ON THE FOURTH-ORDER CUMULANTS ESTIMATION FOR THE HO BLIND SEPARATION OF CYCLOSTATIONARY SOURCES

Anne Férréol and Pascal Chevalier

Thomson-CSF Communications, TTC/TSI, 66 rue du Fossé Blanc, 92231 Gennevilliers, France Tel: (33)-01-46-13-26-98, Fax: (33)-01-46-13-25-55 E-Mail : Anne Férréol@tcc.thomson.fr; pascal.chevalier@tcc.thomson.fr

n

ABSTRACT

Most of the HO blind source separation methods developed this last decade aim at blindly separating statistically independent sources, assumed stationary and ergodic. Nevertheless, in many situations such as in radiocommunications, the sources are non stationary and very often (quasi)-cyclostationary (digital modulations). In these contexts, it is important to wonder if the performance of these HO blind source separation methods may be affected by the potential non stationarity of the sources. The purpose of this paper is to bring some answers to this question through the behaviour analysis of the empirical fourth-order cumulant estimators in the presence of (quasi)-cyclostationary sources.

1. INTRODUCTION

Since more than a decade, the HO blind source separation methods have been strongly developed [1-3]. These methods aim at blindly separating several statistically independent sources, assumed stationary and ergodic. Under these assumptions, the separators presented in [1-3] have been shown to be very powerful in many situations borrowed from the radiocommunications [4-8] and the radar [8] field. Nevertheless, in many situations such as in digital radiocommunications, the sources are non stationary and more precisely (quasi)-cyclostationary. In these contexts, it becomes important to wonder if the performance of the developed HO blind source separation methods [1-3] may be modified by the potential non stationarity of the sources, which appears explicitely at the processing level as soon as the sources are oversampled. The purpose of the paper is to bring some answers to the previous question through the behaviour analysis of the empirical fourth-order cumulants estimators in the presence of (quasi)-cyclostationary sources.

2. HYPOTHESIS AND PROBLEM FORMULATION

Consider an array of *N* Narrow-Band (NB) sensors and let us call x(t) the vector of the complex envelopes of the

signals present at time t at the output of the sensors. Each sensor is assumed to receive a noisy mixtures of P zeromean, (quasi)-cyclostationary and NB independent sources. Under these assumptions, the observation vector $\mathbf{x}(t)$ can be written as

$$\mathbf{x}(t) = \sum_{i=1}^{P} m_i(t) e^{\mathbf{j}(\Delta \omega_i t + \phi_i)} \mathbf{a}_i + \mathbf{b}(t) \stackrel{\Delta}{=} A \mathbf{m}(t) + \mathbf{b}(t) \quad (2.1)$$

where b(t) is the noise vector, assumed zero-mean, stationary, Gaussian and spatially white, $m_i(t)$, $\Delta \omega_i$, ϕ_i and a_i correspond to the complex envelope, the carrier residu, the phase and the steering vector of the source i, m(t) is the vector which components are the signals $m_i(t) \exp[j(\Delta \omega_i t + \phi_i)]$ and A is the $(N \times P)$ matrix which columns are the vectors a_i .

Under the previous assumptions, the correlation matrices of the observations, $R_x(t) \stackrel{\Delta}{=} [\mathbf{x}(t)\mathbf{x}(t)^{\dagger}]$ and $C_x(t) \stackrel{\Delta}{=} E[\mathbf{x}(t)\mathbf{x}(t)^{T}]$, can be written as

$$R_{\chi}(t) = A R_{m}(t) A^{\dagger} + \eta_{2} I$$
 (2.2)

$$C_{\chi}(t) = A C_m(t) A^{\mathrm{T}}$$
(2.3)

where [†] means transpose and complex conjugate, ^T means transpose, η_2 is the mean power of the noise per sensor, I is the identity matrix and $R_m(t) \stackrel{\Delta}{=} E[\boldsymbol{m}(t)\boldsymbol{m}(t)^{\dagger}]$ and $C_m(t)$ $\stackrel{\Delta}{=} E[\boldsymbol{m}(t)\boldsymbol{m}(t)^{T}]$ are the first and second correlation matrix of the vector $\boldsymbol{m}(t)$. The fourth-order statistics of the observations are characterized by the quadricovariance $Q_x(t)$, which elements, $Q_x(i, j, k, l)(t) \stackrel{\Delta}{=} \operatorname{Cum}(x_i(t), x_j(t)^*, x_k(t)^*, x_l(t))$, are the fourth-order cumulants of the vector $\boldsymbol{x}(t)$ and which is defined by

$$Q_{X}(t) = (A \otimes A^{*})Q_{m}(t)(A \otimes A^{*})^{\top}$$
(2.4)

where $Q_m(t)$ is the quadricovariance of the vector m(t) and where \otimes corresponds to the Kronecker product.

Under the previous assumptions, although in the presence of (quasi)-cyclostationary sources it may be useful to use Polyperiodic (PP) and Widely Linear structure of separation [9], the problem of source separation we address in this paper is to blindly implement a $(N \ge P)$ Linear and Time Invariant source separator, W, outputing at time t the $(P \ge 1)$ vector $\mathbf{y}(t) \stackrel{\Delta}{=} W^{\dagger} \mathbf{x}(t)$, corresponding, to within a diagonal Λ and a permutation matrix Π , to the best estimate of the vector $\mathbf{m}(t)$.

3. BLIND SEPARATION OF (QUASI)-CYCLOSTATIONARY SOURCES

3.1 Possible Philosophies

For (quasi)-cyclostationary observations, the matrices (2.2) et (2.3), PP, have a Fourier serial expansion showing off the cyclic frequencies of the observations. The exploitation of the information contained in all the cyclic frequencies of the observations for the blind separation of (quasi)-cyclostationaty sources may be very useful in some situations as it has been shown in [10]. Nevertheless, for particular reasons such as the complexity of the implementation, on may prefer to still implement the classical HO blind methods of source separation [1-3] even for (quasi)-cyclostationary sources, which is the choice done in this paper.

In (quasi)-cyclostationary contexts, these classical methods aim at exploiting the information contained in the zero cyclic frequency of the matrices $R_x(t)$ and $Q_x(t)$, i.e. in the temporal means $R_x \triangleq \langle R_x(t) \rangle$ and $Q_x \triangleq \langle Q_x(t) \rangle$ of the matrices $R_x(t)$ and $Q_x(t)$ respectively. The matrices R_x and Q_x are defined by (2.2) and (2.4) respectively, where $R_m(t)$ and $Q_m(t)$ are replaced by their temporal mean noted R_m and Q_m respectively. Then, it becomes obvious that the temporal mean operation preserves the algebraic structure of $R_x(t)$ and $Q_x(t)$ and the second and fourth-order statistical independence of the sources in particular (R_m is still diagonal and the non zero elements of Q_m are still the coefficients $Q_m[i, i, i, i]$, $1 \leq i \leq P$).

3.2 Statistics estimation

It is well-known that for zero-mean, stationary and ergodic observations, the empirical estimators of the second and fourth-order cumulant of these observations provide asymptotically unbiaised estimates of the latter, which variance tends to zero as the observation time increases. However, for (quasi)-cyclostationary and cycloergodic observations, one may wonder if these empirical estimators still generate asymptotically unbiaised estimates of the temporal means of the second and fourth-order observations statistics. The answer to this question is obviously positive for the second order statistics but is generally negative for the fourth-order ones. To show this, let us write the expression of $Q_x[i, j, k, l]$, given by

$$Q_{X}(i, j, k, l) = M_{X}^{0}(i, j, k, l) - \langle R_{X}(i, j)(t) R_{X}(l, k)(t) \rangle - \langle R_{X}(i, k)(t) R_{X}(l, j)(t) \rangle - \langle C_{X}(i, l)(t) C_{X}(j, k)(t)^{*} \rangle$$
(3.1)

where $M_x^0(i, j, k, l) \stackrel{\Delta}{=} \langle E[x_i(t) \ x_j(t)^* x_k(t)^* x_l(t)] \rangle$, $R_x(i, j)(t) \stackrel{\Delta}{=} E[x_i(t)x_j(t)^*]$, $C_x(i, j)(t) \stackrel{\Delta}{=} E[x_i(t)x_j(t)]$. Noting $F(\alpha)$ and $G(\alpha)$ the Fourier Transforms of two arbitrary functions f(t) and g(t), we have the following result

$$\langle f(t) g(t) \rangle = [F(\alpha) * G(\alpha)]_{\alpha=0} = \int_{\alpha} F(\alpha) G(-\alpha) \, \mathrm{d}\alpha \quad (3.2)$$

where * is the convolution product. Then, using (3.2) into (3.1) and the fact that $R_{\chi}(t)$ and $C_{\chi}(t)$, PP, have a Fourier serial expansion, we obtain the expression (3.3) given by

$$Q_{X}(i, j, k, l) = M_{X}^{0}(i, j, k, l) - \sum_{\alpha} R_{X}^{\alpha}(i, j) R_{X}^{-\alpha}(l, k) - \sum_{\alpha} R_{X}^{\alpha}(i, k) R_{X}^{-\alpha}(l, j) - \sum_{\beta} C_{X}^{\beta}(i, l) C_{X}^{\beta}(j, k)^{*} (3.3)$$

where $R_{x}^{\alpha}(i, j)$ and $C_{x}^{\beta}(i, j)$ are the coefficients associated to the cyclic frequencies α and β respectively in the Fourier serial expansion of $R_{x}(i, j)(t)$ and $C_{x}(i, j)(t)$. This expression has also be obtained in [11] in an other context. On the other hand, the empirical estimators of the fourthorder cumulants of the observations asymptotically generate an apparent cumulant temporal mean, noted $Q_{xa}(i, j, k, l)$ and given by

$$Q_{xa}(i, j, k, l) \stackrel{\Delta}{=} M_x^0(i, j, k, l) - R_x^0(i, j) R_x^0(l, k) - R_x^0(i, k) R_x^0(l, j) - C_x^0(i, l) C_x^0(j, k)^*$$
(3.4)

Using (3.3) into (3.4) we obtain a new expression of $Q_{xa}(i, j, k, l)$, given by

$$Q_{Xa}(i, j, k, l) = Q_X(i, j, k, l) + \sum_{\alpha \neq 0} R_X^{\alpha}(i, j) R_X^{-\alpha}(l, k) + \sum_{\alpha \neq 0} R_X^{\alpha}(i, k) R_X^{-\alpha}(l, j) + \sum_{\beta \neq 0} C_X^{\beta}(i, l) C_X^{\beta}(j, k)^* (3.5)$$

This expression shows that $Q_{xa}(i, j, k, l) = Q_x(i, j, k, l)$ only for observations having no energy at non zero cyclic frequencies, which is in particular the case for stationary observations (which complex envelope is necessarily second order non circular) but which is also the case for the complex envelope of digital modulations sampled at the symbol rate (which may remain non circular). However, in the general case of oversampled (quasi)cyclostationary signals, $Q_{xa}(i, j, k, l) \neq Q_x(i, j, k, l)$ and the empirical fourth-order cumulant estimators generate a modified temporal mean of the fourth-order cumulants, which may induces catastrophic results at the output of the HO blind separators as it will be shown in the following.

3.3 HO blind source separation methods

Let us briefly recall that the HO blind source separators presented in [1-3] aim at separating the received sources from the blind identification of their steering vectors. This identification requires the prewhitening of the data, aiming at orthonormalizing the sources steering vectors so as to search for the latter through a unitary matrix U. If we note z(t) the whitened observation vector, the matrix U is chosen in [1-3] so as to optimize a contrast function, depending on the chosen method and theoretically function of the Q_7 elements, where Q_7 is the temporal mean of the quadricovariance of z(t). However, in practice, taking into account the previous results, the contrast function which is optimized by the separators [1-3] is not a function of the Q_z elements but is a function of the Q_{za} elements, where Q_{za} is the apparent temporal mean of the quadricovariance of z(t), which elements are defined by (3.5) where the indice x is replaced by z. The apparent temporal averaging operation keeps the multilinearity property and we can write

$$Q_{\mathcal{Z}(\mathbf{a})} = (A \otimes A'') Q_{m'(\mathbf{a})} (A \otimes A'')^{\mathsf{T}}$$

$$(3.6)$$

where *A*' is the (*P* x *P*) unitary matrix of the whitened sources steering vectors and where Q_m ' and $Q_{m'a}$ correspond to the true and apparent temporal mean of the quadricovariance of m'(t) respectively, the latter corresponding to the vector which components are the signals $m_i(t)\exp[j(\Delta \omega_i t + \phi_i)]$, $1 \le i \le P$, after a normalization. Thus, the performance of the separators [1-3] in (quasi)-cyclostationary contexts are directly dependent on the matrix $Q_{m'a}$ structure and more precisely on the values of the elements $Q_{m'a}(i, j, k, l)$, such that $(i, j, k, l) \ne (i, i, i, i)$, with respect to the $Q_{m'a}(i, i, i, i)$'s.

4. Q_m 'a STRUCTURE ANALYSIS

4.1 General case

In the general case of P (quasi)-cyclostationary and independent sources, the matrices $Q_{m'}$ and $Q_{m'a}$ are $(P^2 \times P^2)$ matrices which contain P^4 elements. However, if the only non zero elements of the matrix $Q_{m'}$ are the Pelements $Q_m(i, i, i, i)$, $1 \le i \le P$, it is not necessarily the case for the matrix $Q_{m'a}$ which elements $Q_{m'a}(i, i, j, j)$, $Q_{m'a}(i, j, i, j)$ and $Q_{m'a}(i, j, j, i)$ with $i \ne j$, $1 \le i, j \le P$, may also be non zero, in addition to the elements $Q_{m'a}(i, i, i, i)$, $1 \le i \le P$. Taking into account the symetries of the matrix $Q_{m'a}$, we find that $Q_{m'a}(i, i, j, j) = Q_{m'a}(i, j, i, i, j)$ and the analysis of the potential non nullity of the $Q_{m'a}$ elements can be deduced from the analysis of the elements $Q_{m'a}(i, i, i, i, j)$, $q_{m'a}(i, i, j, j)$ and $Q_{m'a}(i, j, j, i)$, with $i \ne j$ ($1 \le i, j \le P$), given from (3.5), by

$$Q_{m'a}(i, i, i, i) = Q_{m'}(i, i, i, i) +$$

$$2\sum_{\alpha \neq 0} R_{m'}^{\alpha}(i, i) R_{m'}^{-\alpha}(i, i) + \sum_{\beta \neq 0} C_{m'}^{\beta}(i, i) C_{m'}^{\beta}(i, i)^{*}$$
(4.1)

$$Q_{m'a}(i, i, j, j) = \sum_{\alpha \neq 0} R_{m'}^{\alpha}(i, i) R_{m'}^{-\alpha}(j, j)$$
 (4.2)

$$Q_{m'a}(i, j, j, i) = \sum_{\beta \neq 0} C_{m'}^{\beta}(i, i) C_{m'}^{\beta}(j, j)^{*}$$
 (4.3)

where $R_m^{\alpha}(i, j)$ and $C_m^{\beta}(i, j)$ are the coefficients associated to the cyclic frequencies α and β respectively in the Fourier serial expansion of the element (i, j) of the first and second correlation matrix of m'(t) respectively.

The expressions (4.1) to (4.3) show that in the general case of (quasi)-cyclostationary sources, the empirical fourth-order cumulant estimators may modify both the autocumulant temporal mean and the fourth order correlation of the sources, which, although statistically independent, may become apparently fourth-order correlated to each other [12]. Note that for second order circular sources such as the M-PSK (M > 2) modulated sources, the expression (4.3) is zero and only the cross terms (4.2) may be non zero. On the other hand, the expressions (4.2) and (4.3) show that for sources which do not share any non zero cyclic frequencies, (4.2) and (4.3)are zero, which means that a necessary condition to obtain apparent fourth-order correlation between two an statistically independent sources is that these two sources have common non zero cyclic frequencies.

4.2 Linearly modulated sources case

To analyse more precisely the conditions under which we obtain an apparent fourth-order correlation between two independent sources, we consider in this section the case of linearly modulated sources, which normalized complex signals are given by

$$m'_{i}(t) = \sum_{n} a_{in} v_{i}(t - t_{i} - nT_{i}) e^{j(\Delta \omega_{i}t + \phi_{i})}$$
 (4.4)

where, for each source i, $1 \le i \le P$, the associated symbols a_{in} are i.i.d random variables, independent of the symbols a_{jn} for $j \ne i$ and where t_i , T_i and $v_i(t)$ correspond to the initial time, the symbol duration and the real-valued pulse function of the source i such that $\langle E[|m'_i(t)|^2] \rangle = 1$. Under these assumptions, the coefficients $R_m^{\alpha}(i, i)$ and $C_m^{\beta}(i, i)$ appearing in (4.1) to (4.3) are given by

$$R_{m}^{\alpha}(i,i) = \mathbb{E}[|a_{i}|^{2}] \sum_{k} R_{i\nu}^{\alpha} \, \delta(\alpha - k/T_{i})$$
(4.5)

$$C_{m}^{\beta}(i, i) = \mathbb{E}[a_{i}^{2}] \sum_{k} R_{iv}^{\beta-2\Delta f_{1}} \,\delta(\beta-2\Delta f_{i}-k/T_{i}) \,e^{j2\phi_{i}} \,(4.6)$$

where $\Delta \omega_i \stackrel{\Delta}{=} 2\pi \Delta f_i$, $\delta(.)$ is the Kronecker symbol and where R_{iv}^{α} is defined by

$$R_{iv}^{\alpha} \stackrel{\Delta}{=} < \sum_{n} v_i (t - t_i - nT_i)^2 \exp(-j2\pi \alpha t) > \quad (4.7)$$

In the particular case of non filtered modulations, for which the pulse function $v_i(t)$ is equal to $1/\mathbb{E}[|a_i|^2]^{1/2}$ si $0 \le t \le T_i$ and zero elsewhere, the coefficient R_{iv}^{Ω} is zero for the non zero α and equal to $1/\mathbb{E}[|a_i|^2]$ for $\alpha = 0$, which implys that (4.5) and (4.6) take the form

$$R_m^{\alpha}(i,i) = \delta(\alpha) \tag{4.8}$$

$$C_{m}^{\beta}(i,i) = (\mathbb{E}[a_{i}^{2}] / \mathbb{E}[|a_{i}|^{2}]) \ \delta(\beta - 2\Delta f_{i}) \ e^{j2\phi_{i}}$$
(4.9)

Then, using (4.8) and (4.9) into (4.1) to (4.3), we obtain

$$Q_{m'a}(i, i, i, i) = c_i + [1 - \delta(\Delta f_i)] |\mathbf{E}[a_i^2]|^2 / \mathbf{E}[|a_i|^2]^2 \quad (4.10)$$

$$Q_{m'a}(i, i, j, j) = 0 (4.11)$$

$$Q_{m'a}(i, j, j, i) = [1 - \delta(\Delta f_i)] [1 - \delta(\Delta f_j)] \delta(\Delta f_i - \Delta f_j) \times e^{j2(\phi_i - \phi_j)} E[a_i^2] E[a_j^2]^* / E[|a_i|^2] E[|a_j|^2]$$
(4.12)

 $c_i \stackrel{\Delta}{=} \operatorname{cum}[a_i, a_i^*, a_i^*, a_i] / \operatorname{E}[|a_i|^2]^2$ (4.13)

The expression (4.10) shows that $Q_{m'a}(i, i, i, i) \neq c_i$ for sources i which are second order non circular with a non zero carrier residu. Besides, the expression (4.12) shows that two independent sources, linearly modulated, non filtered, second order non circular and having the same non zero carrier residu become apparently fourth-order correlated. In particular, under the latter conditions, two BPSK sources *i* and *j* are such that $c_i = -2$, $Q_{m'a}(i, i, i, i) = -1$ and $Q_{m'a}(i, j, j, i) = e^{j2(\phi_i - \phi_j)}$, which gives an apparent fourth-order correlation coefficient, $\rho_{4a}(i, j, j, i)$, with a maximal modulus and equal to one, where $\rho_{4a}(i, j, j, i)$ is the ratio between $Q_{m'a}(i, j, j, i)$ and the product of the square roots of $Q_{m'a}(i, i, i, i)$ and $Q_{m'a}(j, j, j, j)$. In these conditions, the two sources, being apparently strongly fourth-order correlated [12], cannot be separated by the separators [1-3], despite of their independence, as it is shown in the next section.

5. SEPARATORS PERFORMANCE

We expect poor performance at the output of the separators [1-3] for high values of the quantities $|\rho_{4a}(i, j, j, i)|$ and relatively unaffected performance for low values of $|\rho_{4a}(i, j, j, i)|$. The previous results are illustrated at figure 1 which shows, for two BPSK sources which pulse function is a ¹/₂ Nyquist filter, the SINRM (Maximal Signal to Interference plus Noise Ratio) [4-5] of the sources (averaged over 200 realizations) at the output of the JADE separator [1], as a function of the number of independent noise snapshots *K*, for sources which are oversampled by a 10 factor and having the same SNR. Note, under the previous assumption, the non separation of the sources, even independent, and the decreasing convergence speed as ($\Delta f_i - \Delta f_j$) decreases.

6. CONCLUSION

The use of the empirical fourth-order cumulant estimators in (quasi)-cyclostationary contexts may generate, in some cases, the non separation, by the separators [1-3], of non Gaussian sources despite of their statistical independence. This result is due to the fact that the information contained in the non zero cyclic frequencies of the sources are not taken into account in the statistics estimation, which shows all the importance of the fourth-order cumulant estimator choice.



Fig.1 - *SINRM as a function of K*, $\Delta f_1 = 0.1$, N = 5, P = 2, $SNR = 10 \ dB$, $\theta_1 = 50^\circ$, $\theta_2 = 100^\circ$, *ULA*

REFERENCES

- J.F. CARDOSO, A. SOULOUMIAC, "Blind Beamforming for Non Gaussian Signals", *IEE Proc-F*, Vol 140, N°6, pp 362-370, Dec 1993.
- [2] P. COMON, "Independent Component Analysis A new concept ?", *Signal Processing*, Vol 36, N°3, Special Issue On Higher Order Statistics, pp 287-314, Apr. 1994.
- [3] P. COMON, E. MOREAU, "Improved Contrast dedicated to Blind Separation in Communications", *Proc. ICASSP*, Munich (Germany), pp 3453-3456, Apr. 1997.
- [4] P. CHEVALIER, "On the Performance of Higher Order Blind Source Separation Methods", Proc. IEEE ATHOS Workshop on Higher Order Stat., Begur (Spain), pp 30-34, June 1995.
- [5] P. CHEVALIER, "Méthodes Aveugles de Filtrage d'Antenne", *Revue d'Electronique et d'Electricité*, SEE, N°3, pp 48-58, Sept. 1995.
- [6] P. CHEVALIER, "Performances des séparateurs aveugles de sources aux ordres supérieurs", *Proc. GRETSI*, Juan-les-Pins (France), pp. 297-300, Sept. 1995.
- [7] P. COMON, P. CHEVALIER, V. CAPDEVIELLE, "Performance of Contrast-Based Blind Source Separation", *IEEE SP Workshop on SP Advances in Wireless Communications*, SPAWC, Paris, pp 345-349, April 1997.
- [8] V. CAPDEVIELLE, P. CHEVALIER, P. CALVARY, P. COMON, "Comparaison des performances de plusieurs méthodes de séparation aveugle de sources aux ordres supérieurs", *GRETSI*, Grenoble, pp 115-118, Sept. 1997
- [9] P. CHEVALIER, A. MAURICE, "Constrained Beamforming for Cyclostationary Signals", *Proc. ICASSP*, Munich (Germany), pp 3789-3792, Apr. 1997.
- [10] A. FERREOL, P. CHEVALIER, "Higher Order Blind Source Separation using the Cyclostationarity Property of the Signals", *ICASSP*, Munich, pp 4061-4064, Apr. 1997.

- [11] P. MARCHAND, D. BOITEAU, "Higher Order Statistics for QAM Signals : A Comparison between cyclic and stationary representations", *EUSIPCO*, Triestre, pp 1531-1534, Sept 1996
- P. CHEVALIER, V. CAPDEVIELLE, P. COMON, "Behaviour of HO blind source separation methods in the presence of cyclostationary correlated multipaths", *IEEE SP Workshop on HOS*, Alberta (Canada), pp 363-367, July 1997. This work has been partly supported by D.R.E.T This work has been partly supported by D.R.E.T

This work has been partly supported by D.R.E.T

•

This work has been supported by D.R.E.T (French Administration)

This work has been supported by D.R.E.T (French Administration)