

A METHOD OF OPTIMIZING SOURCE CONFIGURATION IN ACTIVE CONTROL SYSTEMS USING GRAM-SCHMIDT ORTHOGONALIZATION

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ABSTRACT

In this paper, a method for optimizing the number and the configuration of control sources in an active control system is proposed. In the optimization process, sources are selected one by one so that the corresponding transfer impedance vector is the most linearly independent. From the results of the simulation, it is shown that the optimized configuration yields not only small average control error but also small condition number in the transfer impedance matrix, which contributes to the robustness of the system against the environmental change.

1. INTRODUCTION

In an active control system, the configuration of the control sources is an important factor which greatly affects its performance. In particular, the performance is highly sensitive to the source configuration when the system is used in enclosures, since coupling of the sources to certain modes plays an important role. One approach to optimizing the source configuration is to locate the sources so that the cost function in designing the control system, that is usually the sum of square-errors at the sensors, is minimized [1, 2]. Disadvantage of this method is that it does not necessarily prevent the system from being ill-conditioned. Although an ill-conditioned system may show good performance at the sensors, it may yield some harmful effects such as emitting a high level of energy outside the target region.

The authors proposed an alternative approach, in which the source configuration is optimized in terms of the linear independence of the transfer impedance vector for each source [3]. Sources are selected one by one using Gram-Schmidt orthogonalization so that the corresponding transfer impedance vector becomes the most linearly independent. By employing this method, ill-conditioned configurations are automatically avoided. In this paper, the complete algorithm, including the extension to the broadband case, is described.

2. CONTROL EQUATION

The control equation of an active sound control system can be written in the frequency domain as

$$\mathbf{Z}(\omega_k)\mathbf{q}(\omega_k) = \mathbf{d}(\omega_k). \quad (1)$$

For the sake of simplicity, the frequency ω_k is omitted hereafter. The symbol \mathbf{Z} is the transfer impedance matrix, its element z_{ml} denoting the transfer impedance from the l th control source to the m th sensor. This transfer impedance matrix can be written using its column vectors as $\mathbf{Z} = [\mathbf{z}_1 \cdots \mathbf{z}_{N_q}]$ where N_q denotes the number of sources. Each column vector corresponds to its respective control source. The element q_l in \mathbf{q} is the source strength of the l th source. The element d_m in vector \mathbf{d} denotes the desired complex sound pressure at the m th sensor. The system is controlled so that the sum of square-error at the sensors,

$$E = \frac{1}{M} \mathbf{e}^H \mathbf{e} \quad (2)$$

is minimized. where the error vector, \mathbf{e} , is defined as $\mathbf{e} = \mathbf{Z}\mathbf{q} - \mathbf{d}$. The symbol H denotes a conjugate transpose.

3. OPTIMIZATION PROCEDURE

Let us consider the case when there are N_c possible source location candidates in the space around the target region. It is assumed that the transfer impedance vector corresponding to each source location candidate is known. Let \mathcal{T} denote the set of transfer impedance vectors of the candidates, $\mathcal{T} = \{\mathbf{z}_1, \cdots, \mathbf{z}_{N_c}\}$. The aim of the algorithm described in this section is to determine the number of control sources \hat{N}_c ($\leq N_q$) and the optimum subset of vectors \mathcal{S}_{N_q} in \mathcal{T} , i.e., $\mathcal{S}_{N_q} = \{\hat{\mathbf{z}}_1, \cdots, \hat{\mathbf{z}}_{\hat{N}_c}\}$. In the optimization process, sources are selected one by one from \mathcal{T} at each step so that the corresponding transfer impedance vector is the most linearly independent of the subset of vectors that have already been selected in the previous steps.

3.1. Basic Algorithm

Let us consider the case of the n th step. It is assumed that $n - 1$ candidates have already been selected from the 1st to the $(n - 1)$ th steps. The subset of the transfer impedance vectors selected in the previous steps is denoted by \mathcal{S}_{n-1} . The subset of the unused transfer impedance vectors at the $(n - 1)$ th step is denoted as $\mathcal{T}_{n-1} = \mathcal{T} - \mathcal{S}_{n-1}$. Also, let us denote the orthonormal basis of the subspace spanned by \mathcal{S}_{n-1} as $\mathcal{V}_{n-1} = \{v_1, \dots, v_{n-1}\}$.

In the n th step, the n th impedance vector, \hat{z}_n , is selected so that the component of \hat{z}_n , that is perpendicular to the subspace spanned by \mathcal{S}_{n-1} , is the largest. The perpendicular component, r_i , of an arbitrary vector $z_i \in \mathcal{T}_{n-1}$ is calculated as

$$r_i = z_i - p. \quad (3)$$

where p is the projection of z_i onto the subspace spanned by \mathcal{S}_{n-1} . The projection p is calculated as

$$p = \sum_{j=1}^{n-1} p_j = \sum_{j=1}^{n-1} (v_j^H z_i) v_j, \quad (4)$$

where p_j is the projection of z_i onto the orthonormal basis $v_j \in \mathcal{V}_{n-1}$. The n th source is determined so that the norm of r_i is maximized, i.e.,

$$\hat{z}_n = \arg \max_{z_i \in \mathcal{T}_{n-1}} J(z_i), \quad (5)$$

where

$$J(z_i) = \|r_i\|. \quad (6)$$

The vector r_i is a component of z_i , which is perpendicular to \mathcal{V}_{n-1} , and, thus, is linearly independent of previously determined impedance vectors, \mathcal{S}_{n-1} . The physical meaning of r_i is the independent contribution of the corresponding source to the control, which cannot be substituted by any of the other sources determined in the previous steps. The n th orthonormal basis v_n is then determined as

$$v_n = r / \|r\|. \quad (7)$$

The relation of these vectors when $n = 3$ is shown in Fig. 1 as an example. The maximized cost function at the n th step is denoted as $\hat{J}_n = J(\hat{z}_n)$.

3.2. Initial condition

To initiate the process, initial conditions, i.e., \hat{z}_1 and v_1 , must be given. In this section, how to determine these initial vectors is described. The role of the control equation (1) is to find a linear combination of transfer

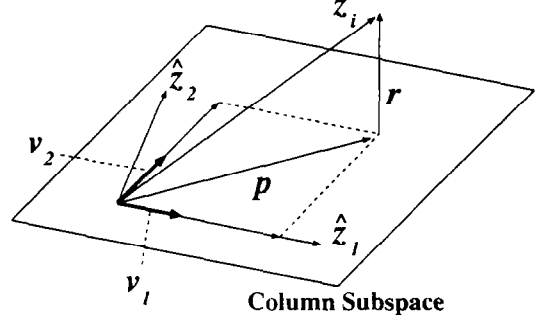


Figure 1: Relation of vectors.

impedance vectors that is closest to the desired vector, d . Therefore, the first vector is selected so that it is closest to d . This is realized by minimizing the angle θ_i between the arbitrary vector $z_i \in \mathcal{T}_0 (= \mathcal{T})$ and d . The cost function to be minimized is defined as

$$F(z_i) = \sin(\theta_i) = \frac{\|z_i - p_d\|}{\|z_i\|} \quad (8)$$

where p_d is the projection of z_i onto d as

$$p_d = \frac{z_i^H d}{d^H d} d. \quad (9)$$

Using these, the first vector is determined as

$$\hat{z}_1 = \arg \min_{z_i \in \mathcal{T}_0} F(z_i). \quad (10)$$

The first orthonormal vector is defined as

$$v_1 = \hat{z}_1 / \|\hat{z}_1\|. \quad (11)$$

The norm of the first impedance vector is then used as the initial value of the cost function for the optimization process as $\hat{J}_1 = \|\hat{z}_1\|$.

3.3. Termination of the optimization process

In the optimization process, the procedure described in Section 3.1 is iterated with n being increased. During this process, the cost function \hat{J}_n monotonically decreases. The optimization process is terminated when the cost function \hat{J}_n becomes smaller than a certain threshold, J_{thr} . The number of sources, \hat{N}_q , is then determined as $\hat{N}_q = n - 1$ when $\hat{J}_n < J_{thr}$.

3.4. Recursive Algorithm

Among all possible configurations, there might be configurations other than the optimum one that satisfy the criterion, $\hat{J}_n < J_{thr}$. These configurations are expected to yield performances similar to that of the optimum

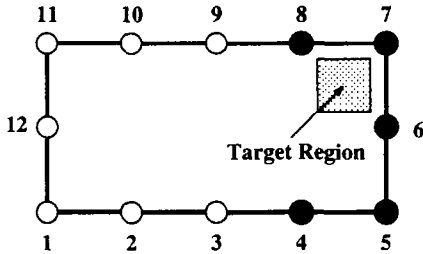


Figure 2: Location of control source candidates. The filled circles are the sources selected by the optimization process for the single frequency case.

one. Using the above procedure recursively with minor modification, these configurations can easily be listed up [4]. Listing up of these configurations widens the choice of configurations.

4. BROADBAND EXTENSION

In this section, the optimization is extended to the broadband case. In the broadband case, each source candidate is represented by a set of transfer impedance vectors of the frequency range of interest, $[\omega_l, \omega_h]$. The set of transfer impedance vectors is denoted as $\bar{z}_i = \{z_i(\omega_l), \dots, z_i(\omega_h)\}$. In the optimization procedure, the optimum set $\mathcal{S}_{N_q} = \{\bar{z}_1, \dots, \bar{z}_{N_q}\}$ is selected.

In the broadband case, the following two cost functions are evaluated.

$$J_{avg}(\bar{z}_i) = \frac{1}{K}(w_l \|\mathbf{r}_i(\omega_l)\| + \dots + w_h \|\mathbf{r}_i(\omega_h)\|) \quad (12)$$

$$J_{min}(\bar{z}_i) = \min(w_l \|\mathbf{r}_i(\omega_l)\|, \dots, w_h \|\mathbf{r}_i(\omega_h)\|) \quad (13)$$

The symbol K denotes the number of discrete frequencies. The symbol w_k denotes an arbitrary weight at the discrete frequency ω_k . The perpendicular component, $\mathbf{r}_i(\omega_k)$, and the orthonormal basis, $\mathbf{v}_i(\omega_k)$, which is required for the calculation of $\mathbf{r}_i(\omega_k)$, are calculated for every discrete frequency separately in the same manner as the single frequency case. The cost function, J_{avg} , is the weighted average norm in the frequency range of $[\omega_l, \omega_h]$ and is maximized during the optimization process, i.e.,

$$\hat{\bar{z}}_n = \arg \max_{\bar{z}_i \in \mathcal{T}_{n-1}} J_{avg}(\bar{z}_i). \quad (14)$$

By using the average norm, the average linear independence of the column vectors are maximized. On the other hand, for determining the required number of source \hat{N}_q and terminating the procedure, the cost function J_{min} is evaluated. When $J_{min} < J_{thr}$, the procedure is terminated. The reason for using \hat{J}_{min} instead of J_{avg} is to avoid low linear independence at all frequencies. The rest of the procedure is the same as that of the single frequency.

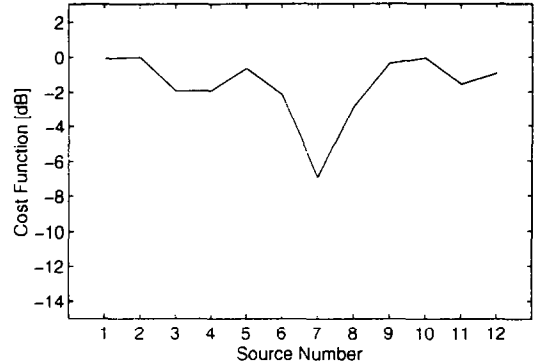


Figure 3: Value of the cost function $F(z_i)$.

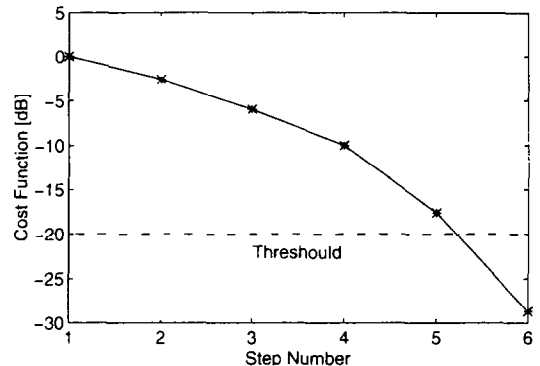


Figure 4: The value of maximized cost function, \hat{J}_n (normalized by \hat{J}_1).

5. SIMULATION

5.1. Condition

As an example of active control, active equalization is treated in this simulation. To obtain uniform sound pressure in the target region (equalization), the desired sound pressure vector was set at $\mathbf{d} = [1 \dots 1]^H$. A two-dimensional enclosure with rigid walls, the size of which is 1.0m (x) \times 0.1m (y) \times 2.0m (z), was employed. The transfer impedance vectors were calculated by using Green's function for a rigid-wall enclosure [5]. As control source candidates, 12 point-sources were located as depicted in Fig. 2. The target region was the shaded square area, 0.3 m \times 0.3 m, where 64 equally-spaced sensors (8×8) were placed at intervals of 4.3 cm.

5.2. Results

The results for the single frequency case (500 Hz) is described. Figure 3 shows the value of the cost function $F(z_i)$ for the determination of the first source. From this figure, source # 7, for which $F(z_i)$ is the smallest, was selected. Figure 4 shows the value of the maximized cost function, \hat{J}_n . The threshold, J_{thr} , was set

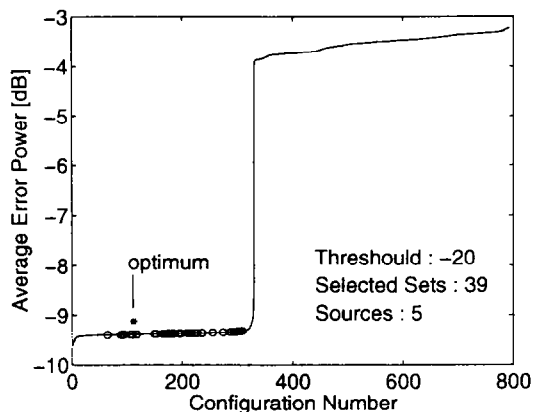


Figure 5: Average error power, \bar{E} , for all possible configurations. The horizontal axis corresponds to the configuration number (each configuration is numbered in increasing order of \bar{E} .) The symbol “*” in the figure shows the performance of the optimum configuration. For the sake of clarity, the symbol was plotted a little off from the curve. The symbol “o” represents the configuration that satisfies the criterion, $\hat{J}_{\hat{N}_q} \geq J_{thr}$.

at -20 dB. From this figure, it can be seen that \hat{J}_n intersects the threshold when $5 < n < 6$. Therefore, the optimum number of sources was determined as $\hat{N}_q = 5$. The optimum configuration selected through this process is indicated by the filled circles in Fig. 2. The performance of all possible configurations are summarized in Fig. 5. From this figure, it can be seen that the optimum configuration yields small average error power. Moreover, all the configurations that satisfy the criterion, $\hat{J}_{\hat{N}_q} \geq J_{thr}$, show small \bar{E} . Figure 6 shows the condition number of the transfer impedance matrix \mathbf{Z} for each configuration. The condition number of the transfer impedance matrix is known as an index of the robustness of the system against environmental change [6]. As can be seen in this figure, all the selected configurations show small condition number. From this figure and Fig. 5, the proposed algorithm selects the configurations that have properties of both small average error and small condition number. The broadband case (400-600 Hz) was also examined by simulations and similar results were obtained.

6. SUMMARY

In this paper, an optimization method of source configuration in active control system was presented. Sources are selected from the candidates one by one so that the corresponding transfer impedance vector becomes the most linearly independent. The results of the simulation show that the selected configuration yields small

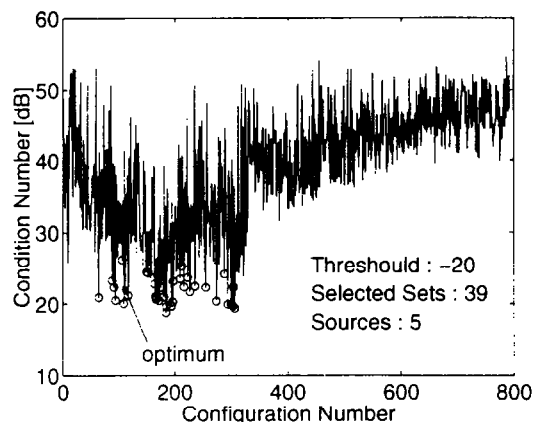


Figure 6: Condition number of the transfer impedance matrix for all possible configurations. The horizontal axis, the symbols “o” and “*” are the same as those in Fig. 5.

average error and small condition number.

7. REFERENCES

- [1] T. C. Yang, C. H. Tseng, and S. F. Ling, “Constrained optimization of active noise control systems in enclosures,” *J. Acoust. Soc. Am.*, vol. 95(6), pp. 3390–3399, 1994.
- [2] E. Benzaria and V. Martin, “Constrained Optimization of secondary source locations: multipolar source arrangements,” In S. Sommerfeldt and H. Hamada, editors, *Proceedings of Active 95*, pp. 499–510. 1995.
- [3] F. Asano, Y. Suzuki, T. Sone, and D. C. Swanson, “Optimization of control source location in active control system,” In *Proceedings of Active 95*, pp. 489–498. 1995.
- [4] F. Asano, Y. Suzuki, and D. C. Swanson, “Optimization of Source Configuration in Active Control System using Gram-Schmidt Orthogonalization,” *being prepared for submission*.
- [5] P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, Princeton, New Jersey, 1968.
- [6] F. Asano, Y. Suzuki, and T. Sone, “Sound equalization using derivative constraints,” *Acustica*, vol. 82(2), pp. 311–320, March/April 1996.