

# BROADBAND FREQUENCY INVARIANT BEAMFORMING METHOD WITH LOW COMPUTATIONAL COST

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## ABSTRACT

This paper presents a method for reducing the computational complexity required for farfield broadband beamforming. First, the propagating wave received by a linear or planar array of sensors is sampled efficiently on the multidimensional frequency plane in spatio-temporal sense so that the so-called non-physical area where no spectrum exists is filled with aliasing component. Next, the derived time-space series is upsampled and processed by a multidimensional filter to derive the target beampattern. Through some considerations and examples, it is shown that this method has some restrictions on the beamwidth and/or maximum beam center angle, but it can reduce the operations to about half the number of conventional one.

## 1. INTRODUCTION

Beamforming is an important array processing technique utilized in many applications such as radar, sonar, geophysical exploration, ultrasonic imaging, etc [1]. Most of these conventional uses have dealt with narrowband waves, but recently, frequency invariant beamforming method for broadband signal is being studied actively for application to acoustic microphone array, directive speaker system, and wideband digital communication [2-5]. While narrowband beamforming is achieved by weighted sum of sensor outputs, we must use frequency dependent weight for each sensor to derive a frequency invariant beampattern for broadband propagating waves. As a consequence, broadband beamforming operation which must use temporal filter of an adequate length instead of simple weight, requires much more computations compared to the narrowband case.

In this paper, in order to relieve this computational load, a novel broadband beamforming approach using an efficient spatio-temporal sampling geometry is proposed. This method is based on a fact that we can consider the whole sensor output as a multidimensional signal, and under the assumption of far-field situation, the support of its spectrum is always restricted to a closed region and the rest of the multidimensional frequency plane is left as a blank area where no spectrum exists. Conventionally, the sampling problem of the beamformer has been examined only from the viewpoint of the adequate sensor location, in spite of temporal dependence of the propagating wave. For instance, hexagonal sensor allocation given in Section 2. is known to be optimal one for circularly bandlimited signals, but once the whole frequency domain including temporal axis is considered, about 70% of 3-D frequency space is yet left as the blank area. This fact means that the sampling

density is excessively high in the time-space domain. That is, if the propagating wave is sampled in a manner that this area is filled with the aliasing component, we can anticipate the reduction of sampling density in the spatio-temporal domain which directly relates to the operation number per unit time. Such a strategy has been used for spatial signal processing [6] and multidimensional filter banks [7], but in the beamformer, extracting only one output of spatial samples [3], somewhat different operation is required. In the following, after the proposed method is given for the both case of linear and planar array, its additional features including the demerit are shown through examination and design examples.

## 2. FARFIELD BROADBAND BEAMFORMING

Since the propagating wave is a function of time and space variables, broadband beamforming results in a multidimensional filtering problem [5]. In this section, based on this interpretation, the principle of the conventional broadband beamforming approach is described. For generality, the case of planar array is mainly explained, but we can always apply the following theory to linear array by setting  $x = 0$  and  $\phi = 90^\circ$ .

Let us consider a plane wave with the velocity  $v$  and the frequency  $F$  arriving from the direction of azimuth  $\theta$  and elevation  $\phi$  received by an array of sensors depicted in Fig. 1. The sensors are assumed to be distributed periodically with the distance  $D$  on (a)  $y$  axis in the case of linear array and (b)  $x$ - $y$  plane in the case of (hexagonal) planar array. On the  $x$ - $y$  plane, this wave is expressed as a function of time  $t$  and position  $(x, y)$  as follows

$$s(t, x, y) = \exp\{j(2\pi Ft + k \sin\phi \cos\theta \cdot x + k \sin\phi \sin\theta \cdot y)\}$$

where  $k = 2\pi F/v$  denotes the wave number. By the Fourier transform to the three dimensional (3-D) frequency domain consists of the temporal frequency  $F_1$  ( $\leftrightarrow t$ ) and spatial frequency  $F_2$  ( $\leftrightarrow x$ ) and  $F_3$  ( $\leftrightarrow y$ ), the spectrum of  $s(t, x, y)$  becomes

$$S(F_1, F_2, F_3) = \delta(F_1 - F) \\ \times \delta(F_2 - F/v \cdot \sin\phi \cos\theta) \delta(F_3 - F/v \cdot \sin\phi \sin\theta).$$

That is, the spectrum of the wave from the  $(\theta, \phi)$  direction always exists on next line:

$$F_1 = \frac{v}{\sin\phi \cos\theta} \cdot F_2 = \frac{v}{\sin\phi \sin\theta} \cdot F_3. \quad (1)$$

This relation is illustrated in Fig. 2 about a wave received by an array (which means spatial sampling) and temporally

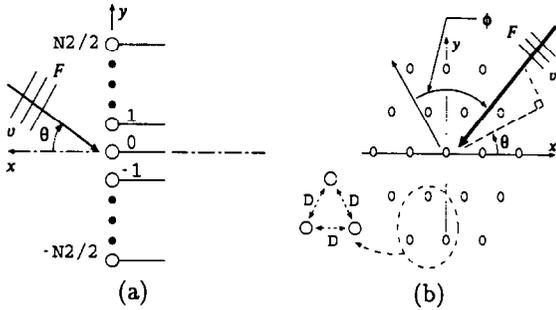


Figure 1: Sensor location of (a) linear and (b) planar array.

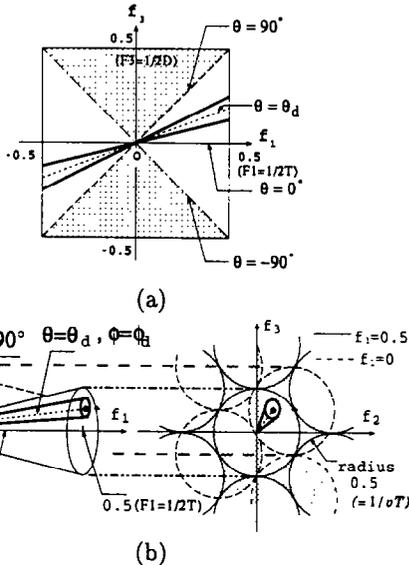


Figure 2: Spectra in proposed method. The bold line denotes the pass band edge of the fan or cone filter. The filled region expresses the aliasing component generated by downsampling.

sampled with a period  $T$  (assumed to be critical value). Here  $f_1 = F_1T$ ,  $f_2 = F_2D$  and  $f_3 = F_3D$  are the normalized frequencies. From this figure, it is seen that we can form a frequency invariant beam in arbitrary direction by using a 3-D cone-shaped digital filter (fan filter for linear array) which has a passband region centered around the line on which the spectrum of the  $(\theta_d, \phi_d)$  direction wave is positioned. In addition, above concept about array of sensors can be easily extended to the case of wave radiation such as in a loudspeaker array.

### 3. PROPOSED METHOD

In the conventional methods, not a small part of the multi-dimensional frequency is not used and left as a blank area. In this section, to avoid this inefficiency as possible, a more effective sampling strategy which fulfills this area is given.

#### 3.1. Linear Array

(i) Sampling of the propagating wave For more general expression of 2-D periodical sampling, 2-by-2 matrix  $V$  is

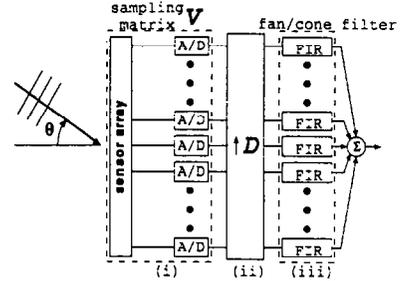


Figure 3: Block diagram of proposed broadband beamformer.

used [6]. When 2-D signal  $s(t, x)$  is sampled by  $V$ , the derived discrete signal  $x(n_1, n_2)$  is given by

$$x(n_1, n_2) = s(Vn), \quad n = [n_1, n_2]^T.$$

By this notation, the conventional method described in Section 2. corresponds to the case of  $V = \text{diag}(T, D)$  and the sampled signal becomes  $x_a(n_1, n_2) = s(n_1T, n_2D)$ . The support of its spectrum  $X_a(\omega_1, \omega_2)$  ( $\omega_i = 2\pi f_i$ ) is restricted to the area between the line of the  $\theta = \pm 90^\circ$  directions containing  $f_1$  axis as shown in Fig. 2. Conversely, in the proposed method, next sampling matrix is used.

$$V = \begin{bmatrix} 2T & -T \\ 0 & D \end{bmatrix}. \quad (2)$$

The resulting signal  $x_b(n_1, n_2)$  is given by

$$x_b(n_1, n_2) = s(2n_1T - n_2T, n_2D). \quad (3)$$

Comparing this sample location with that of  $x_a(n_1, n_2)$  (Fig. 4),  $x_b(n_1, n_2)$  can be interpreted as a signal downsampled from  $x_a(n_1, n_2)$  with a matrix

$$D = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}, \quad (4)$$

and the spectrum of  $x_b(n_1, n_2)$  is expressed as

$$X_b(\omega_1, \omega_2) = 1/2 \cdot \{X_a(\omega_1/2, \omega_1/2 + \omega_2) + X_a(\omega_1/2 - \pi, \omega_1/2 + \omega_2 - \pi)\}. \quad (5)$$

As a result, the blank area is filled with aliasing component as shown in Fig. 2(a).

(ii) Upsampling In usual filtering operations, the sampled data is processed by a filter which has a same sampling geometry as the input signal, but in the beamformer, using only one output of spatial samples, upsampling by matrix  $D$  is required before filtering. This operation is expressed as follows:

$$x_c(n_1, n_2) = \begin{cases} x_b(D^{-1}n) & D^{-1}n \in \mathbf{Z}^2 \\ 0 & D^{-1}n \notin \mathbf{Z}^2 \end{cases} \quad (6)$$

where  $\mathbf{Z}^2$  denotes the set of all 2-D integer vector. After all, the spectrum of  $x_c(n_1, n_2)$  becomes

$$X_c(\omega_1, \omega_2) = 1/2 \cdot \{X_a(\omega_1, \omega_2) + X_a(\omega_1 - \pi, \omega_2 - \pi)\} \quad (7)$$

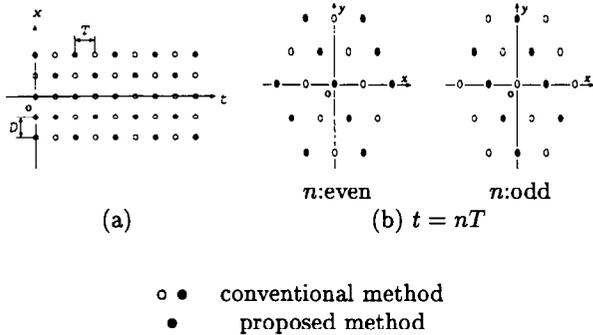


Figure 4: Sampling geometry of proposed method.

as depicted in Fig. 2(a).

(iii) 2-D filtering The frequency response of the 2-D fan filter used here is completely same as the one used in Section. 2., but since about half of the upsampled data is zero, the computational load is reduced by about 50%. The schematic diagram of proposed broadband beamformer is shown in Fig. 3.

The methodology mentioned above adopts the down-sampling of the sensor output, but almost the same operation can be accomplished also by using a fan filter which has similarly downsampled coefficients.

### 3.2. Planar Array

The procedure for planar array is almost same as the case of linear array, except that the sampling matrix has 3-by-3 elements, and  $V$  and  $D$  are changed to

$$V = \begin{bmatrix} 2T & -T & 0 \\ 0 & D/\sqrt{3} & -D/\sqrt{3} \\ 0 & D & D \end{bmatrix}, \quad D = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (8)$$

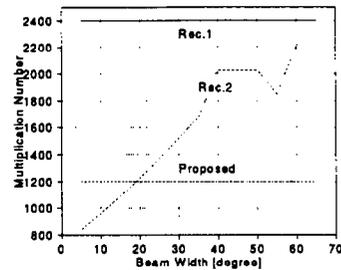
The geometry of sampling points by  $V$  is depicted in Fig. 4. The resulting spectrum is shown in Fig. 2(b). Though the samples are twice ( $= |\det D|$ ) dense than that of conventional one, about 40% of the whole 3-D space is not fulfilled yet. Namely, more efficient sampling matrix may be found through further examination.

## 4. EXAMPLES AND DISCUSSION

### 4.1. Linear Array

The nonrecursive fan filters used here are all designed by using the inverse discrete Fourier transform (IDFT) of the sampled target response. The derived fan filters are evaluated at the pass band edge to suppress the maximum error within a specific value.

First, let us consider a beamformer whose beam center angle is fixed in the  $\theta = 0^\circ$  direction. In this case, there exists another effective method which allows spatial aliasing only in the stop band [3] (abbreviated to Rec. 2). By using this method, the width of the filter transition band is relatively widen, and which leads to the reduction of the filter order ( $=$ sensor number $-1$ ) and operation number. Since this approach can not be used simultaneously with



(a) Multiplication number.

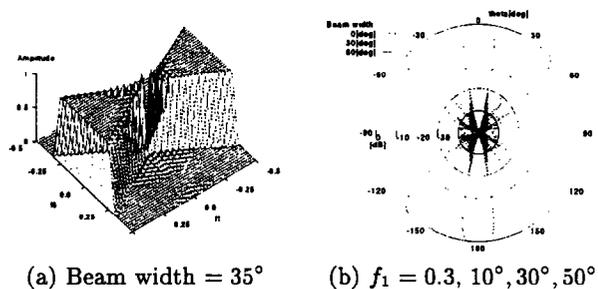
(b) Sensor number.

Beam width	10°	20°	30°	40°	50°	60°
Rec.2	31	35	39	45	45	47
Proposed	49	49	49	49	49	49

Rec. 1 : conventional method

Rec. 2 : spatial aliasing is allowed in stopband area.

Figure 5: Comparison of realization costs.



(a) Beam width = 35°

(b)  $f_1 = 0.3, 10^\circ, 30^\circ, 50^\circ$

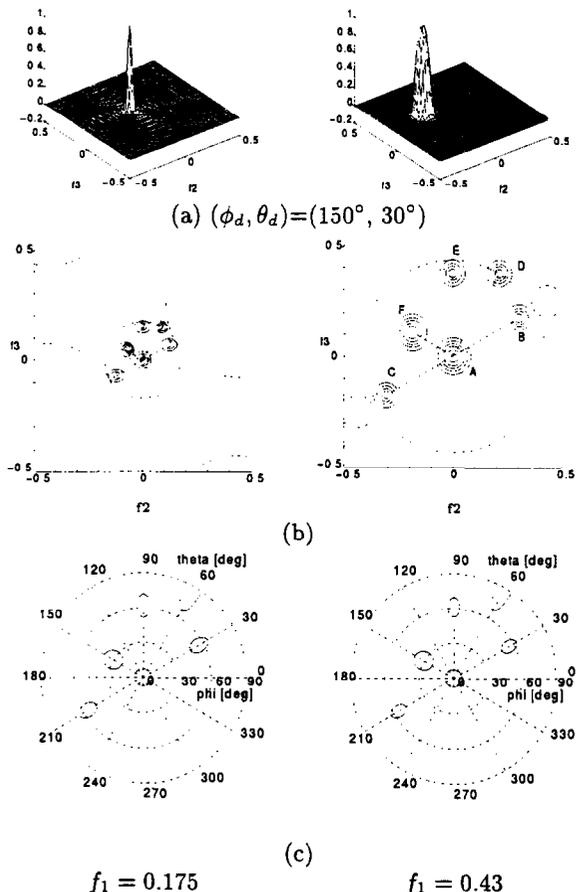
Figure 6: (a) Amplitude response of fan filter and (b) beam-pattern of broadband beamformer.

the proposed method, comparison between both methods are needed. The number of multiplications and sensors are given in Fig. 5. From these graph and table, we can see that the proposed method is preferred in the case of relatively wide beam width, and under this condition, the computational load is reduced by about 50% at the sacrifice of only a few increment of sensor number (compared to Rec. 2 but just same as conventional Rec. 1 method).

If the beam center is steered in various directions, Rec. 2 method cannot be used, and the proposed method fully receive the merit of about 50% reduction of operation number. But due to the newly-generated aliasing component, the beam center angle can not exceed certain value so that the gradient of the upper stop band edge line is kept smaller than that of  $\theta = 90^\circ$ .

### 4.2. Planar Array

When the beam center is fixed to array broadside, unlike the case of linear array, the proposed method is considered to have small advantage even though its beam width is wide, because the difference of the sensor number is expanded in proportion to square of the filter order (ex.  $N = 28, 30 \rightarrow Q = 841, 961$ ). Hence, in the rest of this subsection, only



(a)  $(\phi_d, \theta_d) = (150^\circ, 30^\circ)$

(b)

$f_1 = 0.175$   $f_1 = 0.43$

(c)

$(\phi_d, \theta_d) = A(0^\circ, 0^\circ), B(30^\circ, 55^\circ), C(210^\circ, 55^\circ)$   
 $D(60^\circ, 90^\circ), E(90^\circ, 60^\circ), F(150^\circ, 30^\circ)$ .

Figure 7: (a), (b) Cross-section amplitude response of cone filter and (c) beampattern ( $-6dB$ ) of broadband beamformer.

the case of beamsteering is dealt.

When  $(\theta_d, \phi_d) = (0^\circ, 0^\circ)$ , the cone filters are designed from a fan filter by applying the hexagonal McClellan transformation in [6] about spatial variables ( $f_2 \rightarrow f_2, f_3$ ). In other case ( $(\theta_d, \phi_d) \neq (0^\circ, 0^\circ)$ ), the following improved IDFT design method is introduced to keep the circular symmetry of the filter with respect to the  $(\theta_d, \phi_d)$  line as possible. (i) Design of LPF for each temporal frequency  $f_{1,k} = k/M$   $k = 0, \dots, M-1$ . (ii) Derivation of circular spatial filters which have centers at the origin by the McClellan transformation of the LPFs in (i). (iii) Spatial modulation of these responses. The center is moved onto the line of the  $(\theta_d, \phi_d)$  direction. (iv) Temporal approximation of the spatial filter coefficients in (iii).

The cross-section amplitude response of  $36 \times 36 \times 36$  cone filter is given in Fig. 7. The circles drawn with the broken line around the  $\theta = 30^\circ, 210^\circ$  directions in Fig. 7 (b) denote the aliasing component generated by downsampling, which

must be removed by cone filter. To avoid the leakage of this undesirable components, a large value of the beam center angle  $\phi$  is not allowed around these two azimuth angles. This restriction gets severe as the beam width becomes larger or filter order becomes smaller.

As mentioned above, the proposed method has some restrictions on its beam width and/or maximum beam center angle, but there are many applications in which this fact does not mean the fatal disadvantage. For example, in the case of loudspeaker array mounted on a wall, we usually have no necessity to form a beam along the wall.

## 5. CONCLUSION

A novel farfield broadband beamforming approach which requires small computational load was proposed. In this method, the propagating wave received by a linear or planar array of sensors is sampled efficiently on the multidimensional frequency plane in spatio-temporal sense so that the blank area where no spectrum exists is filled with the aliasing component. The proposed method has some restrictions on the beamwidth and/or maximum beam center angle, but it can reduce the operations to about half the number of conventional one.

## 6. ACKNOWLEDGMENT

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