DEMODULATION OF CPM SIGNALS USING PIECEWISE POLYNOMIAL-PHASE MODELING

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ABSTRACT

In this work we propose a novel approach for demodulating Continuous Phase Modulation (CPM) signals based on the modeling of the instantaneous phase as a piecewise polynomial-phase function. The polynomial modeling can be a good approximation for currently used modulations or it can be exact if the shaping pulse is chosen to be a piecewise polynomial function. The crucial step in the demodulation process is then the estimation of the polynomial coefficients, which is carried out using the so called product high order ambiguity function (PHAF). The proposed approach is suboptimal with respect to the optimal maximum likelihood sequence estimation (MLSE) method, but is much simpler to implement and offers important advantages such as independence of initial phase, tolerance to Doppler shift and time-offset, blind channel identification. We show theoretical results concerning the minimum distance among sequences, which leads to a lower bound on the error probability, together with some simulation results.

1. INTRODUCTION

Continuous Phase Modulation (CPM) [5] has the desirable properties of constant envelope and bandwidth efficiency and for this reason one particular class of CPM, namely the Gaussian Minimum Shift Keying (GMSK) modulation, has been selected as the standard modulation in the GSM and DECT systems [10]. In general, the optimal solution to the demodulation of CPM signals consists in a Maximum Likelihood Sequence Estimator (MLSE), implemented using Viterbi algorithm [5]. In this work we propose a novel approach to CPM signals demodulation based on the modeling of the instantaneous phase as a polynomial function within each symbol interval. We will show that a third order piecewise polynomial function can fit currently used CPM signals (e.g. the GMSK used in the GSM mobile communication system) with negligible error or can model the instantaneous phase exactly, under a proper choice of the shaping pulse. The polynomial modeling allows us to recast the demodulation process in terms of parameter estimation of Polynomial-Phase Signals (PPS), which has received considerable attention in the recent literature (e.g., see [8] and the references therein). More specifically, in this paper we propose a method for demodulating CPM signals using the so called Product High order Ambiguity Function (PHAF) as the basic tool for estimating the PPS parameters. The PHAF was introduced in [2] and [3], as a generalization of the High order Ambiguity Function (see [8]).

Given a finite length sequence $s(n)$, with $|n| \leq (N-1)/2$, its m-th order high order instantaneous moment (HIM) is defined as $s_1(n) = s(n)$ for $m = 1$ and via the following recursive rule [2]: $s_{m+1}(n;\boldsymbol{\tau}_m) = s_m(n+\tau_m;\boldsymbol{\tau}_{m-1})s_m^*(n-\tau_m;\boldsymbol{\tau}_{m-1})$, for $m > 1$, where $\boldsymbol{\tau}_m := (\tau_1, \tau_2, \ldots, \tau_m)$ is the vector containing all the lags. The main motivation for introducing the HIM is that if $s(n)$ is a PPS of degree M, i.e. $s(n) = Ae^{j2\pi\sum_{m=0}^{M} a_m n^m}$, its Mth order HIM is a sinusoid with frequency $f = f_0 = 2^{M-1}M!\prod_{k=1}^{M-1}\tau_k^{(l)}a_M$. The multilag (ml) high-order ambiguity function (HAF) is defined as the discrete Fourier transform of the HIM, i.e. $S_M(f;\boldsymbol{\tau}_{M-1}) = \mathcal{F}\{s_M(n;\boldsymbol{\tau}_{M-1})\}$. The High-order Ambiguity Function (HAF) [8] is a special case of the ml-HAF, corresponding to the situation in which the lags are all equal to each other, i.e. $\boldsymbol{\tau}_{M-1} = (\tau, \tau, \ldots, \tau)$. Therefore the HAF of an Mth degree PPS has a peak at $f = f_0 = 2^{M-1}M!\prod_{k=1}^{M-1}\tau_k^{(l)}a_M$ [2]. The estimation of $\hat{a}_M$ can thus be obtained by searching for the peak of its Mth order HAF [8]. Multiplying $s(n)$ by $e^{-j2\pi a_M n^M}$ we reduce the order of its phase and estimate all the other lower order phase coefficients by reiterating the same procedure. However, the HAF presents spurious peaks when the input signal is given by the sum of PPS's having the same highest order coefficients [2]. Since this is exactly the situation arising when PMSK signals propagate through multipath channels (see Section 4), the use of the HAF would be troublesome in our case. To eliminate the ambiguities and to improve the performance of the HAF in the presence of noise, we introduced the so called product HAF (PHAF) in [2], computed multiplying the ml-HAFs obtained using L different sets of lags, after proper rescaling [2]:

$$S_M^L(f;\boldsymbol{T}_{M-1}^L) = \prod_{l=1}^{L} S_M\left(\frac{\prod_{k=1}^{M-1}\tau_k^{(l)}}{\prod_{k=1}^{M-1}\tau_k^{(1)}}f;\boldsymbol{\tau}_{M-1}^{(l)}\right), \qquad (1)$$

where $\tau_k^{(l)}$ indicates the the k-th component of the l-th set and $\boldsymbol{T}_{M-1}^L$ is the matrix containing all the sets of lags $\boldsymbol{\tau}_{M-1}^{(1)}, \boldsymbol{\tau}_{M-1}^{(2)}, .., \boldsymbol{\tau}_{M-1}^{(L)}$. The main property of the PHAF is that, after rescaling, the useful peaks remain in the same positions, whereas the spurious peaks move along the frequency axis, so that after the multiplication the useful peaks are strongly enhanced with respect to the spurious ones. The combined scaling/multiplication operation provides also a consistent gain with respect to noise terms and cross terms [3]. After having estimated the polynomial coefficients, the transmitted symbols are then obtained through a simple linear transformation. The motivation underlying our proposal is threefold: i) the demodulation procedure is much simpler than MLSE and it permits symbol recovery from partial-response signals without any sequence estimation procedure; ii) the method is tolerant to Doppler shifts, time-offsets and initial phase shifts between transmitter and receiver; iii) the method allows for blind channel identification, within some constraints imposed by the signal bandwidth.

2. POLYNOMIAL-PHASE MODELING

CPM signals assume the following general expression [5]:

$$s(t) = Ae^{j\Phi(t;\boldsymbol{\alpha})}, \tag{2}$$

where $\boldsymbol{\alpha}$ is the vector containing the transmitted symbols and

$$\Phi(t;\boldsymbol{\alpha}) = 2\pi h_F \sum_{i=-\infty}^{\infty} \alpha_i q(t-iT), \tag{3}$$

with $q(t) = \int_{-\infty}^{t} g(\tau)d\tau$. $\{\alpha_i\}$ is the sequence of transmitted symbols, $g(t)$ is the shaping filter impulse response, h_F is the modulation index, T is the symbol period. Transmitting a binary sequence with $\alpha - i \in \{-1, 1\}$, the bandwidth is h_f/T.

In general $g(t) = 0$ for $t < 0$ and $t > MT$, where M is an integer number denoted as *correlation length* [5]. CPM signals with $M = 1$ are called *full response* signals whereas signals with $M > 1$ are referred to as *partial response* signals. In GMSK, the function $g(t)$ is [5]:

$$g(t) = \frac{1}{2T}[Q(2\pi B_b \frac{t-T/2}{\sqrt{(ln2)}}) - Q(2\pi B_b \frac{t+T/2}{\sqrt{(ln2)}})], \tag{4}$$

where $Q(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t} e^{-u^2/2}du$. We define the Polynomial Minimum Shift Keying (PMSK) modulation as in (3), using $g_M(t)$ instead of $g(t)$, where:

$$g_M(t) = \frac{1}{2}\mathcal{F}^{-1}\{sinc^M(\pi T f)\}. \tag{5}$$

$\mathcal{F}^{-1}\{X(f)\}$ denotes the inverse Fourier Transform of $X(f)$ and $sinc(x) = sin(x)/x$. Functions such as (5) are examples of *B-splines* [6]; more specifically, $g_M(t)$ is a piecewise polynomial continuous function of duration mT that can be expressed in closed form as follows:

$$g_M(t) = \frac{1}{2T^M}\sum_{l=0}^{M-1} g_{M,l}(t) rect_T(t-(l-(M-1)/2)T)), \tag{6}$$

where $rect_T(t) = 1$ for $|t| \le T/2$ and $rect_T(t) = 0$ for $|t| > T/2$, and

$$g_{M,l}(t) = \sum_{k=0}^{l}\binom{M}{k}(-1)^k \frac{[t-(k-M/2)T]^{M-1}}{(M-1)!}. \tag{7}$$

Substituting (6) and (7) in (3) we can verify that the instantaneous frequency (phase) is a polynomial of degree $M-1$ (M), in each generic interval $(nT, (n+1)T]$, and the polynomial coefficients are ***linearly*** related to the transmitted symbols through the following identity:

$$\sum_{i=0}^{M-1}(i+1)\beta_{n,i+1}(t-nT)^i = \frac{h_F}{2T^M}\sum_{k=0}^{M-1}\alpha_{n-k} g_M(t-(n-k)T). \tag{8}$$

Using (7), we can find the desired relationship between polynomial coefficients and symbols. For example, for M=1, 2 and 3, we have the following matrix relationships:

$M = 1$

$$\beta_{n,1} = \frac{h_F}{2T}\alpha_n; \tag{9}$$

$M = 2$

$$\begin{pmatrix} \beta_{n,1} \\ \beta_{n,2} \end{pmatrix} = \frac{h_F}{2T^2}\begin{pmatrix} T & 0 \\ -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} \alpha_{n-1} \\ \alpha_n \end{pmatrix}; \tag{10}$$

$M = 3$

$$\begin{pmatrix} \beta_{n,1} \\ \beta_{n,2} \\ \beta_{n,3} \end{pmatrix} = \frac{h_F}{2T^3}\begin{pmatrix} \frac{T^2}{8} & \frac{6T^2}{8} & \frac{T^2}{8} \\ -\frac{T}{4} & 0 & \frac{T}{4} \\ \frac{1}{6} & -\frac{1}{3} & \frac{1}{6} \end{pmatrix}\begin{pmatrix} \alpha_{n-1} \\ \alpha_n \\ \alpha_{n+1} \end{pmatrix}, \tag{11}$$

or, in a matrix form:

$$\boldsymbol{\beta}_n = \boldsymbol{M}\boldsymbol{\alpha}_n. \tag{12}$$

This relationship is the basic step to recast the demodulation of CPM signals in terms of parameter estimation of PPS's.

3. DEMODULATION OF PMSK SIGNALS

The demodulation of PMSK signals can be carried out via the following steps:

1. For each symbol interval $(nT, (n+1)T]$ estimate the vector of PPS parameters $\hat{\boldsymbol{\beta}}_n$;
2. Estimate vector of symbols $\hat{\boldsymbol{\alpha}}_n$ as (see (12)):
$$\hat{\boldsymbol{\alpha}}_n = \boldsymbol{M}^{-1}\hat{\boldsymbol{\beta}}_n; \tag{13}$$
3. Combine estimates made over consecutive intervals and take decisions upon the transmitted sequence.

In [9], we evaluated the covariance matrix $\boldsymbol{C}_{\hat{\beta}}$ of the estimates $\hat{\boldsymbol{\beta}}_n$ obtained using the PHAF. In particular, we proved that $\boldsymbol{C}_{\hat{\beta}}$ tends to the Cramér-Rao lower bound (CRLB), up to a multiplicative constant, for high SNR. The multiplicative factor decreases as the number of products L used in the PHAF increases [2],[3] (in particular for $L = 1$ the PHAF coincides with the HAF). At low SNR, the performance drops down because of the nonlinearity of the PHAF. However, increasing L, the SNR threshold also decreases, although it cannot go below certain lower bounds (around 0 dB) [9]. Moreover, at high SNR and for the number of samples going to infinity, the estimates tend to be Gaussian random variables (rv) $\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{C}_{\hat{\beta}})$. Because of (13), the symbol estimates $\hat{\boldsymbol{\alpha}}_n$ are also asymptotically Gaussian rv $\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{M}^{-1}\boldsymbol{C}_{\hat{\beta}}\boldsymbol{M}^{-T})$, where the subscript n has been dropped due to the stationarity of both transmitted sequence and receiver noise and the superscript $^{-T}$ denotes inverse and transposed. Thus, based on the observation of the only nth interval the decision on the symbols $\boldsymbol{\alpha}(n)$ is the vector is the vector $\boldsymbol{\alpha}_i$ which minimizes the following weighted norm:

$$\boldsymbol{\alpha}_k = argmin_i\{(\hat{\boldsymbol{\alpha}}(n) - \boldsymbol{\alpha}_i)^T \boldsymbol{C}_{\boldsymbol{\alpha}}^{-1}(\hat{\boldsymbol{\alpha}}(n) - \boldsymbol{\alpha}_i)\} \tag{14}$$

where $\hat{\boldsymbol{\alpha}}(n)$ contains the estimates made in the n-th interval, whereas the vectors $\boldsymbol{\alpha}_i$ contain all possible vectors of symbols ($i = 1, 2, \ldots, P^M$, if a P-ary constellation is used and in each interval we estimate M symbols) and $\boldsymbol{C}_{\hat{\alpha}} = \boldsymbol{M}^{-1}\boldsymbol{C}_{\hat{\beta}}\boldsymbol{M}^{-T}$. This decision rule represents the minimum mean square error (MMSE) solution and it approaches the maximum likelihood (ML) solution as the number of samples per symbols tends to infinity and for high SNR.

4. PROPAGATION OF PMSK SIGNALS THROUGH REAL CHANNELS

Transmission over real channels is generally affected by undesired effects like Doppler frequency shift and multipath propagation phenomena.

4.1. Doppler frequency shifts

In case of a Doppler shift f_D, the received signal can be written as:

$$y(t) = Ae^{j2\pi\sum_{n=-\infty}^{\infty}\sum_{i=0}^{M}\beta_{n,i}(t-nT)^i}e^{j2\pi f_D t} + w(t). \tag{15}$$

where $w(t)$ is AWGN. This means that the only effect of the Doppler shift is to alter the first order coefficient $\beta_{n,1}$. The implication of this distortion on the symbols estimate can be evaluated as follows. Denoting by α_n' the altered symbols, we have (we consider the case $M = 3$ only for clarity of the exposition, but the results are valid in general):

$$(\alpha_{n-1}', \alpha_n', \alpha_{n+1}') = (\alpha_{n-1}, \alpha_n, \alpha_{n+1}) + \frac{2Tf_D}{h_F}(1,1,1). \tag{16}$$

Therefore the effect of the Doppler shift is simply to introduce a bias on the symbols estimate directly proportional to

f_D. As a consequence, the shift can be detected by checking the average value of the decoded symbol sequence:

$$\lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\alpha_n' = \frac{2Tf_D}{h_F}. \tag{17}$$

Since the transmitted symbol sequence is usually an ergodic zero mean sequence, an average different from zero reveals the presence of a Doppler effect which can also be estimated as follows:

$$\hat{f}_D = \frac{h_F}{2TN}\sum_{n=1}^{N}\alpha_n'. \tag{18}$$

4.2. Time-offset

If there is a time-offset t_0 between transmitter and receiver, instead of observing an instantaneous frequency

$$f_n(t) = \sum_{i=0}^{M-1}\beta_{n,i+1}t^i \tag{19}$$

in the generic n-th interval, we observe the instantaneous frequency:

$$f_n(t) = \begin{cases} \sum_{i=0}^{M-1}\beta_{n,i+1}(t+t_0)^i, & nT-T/2 < t \le nT-t_0 \\ \sum_{i=0}^{M-1}\beta_{n+1,i+1}(t+t_0)^i, & nT-t_0 < t \le nT+T/2 \end{cases} \tag{20}$$

Manipulating the summations, the observed instantaneous frequency is:

$$f_n(t) = \begin{cases} \sum_{k=0}^{M-1}\gamma_{n,k}t^k, & nT-T/2 < t \le nT-t_0 \\ \sum_{k=0}^{M-1}\gamma_{n+1,k}t^k, & nT-t_0 < t \le nT+T/2 \end{cases} \tag{21}$$

where

$$\gamma_{n,k} = \sum_{i=k}^{M}\beta_{n,i}t_0^{i-k}, \quad \gamma_{n+1,k} = \textstyle\sum_{i=k}^{M}\beta_{n+1,i}t_0^{i-k}. \tag{22}$$

Considering the two highest order coefficients, we obtain:

$$\gamma_{p,M} = \beta_{p,M}, \quad \gamma_{p,M-1} = \beta_{p,M-1} + t_0\beta_{p,M}M, \tag{23}$$

for $p = n, n+1$. Thus, in the absence of noise, if we take the ratio between $\gamma_{p,M-1}$ and $\gamma_{p,M}$, avoiding the cases in which $\gamma_{p,M} = 0$, we obtain:

$$r_n = \frac{\beta_{p,M-1}}{\beta_{p,M}} + t_0 M. \tag{24}$$

Since $\beta_{p,M}$ and $\beta_{p,M-1}$ are zero mean random variables, their ratio $\beta_{p,M-1}/\beta_{p,M}$ is also a zero mean random variable with symmetric distribution. Therefore, proceeding in a similar way as in the Doppler shift case, we observe that the average of the ratio $\beta_{p,M-1}/\beta_{p,M}$ is a constant equal to t_0M so that the estimator of the time offset is:

$$\hat{t}_0 = \frac{1}{NM}\sum_{n=1}^{N}\frac{\gamma_{p,M-1}}{\gamma_{p,M}}. \tag{25}$$

Remark It is important to notice that the time offset and the Doppler frequency shift can be estimated separately, even if they are simultaneously present. In fact, the Doppler shift only alters the first order coefficient, whereas the time offset alters all the coefficients except the highest order one. Therefore we initially compensate for the time offset, using only the high order coefficients, and then we compensate for the Doppler shift.

4.3. Multipath propagation

We consider now the propagation of PPS's through a linear time-invariant (LTI) FIR channel described by the following impulse response:

$$h(t) = \sum_{k=0}^{Q-1}h(k)\delta(t-r_k), \tag{26}$$

where $h(k)$ and r_k are the complex amplitude and the delay of the k-th path, respectively, and $\delta(t)$ denotes Kronecker's delta. The transit of PPS's through LTI-FIR channels was already analyzed in [7], where was shown that the output of an FIR filter driven by a PPS is multiple component PPS whose coefficients satisfy a special relationship. Exploiting that relationship, it is possible to deconvolve the PPS and to get the channel parameters $h(k)$ as well. In [7] the delays r_k were assumed to be known a-priori. In this work we show that all channel parameters can be estimated blindly (the amplitudes up to a multiplicative term), simply exploiting the polynomial-phase behavior of the input signal.

When a PPS having coefficients a_m, $m = 0, \ldots, M$ passes through a channel described by (26) the output is composed by multiple PPS's whose coefficients $b_{k,m}$, $m = 0, \ldots, M$, $k = 0, \ldots, Q-1$, are related to the a_m's via:

$$b_{k,i} = \sum_{l=0}^{M-i}\binom{l+i}{i}a_{l+i}(-r_k)^l. \tag{27}$$

Particularizing (27) to the cases $i = M$ and $i = M-1$, we have:

$$b_{k,M} = a_M, \quad b_{k,M-1} = a_{M-1} - a_M(-r_k). \tag{28}$$

From (28) we notice that all the output signal components have the same highest order phase coefficient. This is exactly the case where the HAF-based approach [7] suffers from an ambiguity problem. Since the PHAF solves such a problem [2], its use is well motivated in this particular scenario. We are now able to describe the algorithm able to estimate all channel parameters blindly (i.e. without assuming any knowledge about the transmitted sequence). We assume for simplicity that the channel order Q is known. Given the observed signal $y(t)$, the overall estimation method is the following:

1. compute the M-th order PHAF of the observed PPS and then obtain $\hat{a}_M$;
2. compensate the M-th order phase term $y_1(t) = y(t)e^{-j2\pi\hat{a}_M t^M}$;
3. compute the $(M-1)$-th order PHAF of $y_1(t)$ and estimate the positions of the highest Q peaks thus obtaining $\hat{b}_{k,M-1}$, $k = 1, \ldots, Q$;
4. estimate the relative delays $\Delta\hat{r}_k$ as:
$$\Delta\hat{r}_k := \hat{r}_k - \hat{r}_0 = \frac{\hat{b}_{k,0} - \hat{b}_{k,M-1}}{M\hat{a}_M}, \quad k = 1, \ldots, Q-1; \tag{29}$$
5. estimate all the phase coefficients $\hat{a}_m, m = 1, \ldots, M$ of the first component;
6. estimate the complex amplitude of each delayed path as follows:
$$\hat{h}(k) = \frac{1}{T}\sum_{t=\hat{r}_k}^{\hat{r}_k+T} y(t)e^{-j2\pi\sum_{i=0}^{M}a_i t^i}, fork = 1, \ldots, Q. \tag{30}$$

If T is sufficiently large and the estimates of the delays and phase parameters are correct, the k-th component is coherently integrated and then enhanced with respect to the other components. In the case of PMSK signals, the previous analysis is only approximate because the PMSK is a piecewise PPS. However, there are two properties which can be advantageously exploited to enhance the useful components with respect to the undesired ones, except the components have a delay exactly equal to the symbol duration: i) the number of samples is N for the useful component and is less than N for the delayed components; hence the peak of the PHAF is higher for the useful component; ii) the PPS parameters of the useful component belong to a finite alphabet (because the symbols belong to a finite alphabet), whereas the other parameters, owing to the delays, do not. Some examples of application of these statements will be shown in the next section.

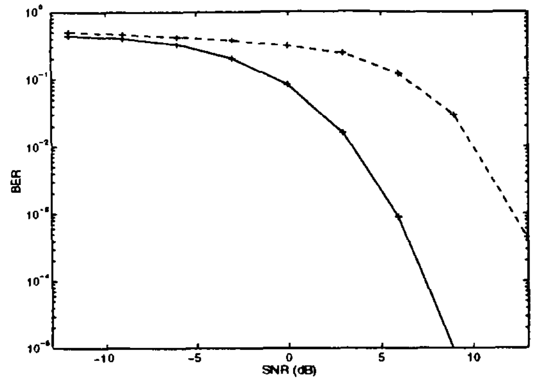

Figure 1. BER vs. SNR (dB) (N=16, M=3, L=1).

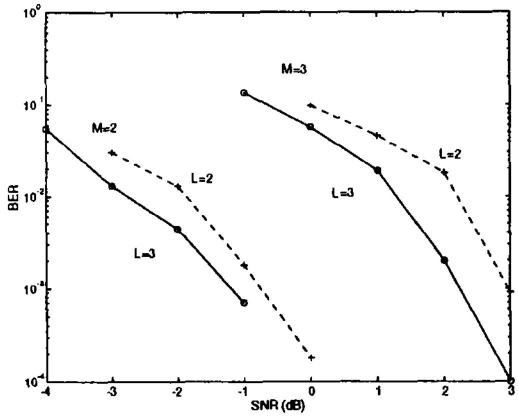

Figure 2. BER vs. SNR (dB) (N=64).

5. PERFORMANCE AND CONCLUSIONS

In this section we evaluate the performance using two sets of parameters: the distance among sequences and the error probability. Using the decision rule expressed in (14), a meaningful performance parameter is the minimum distance among sequences [5]:

$$d^2_{min} = \min_{i \neq j} (\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j)^T \boldsymbol{C}_{\boldsymbol{\alpha}}^{-1} (\boldsymbol{\alpha}_i - \boldsymbol{\alpha}_j). \tag{31}$$

Substituting the Fisher's information matrix instead of $\boldsymbol{C}_{\boldsymbol{\alpha}}^{-1}$ and considering a BPSK transmission (i.e. $\alpha_i \in \{-1, 1\}$), we obtain:

$$d^2_{min} = \frac{\pi^2 h_F^2}{56} \frac{A^2}{\sigma_n^2} = \frac{\pi^2 h_F^2}{56} \frac{E_b}{N_0} \simeq 0.1762 h_F^2 \frac{E_b}{N_0}, \tag{32}$$

where E_b is the energy for binary symbol and N_0 is the noise power spectral density. This minimum distance is achieved when there is only a difference on one bit between the two sequences. It is interesting to observe that the distance depends only upon the integrated SNR and on the signal bandwidth (through h_F).

Combining the estimates made over consecutive intervals, we can show that the minimum distance becomes:

$$D^2_{min} = \frac{457\pi^2 h_F^2}{1260} \frac{E_b}{N_0} \simeq 3.6 h_F^2 \frac{E_b}{N_0}. \tag{33}$$

We can clearly see the gain due to the combined decision. We show now some simulation results reporting the symbol error probability as a function of the SNR.

a) Study case # 1: Single vs. combined decision.

Fig.1 shows the BER obtained in a BPSK transmission as a function of the SNR=A^2/σ_n^2. The number of samples per symbol is 16. The polynomial order is $M = 3$. The number of sets of lags used in the PHAF is $L = 1$. The normalized bandwidth (the bandwidth divided by the symbol rate) is 0.5. Dashed line refers to the decision taken on one interval whereas solid line refers to the combined decision taken according to the minimum distance criterion implemented using Viterbi's algorithm. We can see that there is a gain in terms of SNR of about 6 dB. Indeed this gain is less than what we could predict from the analysis of the minimum distance shown above. This discrepancy is due to the fact that the MMSE solution leads to the maximum likelihood solution only if the estimates are multivariate Gaussian random variables and, for $N = 16$, this assumption is only approximately valid.

b) Study case # 2: Effect of polynomial order and number of products Fig.2 shows the BER vs. SNR obtained using $M = 2$ and $M = 3$. For each M we use a number of products equal to 2 and 3. The number of samples is 64. This would be a high number in a practical application, but we have considered it here to have more degrees of freedom in the choice of the sets of lags used in the PHAF and then better understand the role played by L. We clearly see that the performance is better for $M = 2$. In fact, as far as M increases there is more induced intersymbol interference.

In summary, the method proposed in this paper for demodulating CPM signals is suboptimum with respect to the MLSE, but is much simpler to implement. Moreover, it exhibits better tolerance with respect to Doppler shifts, time offset and distorsions due to multipath propagation phenomena.

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