# A TRULY RECURSIVE BLIND EQUALIZATION ALGORITHM

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## ABSTRACT

This paper describes a new adaptive blind equalization algorithm based on a truly IIR structure that enables the correction of ISI over severely distorted channels. The recursive feedback filter is in lattice form to allow an easy monitoring of the filter stability. During blind training, the adaptation of the equalizer is carried out via the usual stochastic gradient algorithm by minimizing the Shtrom-Fan cost function, a CMA like functional robust to illconvergence. Once in steady state, the algorithm switches automatically into a classical DFE structure adapted via the DD-MMSE criterion. Simulation results show that this new equalizer outperforms most of the traditional blind FIR equalizers.

## I. INTRODUCTION

Several techniques have been devised to cope with channel distortions in high speed data transmissions. One common solution is to rely on the inverse filtering approach specifically dedicated to mitigate Inter Symbol Interferences (ISI) on band-limited digitally modulated signals. This approach simply consists in providing the traditional Maximum Likelihood receiver with a filter designed to inverse in some manner the channel transfer function [1]. In conventional modems, initial acquisition of the equalizer coefficients is usually accomplished using learning sequences transmitted periodically in time. However, it is sometimes desirable to allow receivers to start up without the help of the transmitter. The equalizer must resort only on the received samples and some mild assumptions on the input data to identify the channel.

Blind channel equalization (BCE) has received a lot of attention in the literature starting with the pioneering work of Sato [2] (see [3] for an extensive study of BCE). In this paper, we concentrate on the class of the Bussbang-type equalizers based on the inverse filtering approach. Bussbang equalizers operate on the received signal sampled at the baud rate through a transversal filter which coefficients are adaptively adjusted to minimize an explicit or implicit Higher Order Statistics cost function. Bussbang equalizers are known to suffer from a slow convergence speed due to the use of the stochastic gradient algorithm for adaptation and a potential ill-convergence inherent to the multimodality of the HOS functionals. Another major limitation of Bussbang algorithms lies on their FIR structure that shows poor performance in highly distorted channels.An attempt to overcome this limitation was recently proposed by Labat *et al.* from the initial work of Macchi [5, 6]. During blind training, the equalizer is composed of a recursive error prediction filter followed by a transversal Constant Modulus Algorithm (CMA) equalizer [4]. Once in steady state, the two filters are put in reverse order to lead to the traditional Decision Feedback Equalizer (DFE) adapted via the Decision Directed-MMSE criterion. This algorithm is satisfactory in most situations but it shows a functional weakness. The MMSE DFE equalizer separates the channel into its minimum and maximum phase components while the blind structure proposed in [6] equalizes the channel in amplitude and phase. Thus, the equalizer exhibits a transition period to compensate the influence of poles introduced by the whitening operation, which may lead to instability.

In this paper, we propose another alternative based on a true IIR structure that makes the equalizer able to cope with severely degraded channels when most of the traditional FIR equalizers fail. Unlike most of the recursive blind equalizers recently proposed, this equalizer integrates a direct and simple monitoring of the filter stability. The adaptation of the equalizer coefficients is carried out by minimizing the functional proposed by Shtrom-Fan [7] which shows a better robustness to ill-convergence than the CMA cost function. Once convergence is established, the equalizer switches automatically to the DFE mode via the DD-MMSE criterion. The structure includes an AGC and a second order PLL to deal with amplitude and phase residuals.

The paper is organized as follows: Section II describes the structure of the proposed equalizer and formulates the stochastic gradient update equation both in blind and tracking mode. Section III contains some computer simulation results and section IV presents a discussion followed by the conclusion.

## **II. PROPOSED ALGORITHM**

#### A - Equalizer structure

The introduction of poles in the equalizer transfer function gives IIR filters a better ability to equalize severely distorted channels than their FIR counterparts. However, adaptive IIR structures suffer from some major drawbacks such as a slow convergence speed, a possible ill-convergence and the potential instability of the recursive part. A lot of attention has been dedicated in recent years to overcome these limitations in the context of system identification [8].

In particular, it was shown that the stability issue greatly depends on the structure of the recursive part. The lattice form appears as much more robust than the two other classical structures that are the direct and the parallel forms. Moreover, while monitoring the stability of direct form IIR filter is difficult and computationally expensive, stability monitoring of lattice structure is simple and requires almost no computation [9].

The equalizer proposed in this paper is derived from the structure studied in [9]. It is composed of a transversal feedforward filter followed by a feedback recursive lattice filter. Figure 1 depicts the block diagram of the equalizer. Monitoring the stability requires only to maintain the feedback coefficients below unity in module. This condition guarantees BIBO stability of the filter only in steady state. However, extensive simulations have shown that this condition performs well also during adaptation. Due to its recursive structure, the filter can be easily turned into a DFE filter simply by feeding back the decisions associated to the filter output.



Figure 1. Equalizer structure

## **B** - Equalizer output

To start, define the following vectors

$$H(n) = [h_0(n), h_1(n), \dots, h_{N-1}(n)]^T$$

$$K(n) = [-k_0(n), -k_1(n), -k_2(n) \dots -k_{M-1}(n)]^T$$
(1)

From the figure 1 and (1), the output of the equalizer is given by

$$y(n) = H^{T}(n-1)S(n) + K^{T}(n-1)X(n)$$
(2)

where  $S(n) = [s(n), s(n-1), s(n-2) \dots s(n-N+1)]^T$  is the input vector and  $X(n) = [x_0(n), x_1(n), x_2(n) \dots x_{M-1}(n)]^T$  the state vector of the lattice filter updated through the equation

$$X(n+1) = QX(n) + Ru(n)$$
(3)

where 
$$R = [1, k_0, k_1 \dots k_{M-2}]^T$$
 (4)

$$Q = \begin{bmatrix} -k_0 & -k_1 & -k_2 & \dots & \dots & -k_{M-1} \\ (1-k_0^2) & -k_0k_1 & -k_0k_2 & \dots & \dots & -k_0k_{M-1} \\ 0 & (1-k_1^2) & -k_1k_2 & -k_1k_3 & \dots \\ 0 & 0 & 0 & (1-k_{M-2}^2) & -k_{M-2}k_{M-1} \end{bmatrix}$$
(5)

#### C- Starting-up mode

The equalizer coefficients are adjusted to minimize the Shtrom-Fan cost function [7] defined as

$$SF(W_l) = \frac{1}{E^2 \{|c_i|^2\}} E^2 \{|y_i|^2\} - \frac{E\{|y_i|^4\} - 2E^2 \{|y_i|^2\}}{E\{|c_i|^4\} - 2E^2 \{|c_i|^2\}}$$
(6)

with 
$$W_l(n) = [H^T(n), K^T(n)]^T$$
 (7)

The corresponding stochastic gradient algorithm is given by

$$W_{l}(n+1) = W_{l}(n) - \mu \frac{\partial}{\partial W_{l}} \overline{SF}(W_{l}) \bigg|_{W_{l} = W_{l}(n)}$$
(8)

with 
$$\frac{\partial}{\partial W}\overline{SF}(W) = ((\alpha + 2\beta)E\{|y(n)|^2\} - \beta|y(n)|^2)\frac{\partial|y_i|^2}{\partial W}$$
 (9)

where 
$$\alpha = \frac{1}{E^2 \{ |c_i|^2 \}}$$
  $\beta = \frac{1}{E \{ |c_i|^4 \} - 2E^2 \{ |c_i|^2 \}}$  (10)

The major difficulty to adaptively compute the equalizer coefficients lies in the complexity of the lattice feedback structure. As proposed in [9], a solution is to split the derivation into two parts

$$\frac{\partial}{\partial W_l} \overline{SF}(W) = \left[ \frac{\partial W_d}{\partial W_l} \right]^T \frac{\partial}{\partial W_d} \overline{SF}(W)$$
(11)

where  $W_d = [H^T, A^T]^T$  is the direct form associated with the lattice filter (see [9] for the relation between the two forms). Using typical results in complex derivation, the derivative of the output power of the filter in (9) can be expressed as

$$\frac{\partial}{\partial W} |y(n)|^2 = Re\left\{ y^*(n) \frac{\partial}{\partial W_R} y(n) \right\} + jRe\left\{ y^*(n) \frac{\partial}{\partial W_I} y(n) \right\}$$
(12)

It remains to differentiate the output of the equalizer with respect to the real and the imaginary part of the equalizer coefficients. At this stage, we apply the standard slow convergence property in differentiation to obtain

$$\frac{\partial}{\partial W_d} \overline{SF}(W) = \begin{vmatrix} \frac{\partial}{\partial H} \overline{SF}(W) \\ \frac{\partial}{\partial A} \overline{SF}(W) \end{vmatrix} = \begin{bmatrix} S_A(n) \\ -Y_A(n) \end{bmatrix}$$
(13)

with 
$$S_A(n) = [\xi_0(n), ..., \xi_{N-1}(n)]^T$$
 (14)

$$Y_A(n) = [\Psi_1(n), \dots, \Psi_M(n)]^{T}$$
(15)

$$\xi_{i}(n) = s^{*}(n-i) - \sum_{\substack{p=1\\N}}^{M} a_{p}(n-1)\xi_{i}(n-p)$$

$$\psi_{i}(n) = y^{*}(n-i) - \sum_{p=1}^{N} a_{p}(n-1)\psi_{i}(n-p)$$
(16)

We now rely on the on-line approximation described in [9] which gives from (13) and a simplification of the term  $\partial W_i / \partial W_d$  the following expression for the gradient (11)

$$\frac{\partial}{\partial W_l} \overline{SF}(W_l) = \begin{vmatrix} \frac{\partial}{\partial H} \overline{SF}(W_l) \\ \frac{\partial}{\partial K} \overline{SF}(W_l) \end{vmatrix} = \begin{bmatrix} S_A(n) \\ -X_A(n) \end{bmatrix}$$
(17)

where  $X_A(n)$  is the state vector of the lattice filter with input y(n) and  $S_A(n) = [\xi_0(n), ..., \xi_{N-1}(n)]^T$  as defined above.

The computation of the vector  $S_A(n)$  requires to update at each iteration *N* recursive filters. In this equalizer we use once again the slow convergence assumption to simplify the algorithm replacing  $S_A(n)$  by the following vector

$$\overline{S}_A(n) = \left[\xi_0(n) \dots \xi_0(n-N+1)\right]^T$$
 (18)

Finally, the stochastic gradient update equation during blind startup takes the general form

$$\begin{bmatrix} H(n) \\ K(n) \end{bmatrix} = \begin{bmatrix} H(n-1) \\ K(n-1) \end{bmatrix} - \mu S(n) \begin{bmatrix} \overline{S}_A(n) \\ -X_A(n) \end{bmatrix}$$
(19)

• 
$$\overline{S}_A(n) = \left[\xi(n) \dots \xi(n-N+1)\right]^T$$
 (20)

where 
$$\begin{cases} X_{S}(n) = Q(n-1)X_{S}(n-1) + R(n-1)s^{*}(n) \\ \xi(n) = K^{T}(n-1)X_{S}(n) + s^{*}(n) \end{cases}$$

• 
$$X_A(n) = Q(n-1)X_A(n-1) + R(n-1)y^*(n)$$
 (22)

• 
$$S(n) = y(n)((\alpha + 2\beta)E\{|y(n)|^2\} - \beta|y(n)|^2)$$
 (23)

The output power is estimated according to

$$P_2(n) = \lambda_2 P_2(n-1) + (1-\lambda_2) |y(n)|^2$$
 where  $0 < \lambda_2 < 1$  (24)

## **D** - Commutation rule

After convergence, the algorithm is switched into a classical DFE equalizer adapted via the DD-MMSE criterion. The commutation rule is based on the estimated value of the Shtrom-Fan error and the MSE error according to

$$D(n) = \lambda_d D(n-1) + (1 - \lambda_d) |y(n) - \hat{y}(n)|^2 \text{ where } 0 < \lambda_d < 1$$
(25)

$$E_{SF}(n) = \frac{1}{E^2 \{|c_i|^2\}} P_2^2(n) - \frac{P_4(n) - 2P_2^2(n)}{E\{|c_i|^4\} - 2E^2 \{|c_i|^2\}}$$
(26)

where 
$$P_4(n) = \lambda_4 P_4(n-1) + (1-\lambda_4)|y(n)|^4$$
 with  $0 < \lambda_4 < 1$  (27)

When one of these two parameters decreases below a determined threshold, the equalizer switches into the DFE mode. In the same way, if one of the parameters increases up to a another threshold, the equalizer is switched back into the blind set-up.

## E - Tracking mode

Once in steady state, the equalizer coefficients are updated to minimize the MSE cost function defined as follows

$$\varepsilon(W_l) = E\left\{ \left| y_i - \hat{c}_i \right|^2 \right\}$$
(28)

where  $\hat{c}_i$  is the output of the decision device at time *i*. Following a similar approach to that developed for the blind adaptation, we can show that the updating equation in tracking mode is strictly identical to (17) simply taking

$$S(n) = (y(n) - \hat{c}(n))y(n)$$
 (29)

It is worth noting that the equalizer is able to work in DFE mode even during blind setup. This mode shows better ability to converge quickly but at the expense of some stability problems.

# **F** - Equalizer constraining

The all-zero setting appears as a undesirable local minimum of the Shtrom-Fan cost function. It is required to impose some constraint on the updating equation in order to avoid convergence to this unwanted stable minima. A common solution is to properly initialize setting all parameters to zero except for the last parameter of the feedforward filter [10]. This solution works well in most situations but fails for a distortions free channel, where the only solution to the Shtrom Fan function is to cancel the equalizer output. Another solution is to fix the central tap and update the remaining coefficients. However, this solution appeared as too constraining in simulation. For testing the equalizer, we used the first solution. It remains to develop a constraining condition adapted to the IIR structure.

#### **III. COMPUTER SIMULATIONS**

#### Simulation 1

(21)

This simulation intends to show the greater ability of the cascade IIR equalizer to deal with severely distorted channels in comparison with traditional FIR equalizers. A channel is generally regarded as severely distorted when its transfer function has one or several zeros onto or near the unit circle. We refine this condition, considering a channel as greatly distorted when the ZF equalizer required to remove completely ISI extends over several tens of symbols. The input data is an i.i.d. 16QAM sequence passed through a channel distortion filter with transfer function

$$H(z) = (1 - 0.97z^{-1})(1 - 1.4z^{-1})$$
(30)

The SNR is set to 30 dB. The cascade IIR equalizer has 10 taps in the forward part and 5 taps in the feedback part. As depicted in figure 2, the equalization in ZF sense requires more than 100 coefficients. As a result, no FIR equalizer was able to converge in less than 50000 symbol periods with a 60 taps filter. In the other hand, the cascade IIR equalizer converged after 30000 iterations.



Figure 2. ZF equalizer for simulation 1

## Simulation 2

We now compare the new equalizer and the traditional CMA equalizer in a typical transmission scenario. Consider a pulse-amplitude modulated system using a raised cosine pulse limited with roll off factor 0.2. The data input signal is an iid 16QAM. The channel is a 2 rays multipath channel with impulse response  $h(t) = \delta(t) - j0.7\delta(t - 4.25T)$ , with additive noise of SNR = 20 dB and a frequency offset equal to 1/1000T.

The received signal is low-pass filtered to remove out of band noise and is sampled at the baud rate. The cascade IIR equalizer has 10 taps in the forward part and 5 taps in the feedback part while the CMA is 30 taps long. The parameters are adjusted to achieve a maximum convergence speed for a reasonable residual ISI. Figure 3 depicts the output constellation of both equalizers after convergence. It is clear that the new equalizer outperforms the CMA in convergence speed and steady state performance.



#### IV DISCUSSION AND CONCLUSION

In this paper, we used a cascade IIR structure to develop a truly recursive blind algorithm able to cope with severely distorted channels. This equalizer overcomes two of the major limitations of IIR filters that are the ill-convergence and stability problems. The ill-convergence is avoided using the Shtrom-Fan cost function which shares with the CMA functional the advantage of being independent of the signal phase but with a much better robustness to ill-convergence. This property was only discussed in the case of FIR structures in [7]. The use of the Shtrom-Fan functional into a recursive structure much more sensible to ill-convergence than FIR filters tends to confirm this property. Another important aspect of the new equalizer lies on the lattice structure of the recursive filter. This structure allows to control in a simple manner the stability of the equalizer, a recurrent problem in blind IIR equalization.

Simulations show that the cascade IIR equalizer outperforms most of the traditional FIR equalizers in two aspects: first, it is able to reduce ISI on severely distorted channels where FIR algorithms fail. Secondly, the cascade IIR filter requires much less coefficients than FIR equalizers in order to inverse the minimum phase part of the channel. Moreover, the truly recursive structure of the cascade IIR equalizer makes possible to switch naturally to DFE mode and even to work with a DFE setting during blind start-up. From this point of view, the new algorithm is more coherent than the SA-DFE recently proposed in [6]. Indeed, unlike the SA-DFE, the cascade IIR filter converges directly to the DD-MMSE solution.

It remains to demonstrate properly the robustness of the new algorithm. However, extensive simulations have shown the good behavior of the new algorithm. Improvements are under study in the field of convergence speed, stability monitoring and equalizer constraining.

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