Minimum-Noise-Variance Beamformer with an Electromagnetic Vector Sensor

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Abstract: We develop a minimum-noise-variance beamformer employing one electromagnetic vector sensor, capable of measuring the complete electric and magnetic fields induced by electromagnetic signals at one point. Two types of signals are considered: one carries a single message, and the other carries two independent messages simultaneously. The state of polarization of the interference under consideration ranges from completely polarized to unpolarized. To analyze the performance, we first obtain explicit expressions for the signal to interference-plus-noise ratio (SINR) in terms of the parameters of the desired signal, interference and noise. Then we discuss some physical implications associated with the SINR expressions. Our SINR expressions provide a basis for effective interference suppression, as well as generation of dual-message signals of which the two message signals have minimum interference effect on one another.

1. Introduction

Direction-of-arrival (DOA) estimation and beamforming are two common objectives of array processing. For applications concerning electromagnetic (EM) waves, early work on DOA estimation and beamforming was based on scalar sensors, each of which provides measurements of only one component of the electric or magnetic field induced by the signals. Recently, [1], [2] proposed the use of EM vector sensors, measuring the complete electric and magnetic fields induced by the source signals, for DOA estimation.

DOA estimation with EM vector sensors has attracted considerable research interest. A few studies of uniqueness in DOA estimates have been reported in [3]-[7], and various DOA estimation algorithms have also been suggested in [8]-[11], which have indicated the superiority of EM vector sensors over scalar sensors. In particular, it was revealed in [4]-[6] that with just one EM vector sensor, the DOA's of two or even three signals can be uniquely determined (7 or more appropriately spaced scalar sensors would be needed for the same purpose [12]).

Based on the results reported in [4]-[6] on uniqueness in DOA estimates, EM vector sensors should handle more signals in beamforming applications as compared with (the same number of) scalar sensors. In addition, the findings reported in [1], [2] shed light on vector sensors' ability to receive/reject signals based on both their polarizations and DOA's.

Beamforming with EM vector sensors, however, has received little attention. This motivates us to investigate the performance of minimum-noise-variance (MNV) beamformer [13] for EM vector sensors. We are concerned with one EM vector sensor, and restrict our investigation to scenarios where there exist one desired signal and one interference. Two types of signals are considered: one carries a single message, and the other carries two independent messages simultaneously [1], [2]. We shall call the former single-message (SM) signal, and the latter dual-message (DM) signal. On the other hand, the interference under consideration can be a completely polarized (CP) signal, a partially polarized (PP) signal, or an unpolarized (UP) signal (the state of polarization of a CP signal is constant while that of a PP or UP signal varies with time).

We first obtain explicit expressions for the signal to interference-plus-noise ratio (SINR) in terms of the parameters of the desired signal, interference and noise, for both SM signals and DM signals. Then we discuss some physical implications associated with the SINR expressions. In particular, we deduce that for the two types of signals of interest, the SINR rises with an increase in the separation between the DOA's, and/or the polarizations, of the desired signal and the interference (scalar-sensor arrays do not have such properties). Moreover, we identify strategies for effectively suppressing an interference with an EM vector sensor. The SINR expression for the SM signal that we derive also provides a basis for generating a DM signal in which the two message signals have minimum interference effect on one another.

2. Data Model and Preliminary Discussion

We focus on beamforming using one EM vector sensor in the presence of one desired signal and one interference. We shall adopt the data model proposed in [1], [2]. We first address desired signal of SM type and then of DM type.

2.1 Desired Signal of Single Message Type

We shall use the subscript 's' to indicate that a symbol is associated with the desired SM signal, and 'i' to indicate that a symbol is associated with the interference. Let $\mathbf{y}_s(t)$ be the complex (phasor) sensor measurement obtained with an EM vector sensor at time t, induced by a SM signal in the presence of an interference and additive noise, $\mathbf{e}(t)$. Then we have

$$\mathbf{y}_s(t) = \mathbf{a}(\boldsymbol{\theta}_s)s_s(t) + \mathbf{B}(\phi_i, \psi_i)\boldsymbol{\xi}_i(t) + \mathbf{e}(t), \tag{2.1}$$

where

$$\mathbf{a}(\boldsymbol{\theta}) = \mathbf{B}(\phi, \psi)\mathbf{Q}(\alpha)\mathbf{h}(\beta), \qquad \boldsymbol{\theta} = [\phi, \psi, \alpha, \beta]^T,$$

$$\mathbf{B}(\phi, \psi) = \begin{pmatrix} \mathbf{v}_1(\phi, \psi) & \mathbf{v}_2(\phi, \psi) \\ \mathbf{v}_2(\phi, \psi) & -\mathbf{v}_1(\phi, \psi) \end{pmatrix},$$

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$$(\mathbf{v}_1(\phi,\psi) \quad \mathbf{v}_2(\phi,\psi)) = \begin{pmatrix} -\sin\phi & -\cos\phi\sin\psi \\ \cos\phi & -\sin\phi\sin\psi \\ 0 & \cos\psi \end{pmatrix},$$

$$\mathbf{Q}(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}, \qquad \mathbf{h}(\beta) = \begin{pmatrix} \cos\beta \\ j\sin\beta \end{pmatrix},$$

$s_s(t) \in \mathbb{C}^1$, $\boldsymbol{\xi}_i(t) \in \mathbb{C}^{2\times1}$, $\mathbf{y}_s(t)$, $\mathbf{e}(t) \in \mathbb{C}^{6\times1}$, and '$T$' is the transpose operator.

The first, second and third terms on the right hand side of (2.1) correspond to measurements induced by, respectively, the desired signal, interference and noise. The parameters $\phi \in (-\pi,\pi]$ and $\psi \in [-\pi/2,\pi/2]$ are respectively the azimuth and elevation of the signal, and $\alpha \in (-\pi/2,\pi/2]$ and $\beta \in [-\pi/4,\pi/4]$ are polarization parameters referred to as the orientation angle and ellipticity respectively. The vector $\mathbf{a}(\boldsymbol{\theta})$ is the steering vector of an EM vector sensor associated with a SM signal with parameter $\boldsymbol{\theta}$. The variable $s_s(t)$ is the complex envelope of the desired signal and $\boldsymbol{\xi}_i(t)$ the complex envelopes of the interference.

The covariance of $\boldsymbol{\xi}_i(t)$ determines the state of polarization of the interference. Indeed, the interference covariance matrix, $\mathbf{R}_i \triangleq E(\boldsymbol{\xi}_i(t)\boldsymbol{\xi}_i^H(t))$, where '$H$' is the hermitian operator, can be expressed as (see Lemma 1 of [14])

$$\mathbf{R}_i = \frac{\sigma_{i,u}^2}{2}\mathbf{I}_2 + \sigma_{i,p}^2\mathbf{Q}(\alpha_i)\mathbf{h}(\beta_i)\mathbf{h}^H(\beta_i)\mathbf{Q}^H(\alpha_i), \qquad (2.2)$$

where $\mathbf{I}_2$ is the 2×2 identity matrix. The first term on the right hand side of (2.2) is the UP component with power $\sigma_{i,u}^2$, and the second term is the CP component with power $\sigma_{i,p}^2$. The interference is said to be CP if $\sigma_{i,p}^2 \neq 0$ but $\sigma_{i,u}^2 = 0$, PP if $\sigma_{i,p}^2 \neq 0$ and $\sigma_{i,u}^2 \neq 0$, and UP if $\sigma_{i,u}^2 \neq 0$ but $\sigma_{i,p}^2 = 0$.

The output of a beamformer is $\hat{s}_s(t) = \mathbf{w}_s^H\mathbf{y}_s(t)$, where $\mathbf{w}_s \in \mathbb{C}^{6\times1}$ is the weight vector. Suppose the DOA and polarization parameters of the desired signal are known, then, for the MNV beamformer, the weight vector is obtained through the following constrained minimization:

$$\mathbf{w}_s = \arg\min_{\mathbf{w}\in\mathbb{C}^{6\times1}} \mathbf{w}^H\mathbf{R}_s\mathbf{w}, \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}_s = 1,$$

where $\mathbf{R}_s = E(\mathbf{y}_s(t)\mathbf{y}_s^H(t))$ is the data covariance matrix, and $\mathbf{a}_s$ denotes $\mathbf{a}(\boldsymbol{\theta}_s)$. The beamformer attempts to suppress all incoming signals except for the desired one with steering vector $\mathbf{a}_s$.

2.2 Desired Signal of Dual Message Type

We shall use the subscript 'd' to indicate that a symbol is associated with the desired DM signal. The complex (phasor) sensor measurement obtained by an EM vector sensor at time t, induced by a DM signal in the presence of an interference and additive noise is given by:

$$\mathbf{y}_d(t) = \mathbf{a}(\boldsymbol{\theta}_{d,1})s_{d,1}(t) + \mathbf{a}(\boldsymbol{\theta}_{d,2})s_{d,2}(t) + \mathbf{B}(\phi_i,\psi_i)\boldsymbol{\xi}_i(t) + \mathbf{e}(t) \qquad (2.3)$$

where

$$\boldsymbol{\theta}_{d,k} = (\phi_d,\psi_d,\alpha_{d,k},\beta_{d,k}).$$

The first and second terms on the right hand side of (2.3) correspond to measurements induced by, respectively, the first and second message signals associated with the DM signal. The variables $s_{d,k}(t)$ and $\mathbf{a}(\boldsymbol{\theta}_{d,k})$, where $k = 1,2$, are respectively the complex envelope and the steering vector of the kth message signal. Note that the two steering vectors $\mathbf{a}(\boldsymbol{\theta}_{d,1})$ and $\mathbf{a}(\boldsymbol{\theta}_{d,2})$ correspond to the same DOA (ϕ_d,ψ_d) but distinct polarizations $(\alpha_{d,1},\beta_{d,1})$ and $(\alpha_{d,2},\beta_{d,2})$ respectively. We shall propose in Section 4 an appropriate choice of $(\alpha_{d,1},\beta_{d,1})$ and $(\alpha_{d,2},\beta_{d,2})$ that minimizes the interference effect on one desired message signal due to the other.

The outputs of a beamformer for the first and second message signals are, respectively,

$$\hat{s}_{d,1}(t) = \mathbf{w}_{d,1}^H\mathbf{y}_d(t) \qquad \text{and} \qquad \hat{s}_{d,2}(t) = \mathbf{w}_{d,2}^H\mathbf{y}_d(t),$$

where $\mathbf{w}_{d,1}, \mathbf{w}_{d,2} \in \mathbb{C}^{6\times1}$ are the weight vectors. Note that in order to optimize the recovering of the message signals, a specific weight vector is used for each message signal separately (i.e., $\mathbf{w}_{d,1}$ is not necessarily identical to $\mathbf{w}_{d,2}$). Suppose the DOA and polarization parameters of the desired signal are known, then, for the MNV beamformer, the weight vector for the kth message signal, where $k = 1,2$, is obtained through the following constrained minimization:

$$\mathbf{w}_{d,k} = \arg\min_{\mathbf{w}\in\mathbb{C}^{6\times1}} \mathbf{w}^H\mathbf{R}_d\mathbf{w}, \quad \text{subject to} \quad \mathbf{w}^H\mathbf{a}_{d,k} = 1,$$

where $\mathbf{R}_d = E(\mathbf{y}_d(t)\mathbf{y}_d^H(t))$ is the data covariance matrix, and $\mathbf{a}_{d,k}$ denotes $\mathbf{a}(\boldsymbol{\theta}_{d,k})$.

2.3 Assumptions and Some Useful Measures

The analyses to be carried out are based on the following assumptions:

Assumption 1. The complex envelopes of the signals and noise are all zero-mean Gaussian random variables.

Assumption 2. The DOA and polarization parameters of the desired signal are known.

Assumption 3. The desired signal is uncorrelated with the interference.

Assumption 4. The various components of the noise are uncorrelated among themselves, and also uncorrelated with both the desired signal and interference.

Assumption 5. The powers of the electric noise and magnetic noise are all equal to σ_e^2.

For a performance measure, we use the ratio of the output power of the desired signal to the output power of the interference and noise (SINR). The SINR measure has been used as performance indicator for beamformers in many studies. In our case, for SM signal, the SINR is given by

$$SINR_s \triangleq \frac{\sigma_s^2\mathbf{w}_s^H\mathbf{a}_s\mathbf{a}_s^H\mathbf{w}_s}{\mathbf{w}_s^H(\mathbf{R}_s - \sigma_s^2\mathbf{a}_s\mathbf{a}_s^H)\mathbf{w}_s}, \qquad (2.4)$$

where $\sigma_s^2 = E(s_s(t)s_s^*(t))$ is the power of the desired signal and '$*$' is complex conjugate operator. For DM signal, the SINR for the kth message signal, $\hat{s}_{d,k}(t)$, is

$$SINR_{d,k} \triangleq \frac{\sigma_{d,k}^2\mathbf{w}_{d,k}^H\mathbf{a}_{d,k}\mathbf{a}_{d,k}^H\mathbf{w}_{d,k}}{\mathbf{w}_{d,k}^H(\mathbf{R}_d - \sigma_{d,k}^2\mathbf{a}_{d,k}\mathbf{a}_{d,k}^H)\mathbf{w}_{d,k}}, \qquad (2.5)$$

where $\sigma_{d,k}^2 = E(s_{d,k}(t)s_{d,k}^*(t))$ is the power of the kth desired message signal, $k = 1,2$.

In this work, we shall obtain explicit expressions for $SINR_s$, $SINR_{d,1}$ and $SINR_{d,2}$, and investigate their characteristics in terms of the various parameters of the desired signal and the interference.

To interpret the SINR expressions, we define the *difference between the polarizations of two signals*, (α_1,β_1) and

(α_2, β_2), to be Δ_2^1, the shorter arc length joining p_1 and p_2, where p_1 and p_2 are respectively the representations for the polarizations (α_1, β_1) and (α_2, β_2) on the Poincaré sphere (see [15] for a detailed justification). Note that Δ_2^1 is related to (α_1, β_1) and (α_2, β_2) by

$$\cos^2\left(\Delta_2^1/2\right) = |\mathbf{h}^H(\beta_2)\mathbf{Q}^H(\alpha_2)\mathbf{Q}(\alpha_1)\mathbf{h}(\beta_1)|^2.$$

It can be shown that $SINR_s, SINR_{d,1}$ and $SINR_{d,2}$ as defined in (2.4) and (2.5) are invariant under coordinate rotations. Moreover, the separation between the DOA's, the difference between the orientation angles of the desired signal and interference, as well as the difference between the ellipticity angles of the desired signal and interference all remain unchanged. Consequently, we shall assume hereafter that $(\phi_s, \psi_s) = (\phi_{d,1}, \psi_{d,1}) = (\phi_{d,2}, \psi_{d,2}) = (0,0)$ and $\psi_i = 0$ (i.e., the DOA of the desired signal is parallel to the x-axis and that of interference is on the x-y plane), and this would lead to considerable simplification of the analyses of SINR expressions. With such a set-up, the separation between the DOA's of the desired signal and interference is simply ϕ_i.

3. SINR for Desired Signal of Single Message Type

For convenience, we shall refer to DOA separation as the separation between the DOA's of the desired signal and interference, and denote it by γ. Moreover, we shall refer to polarization difference as the difference between the polarizations of the desired signal and interference. We have succeeded in expressing $SINR_s$ explicitly in terms of the DOA separation, the polarization difference, and the powers of the desired signal, interference and noise. This facilitates the deduction of the dependence of $SINR_s$ on each of the above parameters.

Theorem 1: The expression of $SINR_s$, as given in (2.4), can be expressed as

$$SINR_s = \sigma_s^2\left(\frac{2}{\sigma_e^2} - \frac{(1+\cos\gamma)^2}{(\sigma_e^2+\sigma_{i,u}^2)}\left(\frac{\sigma_{i,u}^2}{2\sigma_e^2} + \frac{\sigma_{i,p}^2\cos^2\frac{\Delta_i^s}{2}}{2\sigma_{i,p}^2+\sigma_e^2+\sigma_{i,u}^2}\right)\right).$$

Proof: See [15].

Clearly, $SINR_s$ increases with an increase in the desired signal's power, σ_s^2, but decreases with an increase in the noise power, σ_e^2, as well as an increase in the power of the CP (i.e., $\sigma_{i,p}^2$) or UP (i.e., $\sigma_{i,u}^2$) component of the interference. However, the dependencies of $SINR_s$ on polarization difference and DOA separation are non-trivial, and these are established in the following corollaries.

Corollary 1: If $\sigma_{i,p}^2 \neq 0$ and $\gamma \neq \pi$, then $SINR_s$ is an increasing function of Δ_i^s.
Corollary 2: If $\sigma_{i,u}^2 \neq 0$ or $\Delta_i^s \neq \pi$, then $SINR_s$ is an increasing function of γ.
Corollary 3: If $\sigma_{i,p}^2 = 0$, then $SINR_s$ is independent of Δ_i^s.
Corollary 4: $SINR_s$ attains the maximum value, $SINR_s^{max} = 2\sigma_s^2/\sigma_e^2$, when either $\gamma = \pi$, or both $\Delta_i^s = \pi$ and $\sigma_{i,u}^2 = 0$ are true. Moreover, $SINR_s^{max}$ simply takes the value of $SINR_s$ in the absence of interference.

Corollary 5: For given (fixed) values of σ_s^2, $\sigma_{i,u}^2 + \sigma_{i,p}^2$, σ_e^2, and γ, the minimum of $SINR_s$ is attained when $\Delta_i^s = 0$ and $\sigma_{i,u}^2 = 0$.

Proof: See [15].

Remarks: 1. Corollary 1 means that $SINR_s$ generally increases with an increase in the the polarization difference Δ_i^s, except for two special cases: (a) $\sigma_{i,p}^2 = 0$, or (b) $\gamma = \pi$. Note that case (a) corresponds to scenarios where the interference is UP, and case (b) corresponds to scenarios where the DOA of the desired signal is exactly opposite to that of interference. For case (a), the interference has no CP component and thus the polarization difference should not affect $SINR_s$ (see Corollary 3). On the other hand, by Corollary 4, $SINR_s$ for case (b) always attains the maximum value $SINR_s^{max}$ regardless of the other signal parameters.

2. Corollary 2 means that $SINR_s$ generally increases with an increase in the DOA separation γ, except for the case where both $\sigma_{i,u}^2 = 0$ and $\Delta_i^s = \pi$ are true. For the exceptional case, $SINR_s^{max}$ can always be attained regardless of the other signal parameters (see Corollary 4). Note that $\sigma_{i,u}^2 = 0$ means that the interference is CP, and $\Delta_i^s = \pi$ means that the polarization difference is the largest possible. Such a polarization difference can be effected if the polarizations associated with the desired signal and interference satisfy $(\alpha_s, \beta_s) = (\alpha_i \pm \pi, -\beta_i)$. Physically, the two polarization ellipses associated with the polarizations (α_s, β_s) and (α_i, β_i) have the same shape but have their major axes orthogonal to each other, and at the same time the directions of spin of the electric fields associated with the two polarizations are opposite.

3. By Corollary 3, if the interference is UP, then it is not possible to increase $SINR_s$ by varying the polarization of the desired signal (α_s, β_s).

4. Corollary 4 means that $SINR_s$ attains the same maximum value, $SINR_s^{max}$, when either the DOA's of the desired signal and the interference are opposite, or the interference is CP with largest possible polarization difference, π. In either case, $SINR_s^{max}$ obtained is equivalent to the $SINR_s$ when there is no interference regardless of the interference's power (i.e., the interference becomes completely ineffective).

5. Corollary 5 means that, for any DOA separation, $SINR_s$ attains its lowest value when the interference is CP with polarization being identical to that of the desired signal.

The corollaries are potentially useful in some applications. For example, one can exploit the fact that $SINR_s$ increases with an increase in the polarization difference (Corollary 1) to effectively suppress an interference if the CP component of the interference is known. Indeed, for a fixed DOA separation γ, one can obtain the largest $SINR_s$ by transmitting the desired signal with polarization such that the polarization difference is the largest possible, i.e., $\Delta_i^s = \pi$. Clearly, if the interference is CP (i.e., $\sigma_{i,u}^2 = 0$), then $SINR_s$ attains $SINR_s^{max} = 2\sigma_s^2/\sigma_e^2$, which is the value when there is no interference, regardless of the DOA separation and the interference's power.

4. SINR for Desired Signal of Dual Message Type

To transmit a DM desired signal (consisting of two message signals), it is desirable that the interference effect of one message signal on the other would be minimal. Since the DOA parameters associated with the two message signals are identical, it is possible to exploit the difference only in the polarization parameters to reduce the interference effect. In this connection, Corollary 4 of Theorem 1 provides a good way for choosing the polarizations. Indeed, consider the scenario where there is no external interference, and view one desired message signal as the desired CP signal, and the other as a CP interference. Then, by Corollary 4 of Theorem 1, both $SINR_{d,1}$ and $SINR_{d,2}$ attain their maximum values if the difference between the polarizations of the two message signals is equal to π (i.e., when extracting one message signal, there is theoretically no interference effect due to the other). Therefore, we shall assume hereafter that the polarizations of the two message signals are chosen in such a way that the polarization difference is π, meaning that the polarizations satisfy $(\alpha_{d,1}, \beta_{d,1}) = (\alpha_{d,2} \pm \pi, -\beta_{d,2})$ (refer to Remark 2 of the corollaries to Theorem 1 for a relevant physical meaning). Similar to the case of SM signal, we are able to express $SINR_{d,1}$ and $SINR_{d,2}$ explicitly in terms of the DOA separation, the difference between the polarization of the first/second desired message signal and the interference, and the powers of the two desired message signals, interference and noise.

Theorem 2: If $(\alpha_{d,1}, \beta_{d,1}) = (\alpha_{d,2} \pm \pi, -\beta_{d,2})$, then

$$SINR_{d,k} = \sigma_{d,k}^2 \left(\frac{2}{\sigma_e^2} - \frac{(1+\cos\gamma)^2}{\sigma_e^4} \left(\mu + \nu \cos^2 \frac{\Delta_i^{d,k}}{2} + \frac{\nu^2}{4\sigma_e^4 \delta_k} (1+\cos\gamma)^2 \sin^2 \Delta_i^{d,k} \right) \right),$$

where

$$\mu = \frac{\sigma_e^2 \sigma_{i,u}^2}{2(\sigma_e^2 + \sigma_{i,u}^2)}, \qquad \nu = \frac{\sigma_e^4 \sigma_{i,p}^2}{(\sigma_e^2 + \sigma_{i,u}^2)(2\sigma_{i,p}^2 + \sigma_e^2 + \sigma_{i,u}^2)},$$

$$\delta_1 = \frac{\sigma_e^2 + 2\sigma_{d,2}^2}{\sigma_{d,2}^2 \sigma_e^2} - \frac{(1+\cos\gamma)^2}{\sigma_e^4} \left(\mu + \nu \sin^2 \frac{\Delta_i^{d,1}}{2} \right),$$

$$\delta_2 = \frac{\sigma_e^2 + 2\sigma_{d,1}^2}{\sigma_{d,1}^2 \sigma_e^2} - \frac{(1+\cos\gamma)^2}{\sigma_e^4} \left(\mu + \nu \sin^2 \frac{\Delta_i^{d,2}}{2} \right).$$

Proof: See [15].

Corollary 1: If $\sigma_{i,p}^2 \neq 0$ and $\gamma \neq \pi$, then $SINR_{d,k}$ is an increasing function of $\Delta_i^{d,k}$, for $k = 1, 2$.

Corollary 2: If $\sigma_{i,u}^2 \neq 0$ or $\Delta_i^{d,k} \neq \pi$, then $SINR_{d,k}$ is an increasing function of γ, for $k = 1, 2$.

Corollary 3: If $\sigma_{i,p}^2 = 0$, then $SINR_{d,k}$ is independent of $\Delta_i^{d,k}$, for $k = 1, 2$.

Corollary 4: $SINR_{d,k}$ attains the maximum value, $SINR_{d,k}^{max} = 2\sigma_{d,k}^2/\sigma_e^2$, when either $\gamma = \pi$, or both $\Delta_i^{d,k} = \pi$ and $\sigma_{i,u}^2 = 0$ are true, for $k = 1, 2$. Moreover, $SINR_{d,k}^{max}$ simply takes the value of $SINR_{d,k}$ in the absence of interference.

Corollary 5: For given (fixed) $\sigma_{d,1}^2$, $\sigma_{d,2}^2$, $\sigma_{i,u}^2 + \sigma_{i,p}^2$, σ_e^2 and γ, the minimum of $SINR_{d,k}$ is attained when $\Delta_i^{d,k} = 0$ and $\sigma_{i,u}^2 = 0$, for $k = 1, 2$.

Proof: See [15].

The dependence of $SINR_{d,k}$ on $\Delta_i^{d,k}$, γ, $\sigma_{d,k}^2$, σ_e^2, $\sigma_{i,p}^2$ and $\sigma_{i,u}^2$ as presented in Corollaries 1-5 of Theorem 2 is basically identical to that of $SINR_s$ on Δ_i^s, γ, σ_s^2, σ_e^2, $\sigma_{i,p}^2$ and $\sigma_{i,u}^2$ as presented in Corollaries 1-5 of Theorem 1. Therefore, the discussions concerning Corollaries 1-5 of Theorem 1 in Section 3 are applicable to Corollaries 1-5 of Theorem 2.

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