H^∞ FILTERING FOR NOISE REDUCTION USING A TOTAL LEAST SQUARES ESTIMATION APPROACH

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ABSTRACT

A noise reduction algorithm for signals corrupted by additive unknown L_2 white noise is proposed using an H^∞ filtering framework. The proposed algorithm consists of two steps: a signal enhancement step and a parameter estimation step, which are iterated at each instant. To weaken the dependence between the signal enhancement step and the parameter estimation step, a total least squares estimation step for the dynamical model parameters needed in the H^∞ filtering is introduced. The effectiveness of the proposed algorithm under low signal-to-noise ratio environments is demonstrated by simulation.

1. INTRODUCTION

Noise reduction with single sensor measurements are often needed in various applications such as in wireless communication systems. Observation noises generated by multiple noise sources have typically been modeled as white Gaussian processes, based on which a two-step noise reduction algorithm has been proposed in [1]. However, this algorithm exhibits a strong dependence between the signal enhancement step and the parameter estimation step. Hence, if a large bias occurs in one of the two steps due to a large noise power, it is difficult to reduce the bias in a few iterations. As a result, it is difficult to reduce noise in a low signal-to-noise ratio (SNR) environment. To get around this difficulty and to treat colored noise in single sensor measurements, Gannot *et al.* [2] has proposed a 4-th order cumulant-based algorithm. This algorithm assumes a Gaussian distribution for the observation noise but has a very large computational load. In a practical situation, observation noise may not obey the Gaussian distribution. Also, a noise reduction algorithm with a much smaller computational burden even under low SNR environments is preferred. To address these problems, we adopt an H^∞ filtering framework as this approach does not need *apriori* information about observation noise. Shen [3] has proposed an H^∞ filtering-based algorithm for speech enhancement. However, he has applied the expectation-maximization algorithm for training the parameters of the dynamic model as in the algorithm of [1]. Hence, sufficient noise reduction under low SNR environments is not possible by his method. In this paper, we use a total least squares (TLS) algorithm to estimate a part of the parameters as followed in [4]. Since the TLS algorithm estimates dynamic model parameters directly from disturbed signals, it weakens the dependence between the signal enhancement and the parameter estimation steps. Under the above assumption, we propose here an H^∞ filtering for noise reduction with TLS estimation under low SNR environments.

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2. PROBLEM FORMULATION

We consider that a sensor measures the sum of the desired signal $s(t)$ and the observation noise $v(t)$ under low signal-to-noise ratio (SNR) environments. The sensor output $z(t)$ is given by

$$z(t) = s(t) + v(t), \tag{1}$$

where

$$\begin{aligned} s(t) &= -\sum_{k=1}^{p} \alpha_k s(t-k) + u(t) \\ &= -[\alpha_1 \cdots \alpha_p] \begin{bmatrix} s(t-1) \\ \vdots \\ s(t-p) \end{bmatrix} + u(t) \\ &\triangleq -\boldsymbol{a}^T \boldsymbol{s}(t) + u(t), \end{aligned} \tag{2}$$

where α_k $(k = 1, \cdots, p)$ is an autoregressive (AR) parameter and $u(t)$ is a model input signal. The model input $u(t)$ and the observation noise $v(t)$ are L_2 white noise that have unknown probability distributions.

For convenience, we represent Eqs. (1) and (2) in a state-space form as

$$\boldsymbol{x}(t) \;=\; \boldsymbol{\Phi}\,\boldsymbol{x}(t-1)+\boldsymbol{e}_1 u(t), \qquad (3)$$

$$z(t) \;=\; \boldsymbol{e}_1^T\boldsymbol{x}(t)+v(t), \qquad (4)$$

where $\boldsymbol{x}(t)$ is the $(p+1)\times 1$ state vector defined by

$$\boldsymbol{x}(t) \;=\; [s(t)\ s(t-1)\ \cdots\ s(t-p)]^T,$$

$\boldsymbol{\Phi}$ is the $(p+1)\times(p+1)$ state-transition matrix

$$\boldsymbol{\Phi}=\begin{bmatrix} -\alpha_1 & -\alpha_2 & \cdots & -\alpha_p & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & & \vdots \\ \vdots & \ddots & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{bmatrix},$$

and $\boldsymbol{e}_1$ is the $(p+1)\times 1$ unit vector.

$$\boldsymbol{e}_1 \;=\; [1\ 0\ \cdots\ 0]^T.$$

The objective is to recover $\{s(t)\}$ from $\{z(t)\}$ assuming that the probability distribution of $u(t)$ and $v(t)$ are not known. The proposed algorithm is developed next.

3. H^∞ FILTERING FOR NOISE REDUCTION

Assuming $u(t)$ and $v(t)$ to be white Gaussian processes represented in the state-space form discussed earlier, we can apply an expectation-maximization (EM) algorithm as in [1]. The EM-based algorithm is quite effective under high SNR environments. However, it exhibits a strong dependence between the signal enhancement step (E-step) and the needed parameter estimation step (M-step). Hence, under low SNR environments, it is difficult to adjust the early estimation errors in a few iterations. To address this problem, the algorithm of [1] has been extended in [2] using higher-order statistics for the low SNR cases as well as the color noise cases. As this algorithm requires 4-th order cumulant calculations, it is computationally intensive. Now, the noise reduction algorithm for white Gaussian noise under low SNR environments proposed in [4] also consists of a signal enhancement step and a parameter estimation step, but estimates a part of needed parameters using a total least squares (TLS) algorithm to weaken the dependence between the signal enhancement step and the parameter estimation step [4]. Computational complexity of this algorithm is less than that of the cumulant-based algorithm. However, both algorithms of [2] and [4] have assumed that $\{u(t)\}$ and $\{v(t)\}$ are Gaussian processes. In practice, the observation noise $v(t)$ is not always a Gaussian process. To solve this type of noise removal problem, we propose an H^∞ filtering framework with a TLS estimation.

3.1. H^∞ Filtering

Although various algorithms exist for H^∞ filtering, we derive a filter based on the *aposteriori* central filter proposed in [5]. If AR parameter vector $\boldsymbol{a}$, σ_u^2 (power of $u(t)$), and σ_v^2 (power of $v(t)$) are known precisely, then we could compute recursively the enhanced signal using the following H^∞ filtering equations: the first element of $\hat{\boldsymbol{x}}(t|t)$ is the optimal H^∞ estimate $\hat{s}(t)$.

$$\hat{\boldsymbol{x}}(t|t-1)=\boldsymbol{\Phi}\hat{\boldsymbol{x}}(t-1|t-1), \qquad (5)$$

$$\boldsymbol{P}(t|t-1)=\boldsymbol{\Phi}\boldsymbol{P}(t-1|t-1)\boldsymbol{\Phi}^T+\sigma_u^2\boldsymbol{e}_1\boldsymbol{e}_1^T, \qquad (6)$$

$$\begin{aligned}\hat{\boldsymbol{x}}(t|t) \;&=\; \hat{\boldsymbol{x}}(t|t-1) \\ &\quad +\boldsymbol{k}(t)[z(t)-\boldsymbol{e}_1^T\hat{\boldsymbol{x}}(t|t-1)],\end{aligned} \qquad (7)$$

$$\boldsymbol{k}(t)=\frac{\boldsymbol{P}(t|t-1)\boldsymbol{e}_1}{\boldsymbol{e}_1^T\boldsymbol{P}(t|t-1)\boldsymbol{e}_1+\sigma_v^2}, \qquad (8)$$

$$\boldsymbol{P}(t|t)=\boldsymbol{F}(t|t-1)\boldsymbol{P}(t|t-1), \qquad (9)$$

$$\begin{aligned}\boldsymbol{F}(t|t-1) \;&=\; \boldsymbol{I}-\frac{1-\gamma^{-2}}{\sigma_v^2+(1-\gamma^{-2})\boldsymbol{e}_1^T\boldsymbol{P}(t|t-1)\boldsymbol{e}_1} \\ &\quad \cdot\boldsymbol{P}(t|t-1)\boldsymbol{e}_1\boldsymbol{e}_1^T,\end{aligned} \qquad (10)$$

where $\hat{\boldsymbol{x}}(t|t)$ denotes an estimated state vector and $\hat{\boldsymbol{x}}(t+1|t)$ denotes a prediction state vector based on data up to time t. The matrices $\boldsymbol{P}(t|t)$ and $\boldsymbol{P}(t+1|t)$ denote their error covariance matrices, respectively. $\boldsymbol{I}$ denotes an identity matrix, and γ is a positive scalar.

However, we must estimate some parameters needed in the preceding filtering since signal parameters are not known previously. We estimate these parameters $\{\boldsymbol{a},\sigma_u^2,\sigma_v^2\}$ using algorithms described in the next section.

3.2. Parameter Estimation

We first describe an algorithm for obtaining the estimate of $\boldsymbol{a}$. We estimate $\boldsymbol{a}$ from observed signals, not from enhanced signals, to weaken the dependence between the signal enhancement step and the parameter estimation step under low SNR environments. We focus on the problem that all obtained signals are affected by noise and σ_u^2 is less than σ_v^2. In this case, the minimum variance estimate of $\boldsymbol{a}$ can be obtained using a

TLS algorithm [6]. We use the TLS algorithm with a slight modification for the AR parameter estimation:

$$\boldsymbol{a}(t+1) = \boldsymbol{a}(t) - \mu e(t)[-\boldsymbol{h}(t) + \boldsymbol{a}(t)z(t)], \tag{11}$$

where μ is a step gain. Here we have defined :

$$e(t) = \hat{z}(t) - z(t), \tag{12}$$

$$\hat{z}(t) = -\boldsymbol{a}^T(t)\boldsymbol{h}(t). \tag{13}$$

As seen from the preceding equation, this algorithm estimates AR parameters directly from disturbed signals $\{z(t)\}$ and has a small computational burden. Even if a large bias occurs in early stages of estimation or enhancement, the bias is reduced in a few iterations because of the introduction of this AR parameter estimation step. In addition, we have experimentally found that parameters obtained from this algorithm have had small fluctuations in the estimation process. Hence, this algorithm is quite suitable for weakening the dependence between the two steps without increasing much the computational burden.

Next, we describe adaptive algorithms to obtain the estimates of σ_u^2 and σ_v^2 using $\boldsymbol{a}$ obtained from the previous TLS algorithm and estimated state quantities in H^∞ filtering equations. From Eq. (2), σ_u^2 is estimated as follows:

$$\begin{aligned}\widehat{\sigma_u^2}(t+1) &= E\{(s(t) + \boldsymbol{a}^T \boldsymbol{s}(t))^2 | \widehat{\sigma_u^2}(t), \widehat{\sigma_v^2}(t)\} \\ &\cong \frac{1}{t}[\sum_{k=1}^{t}\langle s^2(k)\rangle + 2\boldsymbol{a}^T \sum_{k=1}^{t}\langle s(k)\boldsymbol{x}_{p-1}(k-1)\rangle \\ &\quad + \boldsymbol{a}^T \sum_{k=1}^{t}\langle \boldsymbol{x}_{p-1}(k-1)\boldsymbol{x}_{p-1}^T(k-1)\rangle \boldsymbol{a}],\end{aligned} \tag{14}$$

where $\boldsymbol{x}_{p-1} = [s(t-1)\cdots s(t-p)]^T$ and $\langle\cdot\rangle$ denotes the conditional expectation $E\{\cdot|\widehat{\sigma_u^2}, \widehat{\sigma_v^2}\}$. Similarly, σ_v^2 is estimated from Eq. (1) as follows:

$$\begin{aligned}\widehat{\sigma_v^2}(t+1) &= E\{(z(t) - s(t))^2 | \widehat{\sigma_u^2}(t), \widehat{\sigma_v^2}(t)\} \\ &\cong \frac{1}{t}\sum_{k=1}^{t}[z^2(k) - 2z(k)\langle s(k)\rangle + \langle s^2(k)\rangle].\end{aligned} \tag{15}$$

Using the estimated state vectors and their covariances in the H^∞ filtering, we can calculate $\widehat{\sigma_u^2}(t)$ and $\widehat{\sigma_v^2}(t)$ recursively using Eqs. (14) and (15) as described in [4]. If the evolutional speed of statistical property in the observed signal is slow, we can alter the time-average operation $1/t$ in the preceding algorithm to $(1-\lambda)/(1-\lambda^t)$ using the forgetting factor λ $(0 < \lambda < 1)$ to obtain the tracking capability.

3.3. Lowpass Filtering

Although we have assumed the AR model for the desired signal, an autoregressive moving average (ARMA) model is preferable in more general cases. However, it is difficult to estimate ARMA parameters using the TLS algorithm without modeling the input signals and with lower computational burden. As a result, there could be insufficient noise reduction in signals that are properly expressed by the MA model. For compensating this problem, we apply lowpass filtering followed by the H^∞ filtering-based enhancement. It is expected that this lowpass filtering step will reduce the noise that remains in MA model-like parts of signal.

4. COMPUTER SIMULATIONS

We have carried out simulations to demonstrate the performance of the proposed algorithm using a normalized Japanese male speech sampled at 20 kHz rate. To this signal, we have added a uniformly distributed white noise. This noise-corrupted signal has been used as the observed waveform in the simulations. The SNR has been 4.0 dB. We have compared the proposed algorithm with the another H^∞ filter-based algorithm in [3]. We have also compared the proposed algorithm with the algorithm proposed in [4].

In simulations, the scalar γ in the H^∞ filtering is 3.95. The AR model order is 4. The step size μ in TLS estimation is linearly decreased from 0.1 to 0.011 during the first 1000 sample points. In other sample points, μ is kept at 0.011. The FIR lowpass filter has a passband edge at 0.82 and a stopband edge at 0.92 in the normalized frequency, and is designed by the Remez algorithm with a filter length of 27.

Figure 1(a) shows the observed waveform. Figure 1(b) shows the noise-free waveform. Figure 1(c) shows the enhanced waveform using the proposed algorithm. The median value of Itakura-Saito distortion measurements (IS measurements) of this enhanced waveform is 1.89. Listening test has verified that the enhanced speech has considerably less noise without any loss of smoothness. On the other hand, the median value of IS measurements using the algorithm in [3] is 2.66. In this case, a listening test has indicated that the processed speech still contains significant noise. Finally, the median value of IS measurements using the algorithm in [4] is 2.14. The resultant enhanced speech has like stifling in transient parts. From these measurements, it can be seen that the proposed algorithm reduces the additive noise effectively. In particular, the result of the proposed algorithm is superior than those of other algorithms in the transient parts of speech.

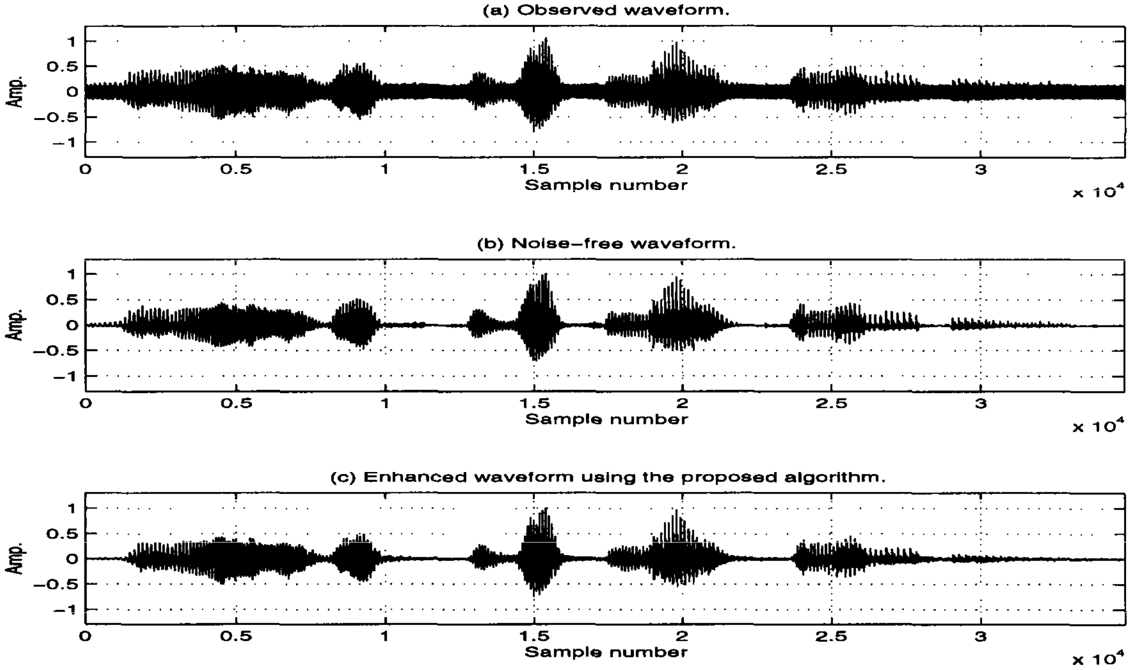

Figure 1: Simulation results using a normalized Japanese male speech.

5. CONCLUSIONS

We have proposed an H^∞ filtering algorithm for noise reduction under low SNR environments. We have formulated the H^∞ filtering in the framework of the noise reduction problem. Next, we developed the adaptive algorithm for estimating the parameters needed in the H^∞ filtering, by estimating first the AR parameters using the TLS estimation method and then estimating the power parameters of the signals using state quantities obtained in the previous steps. In addition, we introduced lowpass filtering to compensate for the noise reduction of signals that are properly represented by an MA model. Computer simulations have verified that the proposed algorithm reduces white noise more effectively than conventional algorithms, especially in the transient parts of the signal.

6. REFERENCES

[1] E. Weinstein, A. V. Oppenheim, M. Feder, and J. R. Buck, "Iterative and sequential algorithms for multisensor signal enhancement," *IEEE Trans. Signal Processing*, vol. 42, pp. 846-859, April 1994.

[2] S. Gannot, D. Burshtein, and E. Weinstein, "Iterative-batch and sequential algorithms for single microphone speech enhancement," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Munich, Ger., May 1997, vol. 2, pp. 1215-1218.

[3] X. Shen, "Discrete H_∞ filter design with application to speech enhancement," in *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, Detroit, MI, May 1995, vol. 2, pp. 1504-1507.

[4] J. Shimizu, Y. Miyanaga, and K. Tochinai, "Signal enhancement using a robust adaptive total least squares algorithm," *Journal of the Acoustical Society of Japan (E)*, vol. 17, pp. 285-293, November 1996.

[5] B. Hassibi, A. H. Sayed, and T. Kailath, "Recursive linear estimation in Krein spaces - Part II: Applications," in *Proc. the 32nd IEEE Conference on Decision and Control*, San Antonio, TX, December 1993, vol. 4, pp. 3495-3500.

[6] K. Gao, M. O. Ahmad, and M. N. S. Swamy, "A constrained anti-Hebbian learning algorithm for total least-squares estimation with application to adaptive FIR and IIR filtering," *IEEE Trans. Circuits and Systems II: Analog and Digital Signal Processing*, vol. 41, pp. 718-729, November 1994.