DESIGN OF BIORTHOGONAL FILTER BANKS COMPOSED OF LINEAR PHASE IIR FILTERS

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Abstract

Since IIR filters have lower computational complexity than FIR filters, some design methods for IIR filter banks have been presented in the recent literatures. Smith et al. have proposed a class of linear phase IIR filter banks. However this method restricts the order of the numerator to be odd and ,moreover ,has some drawbacks. In this paper we present two design methods for linear phase IIR filter banks. One is based on Lagrange-Multiplier method , in which optimal IIR filter banks in least squares sense are obtained. In the other approach , IIR filter banks with the maximum number of zeros are derived analytically.

1. INTRODUCTION

Filter banks are widely used for application to image , speech and communication . A number of design methods for two channel perfect reconstruction (PR) filter banks have been presented[1]-[8].

As is well known ,IIR filters have computational efficiency compared to FIR filters which meet the same specifications. Several design methods for nonlinear phase (or quasi linear phase) IIR filter banks have been proposed [3]-[6]. However nonlinear phase IIR filter banks are not suitable for image coding, because a symmetric extension method can not be employed. For image application, the convolution based on the symmetric extension is preferable to a linear convolution in order to reduce distortion around boundaries , and thus a linear phase characteristic is important.

Since linear phase IIR filter banks are generally nonstable or noncausal, it is not appropriate for infinite length signals such as in speech application. However , for finite length signals such as images , a noncausal filtering is possible. Using the symmetric extension, the implementation is no longer causal. And thus , the linear phase IIR filter banks can be an alternative candidate for subband image coding systems.

Smith et al.[7] have proposed PR linear phase IIR filter banks and Matsumura et al. [8] have presented a design method of linear phase IIR filter banks with quasi power complementary characteristics.

In [7] and [8] ,however, only the IIR filter banks with odd-order numerators were considered and these IIR filter banks have some drawbacks.

In this paper , we propose two design methods for the linear phase IIR filter banks.

One is based on Lagrange-Multiplier technique , in which optimal IIR filter banks in least squares sense are obtained. In the other approach , IIR filter banks with maximum numbers of zeros are derived analytically.

2. CONVENTIONAL LINEAR PHASE IIR FILTER BANKS

The structure of a two channel filter bank is shown in Fig.1.

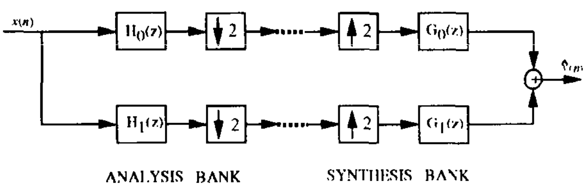

Fig-1. The structure of a filter bank

Since the denominators of the filters in Fig.1 must be functions of z^2, in this paper we deal with the following transfer functions

$$H_0(z) = \frac{E_{00}(z^2) + z^{-1}E_{01}(z^2)}{D_0(z^2)} \quad (1)$$

$$H_1(z) = \frac{E_{10}(z^2) + z^{-1}E_{11}(z^2)}{D_1(z^2)} \quad (2)$$

,where $E_{00}(z^2)+z^{-1}E_{01}(z^2)$ and $E_{10}(z^2)+z^{-1}E_{11}(z^2)$ are symmetric or antisymmetric polynomials and $D_0(z^2)$ and $D_1(z^2)$ are symmetric polynomials whose orders are a multiple of four. When $H_0(z)$ and $H_1(z)$ are specified, the synthesis system $G_0(z)$ and $G_1(z)$,which satisfy the perfect reconstruction , are given by [7]

$$G_0(z) = \frac{2H_1(-z)}{H_0(z)H_1(-z) - H_0(-z)H_1(z)}. \quad (3)$$

$$G_1(z) = \frac{2H_0(-z)}{H_0(z)H_1(-z) - H_0(-z)H_1(z)}. \quad (4)$$

It is known that these filter banks can be designed in the following two ways[7].

Type I.

First $H_0(z)$ and $H_1(z)$ are specified , and then $G_0(z)$ and $G_1(z)$ which satisfy the perfect reconstruction condition are determined from (3),(4) .

In this case, the total computational complexity in the synthesis system is about twice as much as that in the analysis system.

Type II.

The analysis filters $H_0(z)$ and $H_1(z)$ are designed ,wihch satisfy

$$H_0(z) = H_1(-z) \quad (5)$$

,then the synthesis system is determined as

$$G_0(z) = \frac{1}{2}\left(\frac{D_0(z^2)}{E_{01}(z^2)} + \frac{zD_0(z^2)}{E_{00}(z^2)}\right) \quad (6)$$

$$G_1(z) = \frac{1}{2}\left(\frac{D_0(z^2)}{E_{01}(z^2)} - \frac{zD_0(z^2)}{E_{00}(z^2)}\right) \quad (7)$$

The drawback of this class of filter banks is that the order of the numerator is restricted to be odd.

For some applications , it is required that $H_0(z)$ and $H_1(z)$ have the different number of zeros at $z=-1$ and $z=1$, respectively and highpass filters should be compact and lowpass filters should be smooth.

From these reasons, it is desirable that $H_0(z)$ has a different characteristic from $H_1(z)$.

Moreover, it is not necessarily guaranteed that the synthesis filters $G_0(z)$ and $G_1(z)$ have good frequency responses in the both cases[8].

In this paper, in order to overcome the above problems we impose the following condition on the analysis filters.

$$H_0(z)H_1(-z) - H_0(-z)H_1(z) = z^{-(2l+1)} \quad (8)$$

Using $H_0(z)$ and $H_1(z)$ which satisfy (8), $G_0(z)$ and $G_1(z)$ have the same charactaristic as $H_1(-z)$ and $H_0(-z)$, respectively.

3. DESIGN ALGORITHM

3.1. Lagrange-Multiplier method

In this section we describe a design algorithm based on Lagrange-Multiplier method.

In this paper, we consider $H_0(z)$, $H_1(z)$ which have even-order symmetric numerators, but a similar discussion can be applied to the other types of linear phase IIR filters.

In this method, we suppose that a highpass filter $H_1(z)$ has already been designed. Then $H_0(z)$ is designed to satisfy the condition (8).

Now we express the numerators and the denominators of $H_0(z)$ as

$$H_0(z) = \frac{a(0)+a(1)z^{-1}+\cdots+a(1)z^{-(2M-1)}+a(0)z^{-2M}}{d(0)+d(1)z^{-2}+\cdots+d(1)z^{-(4N-2)}+d(0)z^{-4N}}. \quad (9)$$

The frequency response $H_0(\omega)$ is

$$H_0(\omega) = e^{-j(M-2N)\omega}\frac{A_N(\omega)}{d(2N) + A_D(\omega)} \quad (10)$$

where,

$$A_N(\omega) = a(M) + 2\sum_{n=1}^{M} a(M-n)\cos(n\omega)$$
$$A_D(\omega) = 2\sum_{n=1}^{N} d(N-n)\cos(2n\omega)$$

Without a loss of generality, d(2N) can be normalized by 1.

Next , we define an equation error and a cost function to be minimized as

$$\begin{aligned} e(\omega) &= (1+A_D(\omega))\left(H_d(\omega) - \frac{A_N(\omega)}{1+A_D(\omega)}\right) \\ &= H_d(\omega) + H_d(\omega)A_D(\omega) - A_N(\omega) \end{aligned} \quad (11)$$

$$\Phi(a,d) = \sum_{l=0}^{L-1} |W(\omega_l)e(\omega_l)|^2 \quad (12)$$

,where ω_i are discritized frequency points and $H_d(\omega)$ is a real valued desired amplitude response.

$\boldsymbol{E} = [e(\omega_0) \;\; e(\omega_1) \;\; \cdots e(\omega_{L-1})]$ can be written in a matrix form as

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{D} + [\boldsymbol{d}^T \boldsymbol{a}^T]\begin{bmatrix} \boldsymbol{U} \\ -\boldsymbol{V} \end{bmatrix} \\ &= \boldsymbol{D} + \boldsymbol{y}_1{}^T\boldsymbol{X} \end{aligned} \quad (13)$$

where,

$$\boldsymbol{D} = [W(\omega_0)H_d(\omega_0) \;\; W(\omega_1)H_d(\omega_1) \;\; \cdots \;\; W(\omega_{L-1})H_d(\omega_{L-1})]$$

$$\boldsymbol{d} = [2d(N-2) \;\; 2d(N-4) \;\; \cdots 2d(2) \;\; 2d(0)]^T$$

$$\boldsymbol{a} = [a(M) \;\; 2a(M-1) \;\; 2a(M-2)\cdots \;\; 2a(1) \;\; 2a(0)]^T$$

$$\begin{aligned} [\boldsymbol{U}]_{k+1,l+1} &= W(\omega_l)H_d(\omega_l)\cos(2k\omega_l) \\ k &= 1,1,\cdots,N \quad l = 0,1,\cdots,L-1 \end{aligned}$$

$$\begin{aligned} [\boldsymbol{V}]_{k+1,l+1} &= W(\omega_l)\cos(k\omega_l) \\ k &= 0,1,\cdots,M \quad l = 0,1,\cdots,L-1 \end{aligned}$$

($[\boldsymbol{U}]_{k,l}$ indicates k-th row l-th colomn elements)

Further we set

$$\boldsymbol{Q} = \boldsymbol{X}\boldsymbol{X}^T \quad , \quad \boldsymbol{P} = \boldsymbol{X}\boldsymbol{D}^T$$

And then Eq.(12) is rewritten as follows.

$$\Phi(a,d) = \boldsymbol{y}_1{}^T\boldsymbol{Q}\boldsymbol{y}_1 + 2\boldsymbol{P}^T\boldsymbol{y}_1 + \boldsymbol{D}\boldsymbol{D}^T \quad (14)$$

Next, the PR condition is examined. The condition (8) can be rewritten as

$$E_{01}(z)E_{10}(z) - E_{00}(z)E_{11}(z) = \frac{1}{2}z^{-2l}D_0(z)D_1(z) \quad (15)$$

When the high pass filter is specified, (15) can be expressed as the linear combination of the parameters of $\{E_{00}(z), E_{01}(z), D_0(z)\}$ as

$$\boldsymbol{C}\boldsymbol{y}_1 = \boldsymbol{m}. \quad (16)$$

From Eqs.(14), (16), the optimization problem is stated as follows.

◇ optimization problem

$$min \quad \Phi(a,d) = \frac{1}{2}\boldsymbol{y}_1{}^T\boldsymbol{Q}\boldsymbol{y}_1 + \boldsymbol{P}^T\boldsymbol{y}_1 + \frac{1}{2}\boldsymbol{D}\boldsymbol{D}^T$$

$$subject \quad to \qquad \boldsymbol{C}\boldsymbol{y}_1 = \boldsymbol{m}$$

The optimal solution of this problem is uniquely given by

$$\begin{bmatrix} -\boldsymbol{Q} & \boldsymbol{C}^T \\ \boldsymbol{C} & \boldsymbol{o} \end{bmatrix} \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \boldsymbol{P} \\ \boldsymbol{m} \end{pmatrix}. \tag{17}$$

Including these, the design algorithm is constructed as follows.

[design algorithm]

1. $H_1(z)$, N, M, passband edge, stopband edge are given.
2. Set $W^0(\omega) = 1$
3. Solve (17)
4. Set $W_{i+1}(\omega) = 1/(1 + A_D(\omega))$
5. If $|\Phi^{i+1}(a,d) - \Phi^i(a,d)|/\Phi^i(a,d) < \epsilon$ then quit. Otherwise go to 3.

(,where i indicates the number of a iteration.)

3.2. maximally flat linear phase IIR filter banks

It is important for wavelet application that $H_0(z)$ and $H_1(-z)$ have some zeros at $z = -1$. Suppose that $H_0(z)$ and $H_1(-z)$ have a maximum number of zeros at $z = -1$. Then one can derive these filters analytically.

Note that a similar discussion about orthonormal IIR filter banks can be found in [3].

We consider the following analysis system

$$H_0(z) = \frac{(1+z^{-1})^{N_0}}{D_0(z^2)} \tag{18}$$

$$H_1(z) = \frac{(1+z^{-1})^{N_1}}{D_1(z^2)}. \tag{19}$$

Here we assume that $L = (N_0 + N_1)/2$ is odd.

Then the product filter $F(z) = H_0(z)H_1(-z)$ is expressed as

$$F(z) = \frac{(1+z^{-1})^{N_0+N_1}}{D_0(z^2)D_1(z^2)} \tag{20}$$

Eq.(8) means that the product filter $F(z)$ is a half-band filter. Using the Butterworth filter $H_B(z)$ with the a cut-off freqency of 0.5π, $F(z)$ in (20), which has the minimum order of denominator, can be derived from

$$F(z) = H_B(z)H_B(z^{-1}) \tag{21}$$

where the order of $H_B(z)$ is $L(= (N_0 + N_1)/2)$.

Consequently, $F(z)$ can be expressed in a closed form as

$$F(z) = \frac{C(1+z^{-1})^{N_0+N_1}}{\prod_{k=0}^{(L-3)/2}(z^{-1}-p_k)(z^{-1}-p_k^{-1})(z^{-1}-p_k^*)(z^{-1}-p_k^{*-1})} \tag{22}$$

,where

$$p_k = \frac{1+\exp[-j(\frac{\pi}{2}+\frac{\pi}{2L}+\frac{k\pi}{L})]}{1-\exp[-j(\frac{\pi}{2}+\frac{\pi}{2L}+\frac{k\pi}{L})]}$$

, * implies complex conjugate and C is a constant. Note that p_k is purely imaginary.

Similarly ,when $(N_0 + N_1)/2$ is even,

$$F(z) = \frac{Cz^{-1}(1+z^{-1})^{N_0+N_1}}{\prod_{k=0}^{(L-2)/2}(z^{-1}-p_k)(z^{-1}-p_k^{-1})(z^{-1}-p_k^*)(z^{-1}-p_k^{*-1})}. \tag{23}$$

Since the denominator of the Butterworth filter with a cut-off frequency of 0.5π has only even powers of z^{-1}, the order of the denominator of $F(z)$ in (22),(23) is a multiple of four. Thus, one can necessarily distribute the poles among the denominator of $H_0(z)$ and $H_1(z)$ which have linear phase and are the polynomials of z^{-2}.

Although these poles can be freely ditributed among the filters, we have observed that better performances are obtained for an image coding when $H_0(z)$ has larger $|p_k|$ and $H_1(z)$ has smaller $|p_k|$.

4. SIMULATION RESULTS

4.1. Lagrange-Multiplier Method

Solving (17) without any constraints, the 6-th/4-th order highpass filter $H_1(z)$ was designed in advance. Then we designed 12-th/8-th order$H_0(z)$ by the proposed method. Both $H_0(z)$ and $H_1(z)$ have the band edges 0.4π and 0.6π.

In Fig.2, the magnitude responses of the designed filter are illustrated, where the dotted line shows the magnitude response of the FIR filter designed by [2]. These FIR lowpass and highpass filter have the order of 14 and 24 , respectively.

If we use symmetrically extended sequences of input signals, $H_0(z)$ and $H_1(z)$ requires 9.2 and 5.1 multiplies per sample, respectively ,while the above FIR systems requires 8 and 12 multiplies per sample. Note that the above 0.2 and 0.1 multiplies per sample are required to compute initial values of output (for details , see [7]).

And thus it can be seen from the figure that the proposed IIR filter banks can achieve better frequency response with lower computational complexity than the FIR filter banks.

4.2. Maximally flat IIR filter banks

The 6-th/4-th order lowpass filter $H_0(z)$ and the 3-th/4-th order highpass filter $H_1(z)$ were derived from (23). The all zeros of $H_0(z)$ and $H_1(-z)$ are located at $z = -1$. Since the first coefficient of $E_{00}(z)$ is zero from (23), the number of coefficients in the numerator is 6. The freqency responses of the obtained filters are shown in Fig.3. Since the numetator of these filters can be expressed as a product of $1 + z^{-1}$, the filtering of the numerators requires no multiplies. Consequently, the total number of multiplies per sample is 4.

Next we examine image coding performance. This IIR filter bank and (9,7)-tap Daubechies' wavelet [9] were tested in the same environment, that is, 3-level tree structured decomposition was used for

'256 × 256 Lena' and the all 10 bands were quantized uniformly with the same step-size. Fig.4 shows PSNR versus entropy plot derived , where the entropies were calculated by

$$\text{Entropy} = -\sum P \log_2(P). \tag{24}$$

In the figure, '*' and 'o'are the results of the proposed IIR filter banks and the Daubechies' FIR filter ,respectively.

Acknowledgement

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5. REFERENCES

[1] P.P. Vaidyanathan , "Multirate Systems and Filter Banks" Prentice Hall Signal Processing Series, Englewood Cliffs, NJ, 1993.

[2] B.R.Horng and A.N.Willson Jr. "Lagrange Multiplier Approaches to the Design of Two-Channel Perfect-Reconstruction Linear-Phase FIR Filter Banks" *IEEE Trans.Signal Processing*,vol.40,No.2,Feb. 1992

[3] X.Herley and M Vetterli. "Wavelets and Recursive Filter Banks". *IEEE Trans. Signal Processing*,vol.41,No.8,pp.2536-2556, Aug. 1993

[4] S.Basu. C.H.Chiang and H.M.Choi, "Wavelets and Perfect Reconstruction Subband Coding with Causal Stable IIR Filters" *IEEE Trans.Circuits and Systems-II*.vol.42,No.1 Jan.1995

[5] S.M.Phoong, C.W.Kim and P.P.Vaidyanathan. "A New Class of Two-Channel Biorthogonal Filter Banks and Wavelet Bases" *IEEE Trans. Signal Processing*,vol.43,No.3,pp.649-664,Mar.1995

[6] L.L. Persti and G. Olmo. "A realizable paraunitary prefect reconstruction QMF bank based on IIR filters", Signal Processing 49 pp.133-143 (1996)

[7] M.J.T. Smith and S.L. Eddins. "Analysis/Synthesis Techniques for subband image coding", *IEEE Trans.Acoustic, Speech and Signal Processing*,vol.38,No.8,pp.1446-1456, Aug. 1990

[8] S. Matsumura ,F. Murata and T.Takebe, "Perfect Reconstruction Two-CH linear phase IIR Filter Banks with Quasi Power Complementary Characteristic", Proc. of ECCTD'97 Vol.3 pp.543-548 1997.

[9] I. Daubechies. "Orthogonal bases of compactly supported wavelets", Commun. Pure Appl. Math., Vol. XLI pp.909-996.

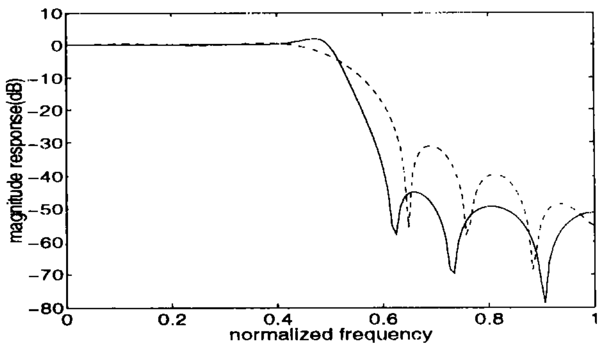
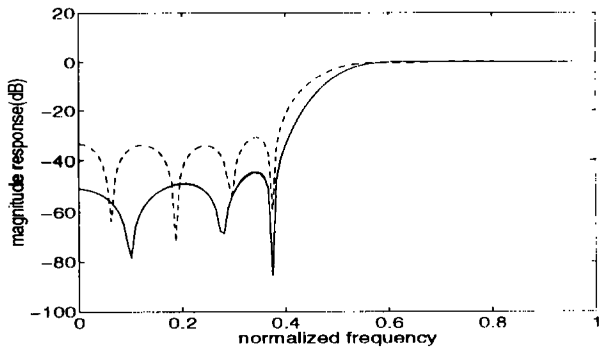


Fig.2(a):magnitude response



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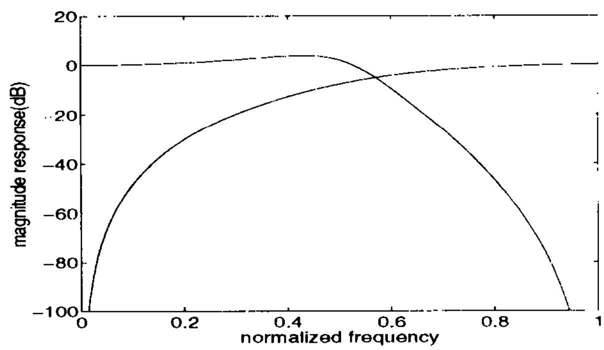

Fig.3:magnitude response

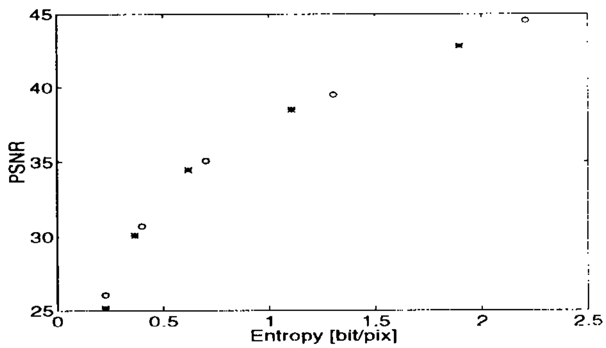

Fig.4: PSNR vs Entropy plot