R_I(f) \geq R_{I,min}(f)$

One of the linear constraints is the non-distortion constraint :

$$A_1(f) = \sum_m G_m(f)^* \alpha_{1,m} e^{-j2\pi f \tau_{1,m}} = 1$$

The other columns of the matrix $\mathbf{C}$ can for example be other propagation vectors (e.g. echo path), the corresponding rows of $\mathbf{e}$ the conjugate of the desired gain for these directions. We obtain :

$$F_D(f)^{-1} = \frac{1}{4\pi} \int_\varphi \int_\theta \left| A_2(f,\varphi,\theta) \right|^2 \sin\theta d\varphi d\theta = \mathbf{G}(f)^H \mathbf{D}(f) \mathbf{G}(f)$$

with the hermitian non negative definite matrix

$$\mathbf{D}(f) = \frac{1}{4\pi} \int \int \mathbf{d}(f,\varphi,\theta) \mathbf{d}(f,\varphi,\theta)^H \sin\theta d\varphi d\theta$$

and the propagation vector

$$\mathbf{d}(f,\varphi,\theta) = \left( \alpha(\varphi,\theta,1) e^{-j2\pi f \tau(\varphi,\theta,1)}, \cdots, \alpha(\varphi,\theta,M) e^{-j2\pi f \tau(\varphi,\theta,M)} \right)^T$$

Using the Lagrange method we derive the optimal solution :

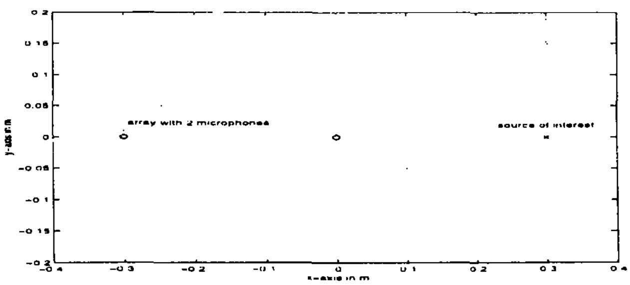
$$\mathbf{G}_{opt}(f) = \mathbf{K}(f)^{-1} \mathbf{C}(f) \left( \mathbf{C}(f)^H \mathbf{K}(f)^{-1} \mathbf{C}(f) \right)^{-1} \mathbf{e}(f)$$

with $\mathbf{K}(f) = \mathbf{D}(f) + \varepsilon(f)\mathbf{I}$

The Lagrange multiplier $\varepsilon(f) \geq 0$ must be chosen big enough to ensure the robustness constraint. $\mathbf{I}$ is the identity matrix.

## 5. EXAMPLES AND SIMULATIONS

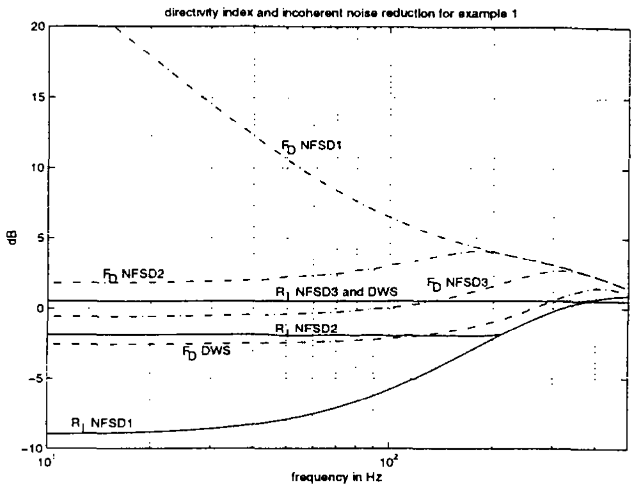
The first example is a very simple one. The source of interest is at a distance of 30cm from the first microphone and at 60cm of the second one. Both microphones are omnidirectional.




$\alpha_{1,1} = 1$, $\alpha_{1,2} = 0.5$, $\tau_{1,1} = 0\text{ms}$, $\tau_{1,2} = 0.88\text{ms}$, $\alpha(m,\varphi,\theta) = 1$

We evaluated the NFSD algorithm with 3 different robustness constraints :

1. NFSD1 : no constraint $(\varepsilon(f) = 0)$
2. NFSD2 : $R_{I,min}(f) = -2\text{dB}$
3. NFSD3 : $R_{I,min}(f) = 0.5\text{dB}$ (identical with DWS)




The comparison of the delay-weight-sum (DWS) beamformer with the NFSD algorithm shows several surprising results (figure above, note that $f_{aliasing} = 340/0.6\text{Hz} = 567\text{Hz}$) :

- the directivity index for DWS is -2.5dB for $f \to 0$ !
- the directivity index for NFSD1 tends to infinity for $f \to 0$ whereas the incoherent noise amplification is limited to 9dB !
- the DWS beamformer with uniform weighting does not achieve the best incoherent noise reduction

The first two results can be explained as follows :
For $f \to 0$, signals coming from arbitrary directions are in phase on all microphones. If we add the signal of interest in phase, then we also add the coherent noise in phase. Since the signal is weaker on the second microphone than on the first one, we finally enhance the coherent noise more than the signal :

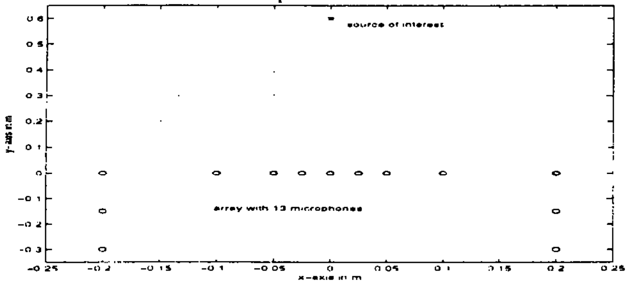
| | | |
|---|---|---|
| DWS weights : | $G_1(f=0) = G_2(f=0) = 2/3$ | |
| signal gain : | $A_1(f=0) = 2/3*1 + 2/3*0.5 = 1$ | 0dB |
| coh. noise gain : | $A_2(f=0,\varphi,\theta) = 2/3*1 + 2/3*1 = 4/3$ | 2.5dB |
| incoh. noise red. : | $R_I(f=0) = (2*2/3*2/3)^{-1} = 9/8$ | 0.5dB |

Since all far field signals are almost identical on both microphones for very low frequencies, it is possible to cancel them by taking the difference. The signal of interest is not identical and is only partially canceled. In order to have a unity gain for the signal of interest, we can choose :

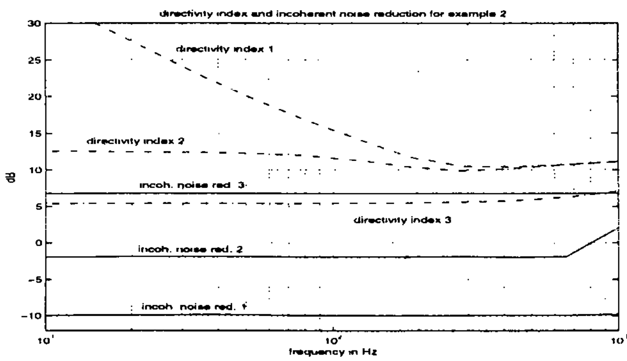
| | | |
|---|---|---|
| NFSD weights : | $G_1(f=0) = 2, \quad G_2(f=0) = -2$ | |
| signal gain : | $A_1(f=0) = 2*1 - 2*0.5 = 1$ | 0dB |
| coh. noise gain : | $A_2(f=0,\varphi,\theta) = 2*1 - 2*1 = 0$ | $-\infty$dB |
| incoh. noise red. : | $R_I(f=0) = (2*2*2)^{-1} = 1/8$ | -9dB |

This example clearly shows the superior performance of NFSD compared with DWS beamforming especially for low frequencies. Even when we impose the same incoherent noise reduction as in the DWS case, we get 2dB higher directivity indices for $f < 300\text{Hz}$. For $f \to 0$ the spatial selectivity of the array is essentially a far/near attenuation, i.e. the transfer function decreases faster than 1/r (see figures for the second example).

The second example is a more complex one. We examined the question whether it is possible to combine NFSD for low frequencies with DWS for high frequencies for a workstation sound pick up. For f>1000Hz, we used 9 cardioid microphones (cardio factor 1.6) nested in 3 sub-arrays with 5 microphones each. The source of interest is supposed to be in the broadside direction at 60cm. 4 additional microphones have been added behind the first line of microphones as shown below.




Note that the total array size is only 40cm*30cm. The performance of NFSD in terms of incoherent noise reduction and directivity index is shown below for 3 functions $R_{I,min}(f)$.




## 6. MEASUREMENTS

Measurements of the far field directivity patterns and of the attenuation in the y-axis direction are given in the last figures. They correspond to the second case of example 2 (with $R_I(f=0)=-2dB$ ). For f<200Hz, the spatial selectivity is essentially of the near/far type, so we have to consider both the directivity patterns (which are normalized to 25dB by the measurement software) and the attenuation in the main axis.

## 7. CONCLUSION

A new fixed beamforming technique has been presented, which exploits not only the phase but also the amplitude information. This approach is of considerable interest if the desired source is in the near field and most other sources are in the far field, e.g. for microphone array sound pick up. The new technique NFSD can then achieve very good results for low frequencies down to 0 Hz, where traditional delay-weight-sum beamforming fails. We proved that a small sized array can cover a large frequency band. Note that it is necessary that the amplitudes of the signal of interest are significantly different on the sensors, whereas the amplitudes of the other sources should be identical.

## 8. REFERENCES


[1] Kennedy, Abhayapala, Ward, Williamson ; *Nearfield Broadband Frequency Invariant Beamforming* ; ICASSP 96, pp. 905-908

[2] Ryan, Goubran ; *Near-field Beamforming for Microphone Arrays* ; ICASSP 1997, pp. 363-366

[3] Cox, Zeskind, Kooij ; *Practical Supergain* ; IEEE Trans. on ASSP 34 No 3 , June 1986 pp. 393-398

[4] Monzingo, Miller ; *Introduction to Adaptive Arrays* ; John Wiley & Sons, 1980

[5] Johnson, Dungeon ; *Array Signal Processing* ; Prentice Hall 1993