

A VOLTERRA MODEL FOR THE HIGH DENSITY OPTICAL DISC

L. Agarossi, S. Bellini**, A. Canella***, P. Migliorati*****

(*) PHILIPS Research, Monza, Italy - (**) Politecnico di Milano - (***) CEFRIEL, Milano
(****) University of Brescia, Via Branze 38, Brescia, Italy, pier@ing.unibs.it

ABSTRACT

This paper presents a study aiming to define a nonlinear model, based on the Volterra series, of the high density optical disc read out process. Under high density condition, because of the high linear density and reduced track pitch, the signal read out is not a linear process and suffers from cross talk. To cope with such a problem the identification of a suitable nonlinear model is required. According to the Hopkins analysis, a physical model based on the optical scalar theory was implemented. The results of this analysis have then been used to identify the kernels of a nonlinear model based on the Volterra series. The obtained results show that a second order bidimensional model is sufficient to accurately describe the read out process. The nonlinear Volterra model is a convenient starting point to devise and analyze nonlinear equalization and cross talk cancellation techniques.

1. INTRODUCTION

The information density on optical discs can be augmented either increasing the operating spatial frequency or decreasing the track pitch (the distance between adjacent tracks). In high density systems the read out signal is significantly affected by InterSymbol Interference (ISI) and cross talk (XT) among adjacent tracks. The recovery of the recorded signal can then be performed only by a suitable signal processing and channel equalization process [1]. The optimization of this process requires the characterization of the channel in order to be able to compensate for signal distortion. The definition of a suitable model of the read out system is therefore a fundamental step, to be discussed in this paper.

In order to determine the input output relationship of an optical disc system, it is necessary to evaluate accurately the read out signal. In case of high density recording, the linear model based on the Modulation Transfer Function (MTF), is no more realistic and a more complex one is required [2]. A model closer to the read out process was firstly developed by Hopkins [3] using a scalar theory approach. The scalar theory can be applied also to high density optical discs, even if pit dimensions of the order of one wavelength are involved, because light reflection is performed through a material with a refractive index that reduces the effective wavelength. In our work an optical physical model has been implemented. This model has then been used to identify a nonlinear analytical model based on the Volterra series. This model has two great advantages: first, it allows to simulate the read out signal, for a given data sequence, much faster than using the optical model; second, and most important, it explicitly brings the dependence of the output on input data, as we shall see shortly.

The experimental results show that a second order nonlinear model is a good approximation of the read out process.

The paper is organized as follows. In Section 2, the implemented

optical model is described. Section 3 is devoted to the description of the procedure used to estimate the Volterra kernels, and shows some experimental results. The following Section presents the second order "bidimensional" model in presence of cross talk. Concluding remarks are given in the final section.

2. THE OPTICAL MODEL

Hopkins's analysis [3] is shortly described as follows. From the laser source the light propagates, through the lens, towards the disc surface. The scalar theory describes mathematically the field propagation as a Fourier transform of the scalar input field. Hopkins modeled the disc reflectivity making use of the Fourier series analysis for periodic structures. The reflected light is simply equal to the phase profile of the disc times the incident field. The photodiode signal is the electro optical conversion of the reflected field after backpropagation to the detector, that is after another Fourier transform. Instead of calculating the time consuming Fourier series coefficients for quite short periodic sequences, the implemented model [4] evaluates the light incident on the detector as a simple multiplication of the incident field by the disc reflectivity function and a single 2D Fast Fourier Transform. For a disc surface of length L and width W , the incident field on the photodetector plane $a'(x, y)$ is

$$a'(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} R_{m,n} e^{-i2\pi \frac{m}{L} \nu t} \cdot P(x, y) f\left(x - \frac{m}{L}, y - \frac{n}{W}\right) \quad (1)$$

where $R_{m,n}$ is the coefficient matrix of the 2D Fourier expansion, ν the tangential velocity of the disc, $P(x, y)$ the "pupil function" taking into account also the geometrical parameters related to lens aberrations (spherical aberration, coma, defocusing, astigmatism and tilt), and $f(x, y)$ the power density of the laser beam. This leads to a physical model where the sensitivity of the system to these parameters can be evaluated. The intensity of the incident field is:

$$I(x, y) = |a'(x, y)|^2. \quad (2)$$

If ξ is the sensitivity function of the photodetector, the output electrical signal is given by

$$s(t) = \iint_{x^2+y^2 < 1} I(x, y) \xi(x, y) dx dy. \quad (3)$$

This model can be used to evaluate the read out signal also if some amount of light impinges neighbouring tracks, and the detector picks up unwanted signal from them.

The described model has been used to simulate the playback of different sequences recorded on the disc. First the pitlength of the Compact Disc Digital Audio (CDDA) system, namely $0.9 \mu m$, has been considered. The general results of the analysis carried out through the physical model, show that a linear model for the optical system is not an accurate approximation for higher density optical discs [5] [6]. If we analyze the details of the scalar theory, we see that the propagation of light can be represented as a chain of linear transformations, followed by the quadratic distortion generated by the photodetection process. This means that a second order analytical model is sufficient to represent the read out process, as long as the Hopkins scalar theory holds.

3. THE MONODIMENSIONAL VOLTERRA MODEL

In order to estimate the nonlinear characteristics of the optical disc, a mathematical model based on a Volterra series is considered [7]. The functional input output relationship $y(t) = f[x(t)]$ is:

$$y(t) = h_0 + \int h_1(\tau)x(t-\tau)d\tau + \iint h_2(\tau_1, \tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \dots \quad (4)$$

The zero order term h_0 accounts for the response to a zero input. The first order kernel $h_1(t)$ is simply the impulse response of a linear system. Higher order kernels can thus be viewed as higher order impulse responses, which characterize the various orders of nonlinearity of the system.

As discussed in Section 2, a second order Volterra model is expected to give an accurate analytical description of the read out process.

3.1. Calculation of the Volterra kernels

Volterra kernels could be evaluated, in principle, by means of Eqs. 1-3. We preferred a different approach. A simple means to identify Volterra kernels of second order systems is to probe the nonlinear system with pairs of impulses [8]. If we associate the amplitude 0 to lands, and 1 to pits, the test sequences consist of two short pits at appropriate locations.

In most cases, e.g. when we analyze the performance of an equalizer, we prefer to consider a "bipolar" input, namely ± 1 , instead of a polar one (0,1). It is straightforward, however, to translate kernels from polar to bipolar representation of data [5]. In the following, primes indicate "bipolar" kernels.

The kernel h_0 is the output when the input is identically equal to 0, that is when a mirror disc is used.

To evaluate the higher order kernels h_1 and h_2 , a small pit of $30nm$ spatial width was chosen as a unitary impulse. As the computation of the h_1 kernel is based on the knowledge of h_2 , the latter is evaluated first.

When two impulses $\delta(t-\tau_1)$ and $\delta(t-\tau_2)$, are applied to a second order system, the output is [8]

$$y_{12}(t) = h_0 + h_1(t-\tau_1) + h_1(t-\tau_2) + h_2(t-\tau_1, t-\tau_1) + h_2(t-\tau_2, t-\tau_2) + 2h_2(t-\tau_1, t-\tau_2). \quad (5)$$

while the response to an impulse in τ_1 alone is

$$y_1(t) = h_0 + h_1(t-\tau_1) + h_2(t-\tau_1, t-\tau_1). \quad (6)$$

Likewise, the response to a single impulse in τ_2 is

$$y_2(t) = h_0 + h_1(t-\tau_2) + h_2(t-\tau_2, t-\tau_2). \quad (7)$$

From the above equations we get the second order kernel

$$h_2(t-\tau_1, t-\tau_2) = \frac{1}{2}(y_{12}(t) + h_0 - y_1(t) - y_2(t)). \quad (8)$$

Finally, the first order kernel is given by

$$h_1(t-\tau_1) = y_1(t) - h_0 - h_2(t-\tau_1, t-\tau_1). \quad (9)$$

Note that $\tau_1 = \tau_2$ is not allowed, as this would require an impulse with amplitude 2, which has no physical meaning. Hence, $h_2(\tau, \tau)$ is obtained by interpolation from the values of $h_2(\tau_1, \tau_2)$ with $\tau_1 \neq \tau_2$. The "polar" Volterra kernels are shown in Figs. 1 and 2.

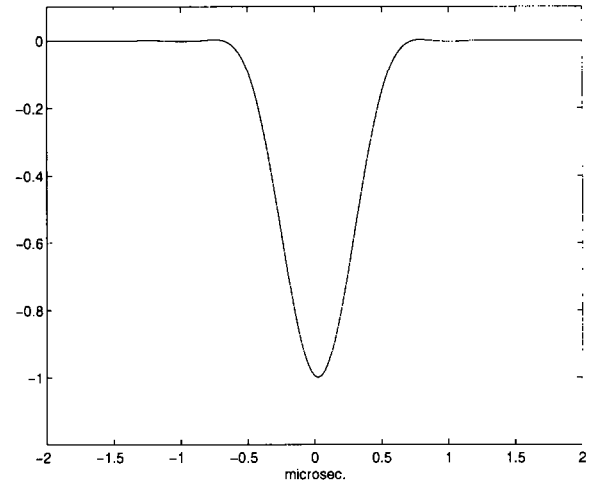


Figure 1: First order normalized Volterra kernel.

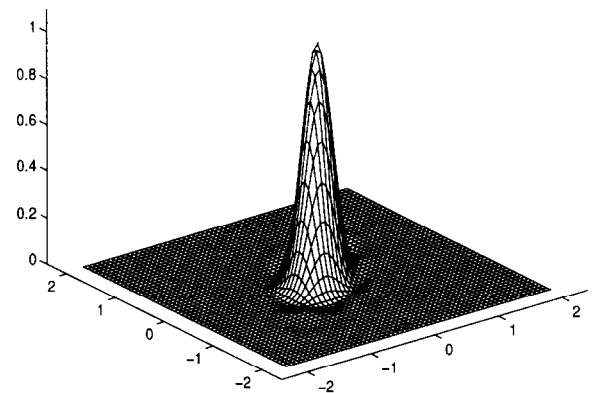


Figure 2: Second order normalized Volterra kernel.

The output signal coming from the physical optical model has then been compared with the output of the nonlinear model based

on Volterra series for an EFM (Eight to Fourteen Modulation) sequence input signal, and the CDDA standard's parameters (the minimum pit or land length $l = 0.9\mu m$). The input signal can be subdivided in small impulses of $30nm$ each. In the CDDA standard, however, pit and land lengths are multiple of $0.3\mu m$, due to the Run Length Limited (RLL) code. Hence, we can also evaluate the first and second order response to rectangles this wide, and superpose them according to the data. The evaluation of the modified kernels requires simply one and two dimensional discrete convolutions. The output signals obtained by the optical model and by the Volterra series are so similar that it is not possible to distinguish between them. This result confirms that the (second order) Volterra model agrees with the scalar Hopkins theory.

If we want to consider a bipolar input signal $x'(t)$ with -1 associated to lands and +1 to pits, we can express the corresponding polar signal as $x(t) = \frac{x'(t)}{2} + \frac{1}{2}$.

Substituting the above expression into Eq. 4 we easily obtain that the "bipolar" kernels are

$$h'_0 = h_0 + \frac{1}{2} \int h_1(\tau) d\tau + \frac{1}{4} \iint h_2(\tau_1, \tau_2) d\tau_1 d\tau_2 \quad (10)$$

$$h'_1(\tau) = \frac{1}{2} h_1(\tau) + \frac{1}{2} \iint h_2(\tau, \tau_2) d\tau_2 \quad (11)$$

$$h'_2(\tau_1, \tau_2) = \frac{1}{4} h_2(\tau_1, \tau_2) \quad (12)$$

It is noteworthy that part of the second order distortion is folded into the linear term $h'_1(t)$. Fig. 3 shows the complete Volterra model's output, compared to the contribution of the first order kernels $h_1(t)$ and $h'_1(t)$.

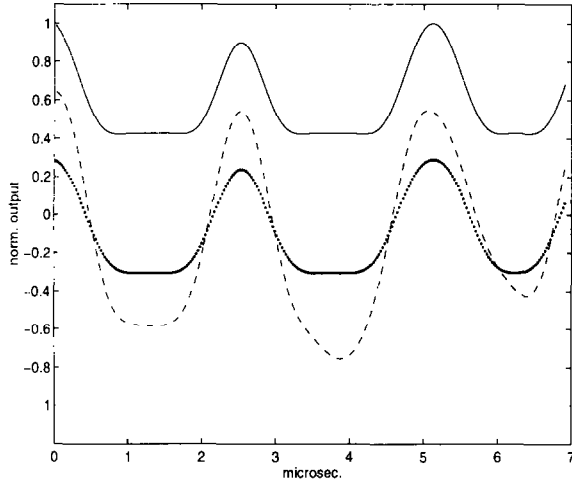


Figure 3: Output signals. Solid line: complete Volterra model. Dotted line: contribution of $h'_1(t)$. Dashed line: contribution of $h_1(t)$.

The contribution of the linear term alone is much closer to the physical model output if the bipolar approach is chosen. Second order terms are clearly exaggerated by polar kernels. Since this conclusion holds for all the examples we worked out, in the following only bipolar kernels are considered. Polar kernels are to be used only for identification purposes.

4. THE BIDIMENSIONAL VOLTERRA MODEL

The expressions derived so far represent the optical model only if XT is negligible.

If $x_1(t)$ and $x_{-1}(t)$ are the inputs corresponding to adjacent tracks, the signal read out in presence of XT is

$$y(t) = h_0 + \sum_{i=-1}^1 \left[\int h_1^i(\tau) x_i(t - \tau) d\tau + \int \int h_2^i(\tau_1, \tau_2) x_i(t - \tau_1) x_i(t - \tau_2) d\tau_1 d\tau_2 \right] + \sum_{i=-1}^1 \sum_{j=-1}^1 \int \int h_2^{ij}(\tau_1, \tau_2) x_i(t - \tau_1) x_j(t - \tau_2) d\tau_1 d\tau_2.$$

We can easily obtain the monodimensional kernels of each track, applying the same method described in Section 3. To obtain the cross kernels h_2^{01} and h_2^{10} we apply the inputs

$$x_0(t) = \delta(t), x_1(t) = \delta(t - T), x_{-1}(t) = 0.$$

Then

$$y(t) = h_0 + y_d^1(t - T) + y_d^0(t) + h_2^{01}(t, t - T) + h_2^{10}(t - T, t) \quad (13)$$

where $y_d^1(t)$ is the signal on track "0" (after subtracting h_0), when an impulse $\delta(t)$ is the input signal on track "1".

All the couples of cross kernels have the following symmetries:

$$h_2^{ab}(t_1, t_2) = h_2^{ba}(t_2, t_1), \quad ab = 10, -10, -11 \quad (14)$$

This means that the coupled cross kernels offer the same contribution to the total output. Then, Eq. 13 can be written as follows:

$$y(t) = h_0 + y_d^1(t - T) + y_d^0(t) + 2h_2^{01}(t, t - T) \quad (15)$$

Exploring all values of T , and subtracting the contribution of both tracks, it is possible to estimate $h_2^{01}(t_1, t_2)$ along the lines $t_2 = t_1 - T$ in the (t_1, t_2) plane.

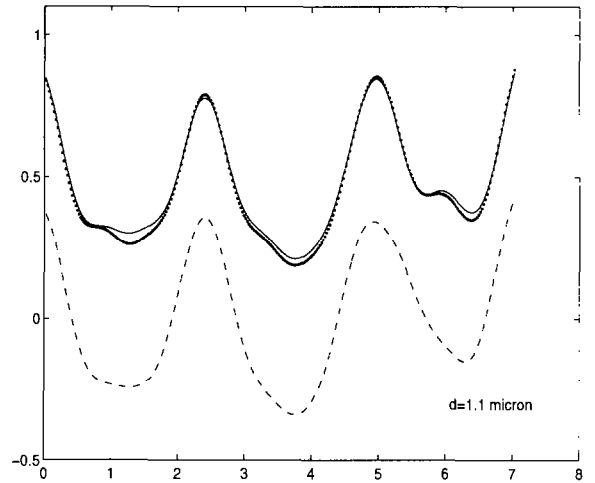


Figure 4: Hopkins model output (solid); Volterra model's output signal without cross kernels (dotted); first order kernels (dashed).

Now it is interesting to evaluate the relative amplitudes of first order, monodimensional second order, and cross terms. Fig. 4 shows the Hopkins model's output (the Volterra's one is an exact copy), and the Volterra model's output without cross kernels. We considered the CDDA standard's parameters, with the only exception that the distance d between adjacent tracks was reduced from $1.6 \mu\text{m}$ to $d = 1.1 \mu\text{m}$. We can see that in this case cross terms are quite small. In the same figure we see that the contribution of first order kernels alone is close to the total output signal, but for a constant. In Fig. 5 the Hopkins model's output is compared to Volterra's model without cross kernels, and to Volterra's first order output, as before, for a shorter track distance $d = 0.7 \mu\text{m}$. Now XT terms are stronger, and also cross kernels are required to produce an exact representation of the read out signal. In this case the first order kernel alone does not give a good approximation of the read out signal. A shorter distance between neighbouring tracks means more information density, but also a growth of nonlinear components.

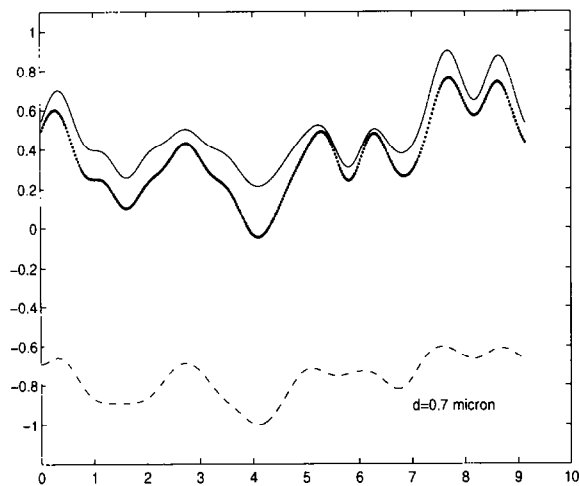


Figure 5: Hopkins model's output signal (solid line); Volterra model's output signal without cross kernels (dotted); first order kernels (dashed).

Considering the parameters of the Digital Video Disc (DVD) standard ($d = 0.74 \mu\text{m}$, $l = 0.4 \mu\text{m}$, wavelength of 650 nm) we obtained the results shown in Fig. 6. In this case the contribution of cross kernels is small. First terms alone, yet, do not give a good replica of the photodetector output.

5. CONCLUSIONS

In this paper a study devoted to the definition of a nonlinear model for high density recording on optical discs has been presented. A model of the read out system based on optical scalar theory and second order Volterra series was developed. The simulation results show a significant nonlinear behaviour of the read out signal. The results also confirm that the second order Volterra model can be adopted to define a non linear analytical representation of the read out process, also in presence of cross talk, and for each set of system parameters. In some cases, e.g. for the CDDA and DVD standards, second order cross kernels are negligible. However, also these terms must be considered if we

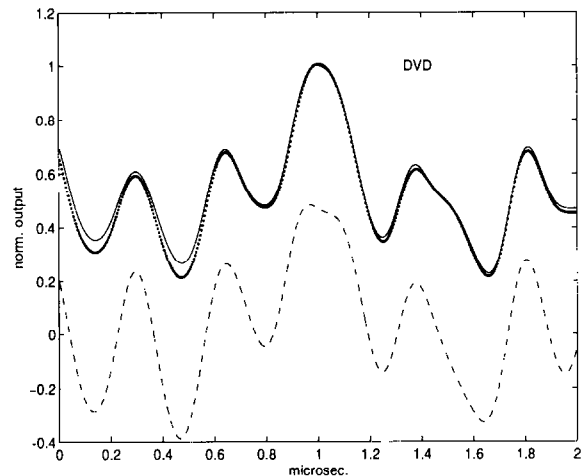


Figure 6: Hopkins model output(solid); Volterra model's output signal without cross kernels (dotted); first order kernels (dashed).

want to increase the data density further.

The nonlinear Volterra series is very fast to evaluate. Besides, it explicitly brings the dependence of the photodetector output on input data. Hence, the Volterra series is a convenient starting point to devise innovative nonlinear equalizers or cross talk cancellers for the optical channel (see, for instance [9]).

6. REFERENCES

- [1] L. Agarossi, S. Bellini, P. Migliorati, S. Olivieri, "Channel Equalization and Cross Talk Cancellation for High Density Optical Recording", in *Proc. GLOBECOM'96*, Nov. 1996, London, UK.
- [2] G. Bouwhuis, J. Braat, A. Huizer, J. Pasman, G. van Rosmalen and K. Shouhamer Immink, *Principles of Optical Disc System*, Adam Hilger Ltd, Bristol and Boston, 1985.
- [3] H. H. Hopkins, "Diffraction Theory of Laser Read Out System for Optical Video Disc", *Journal Opt. Soc. Am.*, Vol. 69, pp. 4-24, 1979.
- [4] V.B. Jipson, C.C. Williams, "Two-Dimensional Modeling of an Optical Disc Read Out", *Applied Optics*, Vol. 22, pp. 2202-2209, 1983.
- [5] L. Agarossi, S. Bellini, C. Beoni, P. Migliorati, "Nonlinear Effects in High Density Optical Recording", in *Proc. Int. Workshop on HDTV'96*, Oct. 1996, Los Angeles, CA.
- [6] A. Canella, "A Volterra Model for the Optical Disc Read-Out Process", CEFRIEL Master Thesis, Jul. 1997, Milano, Italy.
- [7] S. Benedetto, E. Biglieri, V. Castellani, *Digital Transmission Theory*, Prentice Hall International, 1987.
- [8] M. Schetzen, "Nonlinear System Modeling Based on Wiener Theory", *IEEE Proc.*, Vol. 69, pp. 1557-1561, 1981.
- [9] L. Agarossi, S. Bellini, F. Bregoli, P. Migliorati, "An Efficient Nonlinear Receiver for High Density Optical Recording", submitted for publication.