NONMAXIMALLY DECIMATED FILTERBANK BASED PRECODER / POST-EQUALIZER FOR BLIND CHANNEL IDENTIFICATION AND OPTIMAL MMSE EQUALIZATION

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ABSTRACT

A novel nonmaximally decimated multirate filterbank structure is proposed for blind identification of communication channels. This structure is shown to be very similar to a form proposed earlier in the literature. It is presented that the proposed blind channel identification algorithm is not sensitive to the characteristics of unknown channel, including mixed phase and zeros on the unit circle. An optimal minimum mean square error based linear equalizer using the blind channel identification scheme is investigated. It is shown that the proposed system outperforms the existing zero-forcing blind equalization algorithms in literature. It can simultaneously cancel the intersymbol interference (ISI) and suppress the noise enhancement. The reconstructed signal to noise ratio is maximized by the proposed algorithm. Simulation results show the superior performance and robustness of the proposed blind identification and equalization scheme.

1. INTRODUCTION

Telecommunication performance through a channel with multipath or high level of attenuation is severely degraded by intersymbol interference. It is a common problem in many communication systems, including wireless mobile communication applications. The blind channel identification and equalization is of significant interest in the communications field due to its performance insensitivity to the channel properties. Conventional blind channel estimation techniques rely on high-order statistics of the stationary channel output. The second-order cyclostationarity based approaches were proposed by Xu and Tong which make use of fractional oversampling [1,2]. Recently, Xia proposed a new precoding scheme to overcome ISI using nonmaximally decimated multirate filterbank as an ideal FIR equalizer [3]. Giannakis forwarded a new filterbank precoder for blind channel identification and equalization [5]. In fact, Xia and Giannakis independently came up with the same precoding structure from different perspectives.

A nonmaximally decimated multirate filterbank precoder / post-equalizer structure is presented in this paper. It is shown, that the proposed approach turned out to be the same as Giannakis' with respect to the problem of blind channel identification. Additionally, we propose an optimal MMSE linear FIR combiner as a post-equalizer. It jointly maximizes the overall signal to noise ratio of blind identification and equalization steps.

2. NONMAXIMALLY DECIMATED FILTERBANK AS A PRECODER / DECODER: BLIND CHANNEL IDENTIFICATION

Fig. 1 displays a nonmaximally decimated multirate filterbank precoder structure at the transmitter. Similarly, Fig. 2 depicts a nonmaximally decimated multirate filterbank decoder / post-equalizer structure. In these two figures, $\downarrow K$ represents a down-sampling by a factor of K and $\uparrow N$ represents an up-sampling by a factor of N.

In Fig. 1, $x(n)$ is the independent identically distributed (i.i.d.) information sample sequence. $\tilde{x}(n)$ is the precoded data sequence which is transmitted through the channel. $\hat{x}(n)$ is the received data sequence at the receiver.

In Fig. 2, the received data samples go through a decoder / post-equalizer step. The decoder consists of a nonmaximally decimated filterbank. $\overline{x}(n)$ are post-equalized / reconstructed data samples. Whenever the transmission system has perfect reconstruction (PR) property, $\overline{x}(n)$ will be the same as input $x(n)$ with some time delay [4].

The intersymbol interference transmission channel model used in this paper is the baseband discrete model which combines the effects of transmitter shaping filter, transmission channel and receiver's anti-aliasing filter. In this bandlimited ISI channel, there are ISI and additive white Gaussian noise. It is assumed that the unknown ISI channel is a linear time invariant (LTI) system. Also assume that the impulse response of the transmission channel is of order L, $h = [h_0\,,\, h_1\,,\, h_2, \ldots,\, h_L]$. $H(z)$ is its z transform function defined as $H(z) = \sum_{i=0}^{L} h_i z^{-i}$. Without any loss of generality, we can assume that h_0 must not be zero. The received samples $\hat{x}(n)$ can be expressed as a function of the transmitted sequence $\tilde{x}(n)$ as

$$\hat{x}(n) = \sum_i \tilde{x}(i) h_{n-i} + v(n) \tag{1}$$

where $\{v(n)\}$ are samples of additive white Gaussian noise.

Using multirate filterbank theory [4], the channel's transfer function $H(z)$ can be decomposed using polyphase representation as

$$H(z) = \sum_{i=0}^{N-1} H_i(z^N) z^{-i} \qquad (2)$$

where $H_i(z) = \sum_n h(Nn+i)z^{-n}$. Let us form blocks of transmitted and received data samples with a size of N as $\tilde{X}_k$ and $\hat{X}_k$, respectively. Then, Eq.(1) can be expressed in a block form as

$$\hat{X}_k(z) = H(z)\tilde{X}_k(z) + V_k(z). \qquad (3)$$

$H(z)$ is an $(N \times N)$ pseudo-circulant matrix as

$$H(z) = \begin{bmatrix} H_0(z) & z^{-1}H_{N-1}(z) & & \cdot & \cdot & z^{-1}H_1(z) \\ H_1(z) & H_0(z) & & \cdot & \cdot & z^{-1}H_2(z) \\ H_2(z) & H_1(z) & H_0(z) & \cdot & & z^{-1}H_3(z) \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot & \cdot \\ H_{N-1}(z) & H_{N-2}(z) & & \cdot & \cdot & H_0(z) \end{bmatrix}$$

In the proposed precoder structure, every block of K information data symbols X_k is precoded / pre-equalized into a block of N transmitted samples $\tilde{X}_k$ by going through a nonmaximally decimated filterbank precoder at the transmitter. This precoding process can be mathematically expressed as a matrix operation as

$$\tilde{X}_k(z) = F(z)G(z)X_k(z) \qquad (4)$$

where $F(z)$ and $G(z)$ are polyphase matrices of multirate analysis and synthesis filterbanks, respectively.

It is well known in multirate filterbank theory, that there can not be any perfect reconstruction (PR) solution for the case of $K > N$. Therefore, K must be less than N. A nonmaximally decimated filterbank with $K < N$ puts $(N-K)$ zeros as dummy inputs to the channel and $(N \times N)$ $H(z)$ matrix is reduced to $H_{part}(z)_{N\times K}$ where $H_{part}(z)_{N\times K} = H(z)_{(N\times K)}$.

At the receiver side, received signal samples form a block $\hat{X}_k$ of size N. Whenever analysis and synthesis filterbanks are set with a proper time delay, their polyphase matrices $G(z)$ and $F(z)$ become identity matrices of size K and N, respectively. Then, the received signal is expressed in a matrix form as

$$\begin{aligned} \hat{X}_k(z) &= H(z)F(z)[G(z);0_{(N-K)\times K}]^T X_k(z) + V_k(z) \\ &= H_{part}(z)X_k(z) + V_k(z). \end{aligned} \qquad (5)$$

Assume that the following constraints are set as

$$K < N; K \geq L+1; N \geq K+L. \qquad (6)$$

Obviously, any arbitrary integer values of K, N which satisfy the conditions of Eq.(6) will generate a constant $N \times K$ matrix $H_{part}(z)$. The decoder / post-equalizer section of the proposed system becomes a constant $K \times N$ matrix. It is much simpler than the Smith form operation used in [3] which is a $K \times N$ polynomial matrix.

Assume that we pick $K = L+1$ and $N = K+L$ as the critical value of (K, N). Then, $H_{part}(z)$ is a constant matrix as shown

$$H_{part}(z)_{(N\times K)} = \begin{bmatrix} h_0 & 0 & 0 & \cdot & \cdot & 0 \\ h_1 & h_0 & 0 & \cdot & \cdot & 0 \\ h_2 & h_1 & h_0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ h_L & h_{L-1} & h_{L-2} & \cdot & \cdot & h_0 \\ 0 & h_L & h_{L-1} & \cdot & h_2 & h_1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 & h_L \end{bmatrix}$$

Now, let us denote the received signal block of size N as $\hat{X}_k(z)$ and assume that AWGN samples have a variance of σ_n^2. The information symbols are assumed to have variance of σ_x^2. The AWGN is assumed to be independent of the information sequence. The autocorrelation matrix $R_{\hat{X}_k(z)}$ can be derived as

$$\begin{aligned} R_{\hat{X}_k(z)} &= \mathbf{E}[\hat{X}_k(z)\hat{X}_k(z)^{*T}] \\ &= \sigma_x^2 H_{part}(z)[H_{part}(z)]^{*T} + \sigma_n^2 I_{N\times N} \end{aligned} \qquad (7)$$

where $\mathbf{E}$ is the expected value and * denotes the conjugate operation. Using the structure of matrix $H_{part}(z)$, The first raw, $R_{\hat{X}_k}(1)$, of autocorrelation matrix $R_{\hat{X}_k}$ is therefore in the form of

$$R_{\hat{X}_k}(1) = \sigma_x^2 h_0 \left[\; h_0 + \frac{\sigma_n^2}{\sigma_x^2 h_0} \quad h_1 \quad h_2 \quad \cdot \quad h_L \quad 0 \quad \cdot \; \right]$$

Hence, $h_0, h_1, h_2, ..., h_L$ can be blindly identified from the autocorrelation function $R_{\hat{X}_k}(1)$ as

$$\begin{aligned} \hat{h}_0 &= \{\frac{1}{\sigma_x^2}[R_{\hat{X}_k}(1,1) - \sigma_n^2]\}^{\frac{1}{2}} \\ \hat{h}_i &= \frac{1}{\hat{h}_0 \sigma_x^2} R_{\hat{X}_k}(1,i), \; 1 \leq i \leq L \end{aligned} \qquad (8)$$

where $SNR_{in} = \frac{\sigma_x^2}{\sigma_n^2}$ is the signal to noise ratio. Assume that the transmitted signal variance is normalized, $\sigma_x^2 = 1$, and SNR_{in} is estimated. Then, all of the channel coefficients can be blindly identified. The second part of Eq. (8) is the same form as presented in [5].

3. OPTIMAL MMSE EQUALIZATION

As we saw in the previous section, the channel parameters can be blindly identified by the proposed nonmaximally decimated filterbank based precoder structure. After identifying the channel coefficients, the ISI distorted received symbols can be equalized in order to recover the transmitted signal. The ployphase matrices of nonmaximally decimated multirate analysis and synthesis filterbank $\tilde{F}(z)$ and $\tilde{G}(z)$ are set to be identity matrices with sizes of N and K, respectively. Let's assume that a $(K \times N)$ decoder (equalizer) matrix W_{decode} is used. The decoded or reconstructed symbol $\overline{X}_k$ is therefore derived as

$$\begin{aligned} \overline{X}_k(z) &= \tilde{G}(z)W_{decode}(z)\tilde{F}(z)\hat{X}_k(z) \\ &= W_{decode}(z)H_{part}(z)X_k(z) + W_{decode}(z)V_k(z) \end{aligned}$$

E_k is denoted as the reconstruction error vector, which is defined as the difference between the reconstructed data vector and the transmitted data vector; $E_k(i) = x(i) - \hat{x}(i)$ for $1 \leq i \leq K$. Then,

$$\begin{aligned} E_k(z) &= X_k(z) - \overline{X}_k(z) \\ &= [W_{decode}(z)H_{part}(z) - I_{(K\times K)}]X_k(z) \\ &\quad + W_{decode}(z)V_k(z) \end{aligned} \quad (9)$$

where $I_{(K\times K)}$ is an identity matrix of size K. From Eq. (9), it is easy to observe that the first part is due to the intersymbol interference, while the second part is caused by the AWGN noise enhancement. The perfect reconstruction condition implies that there is no ISI such that

$$W_{decode}(z)H(z)F(z)[G(z); 0_{(N-K)\times K}]^T = I_{(K\times K)} \quad (10)$$

In [3], $G(z)$ is set to be $I_{(K\times K)}$. A methodology to find a $(K \times N)$ polynomial matrix $W_{decode}(z)$ in order to obtain a perfectly reconstructed $\overline{x}(n)$ was suggested. It was shown that the $(K \times N)$ polynomial matrix can be derived using the Smith form operations. This solution is indeed a zero-forcing decoder or post-equalizer. The noise enhancement problem of a post-equalizer, which is the second part of Eq. (9) was not considered. Although one can get the $(K \times N)$ PR FIR polynomial matrix through the computationally involved Smith form operation, the SNR_{out} degradation of the system can be very significant.

The equalization scheme of [5] could not suppress any noise enhancement. It is a zero-forcing PR post-equalization method. Additionally, $G(z)$ matrix is set to be an identity matrix of size K. The precoder structures at the transmitter are indeed the same in both [3] and [5].

The Eq. (9) is valid for any integer values of (K, N) in a multirate filterbank structure (analysis / synthesis filterbank configuration). It is a polynomial relationship in z. As we select (K, N) satisfying the conditions of Eq. (6), Eq. (9) becomes a constant matrix relationship.

Due to the interpretation of Eq. (9), we attempt to obtain the optimal MMSE based one tap $(K \times N)$ constant matrix solution to improve SNR_{out}. This MMSE based nonmaximally decimated filter bank decoder attempts to cancel ISI while not enhancing the noise such that the maximum value of SNR_{out} after the post-equalizer is achieved. It is quite similar to the conventional MMSE linear equalizer solution.

Denote $W_{decode} = [W_1, W_2, ..., W_K]^T_{K\times N}$, each raw $W_i = [W_{i,1}, W_{i,2}, ..., W_{i,N}]$; and $H_{part} = [H_1, H_2, ..., H_K]$. Then, the total ISI plus noise energy is written as

$$\begin{aligned} E_{error} &= \mathbf{E}[X_k^T(W_{decode}H_{part} - I_{(K\times K)})^T(W_{decode}H_{part} \\ &\quad - I_{(K\times K)})X_k] + \mathbf{E}[V_k^T W_{decode}^T W_{decode} V_k] \\ &= \sum_{i=1}^{K} \sigma_x^2 \sum_{j=1}^{K} (W_i H_j - \delta_{i-j})^2 + \sigma_n^2 \sum_{i=1}^{K} W_i W_i^T \end{aligned} \quad (11)$$

Now, we want to obtain an optimal $W_{decode}(i, j)$ such that ISI plus noise is minimized to get a maximum SNR_{out}. Since $E_{error} = \sum_{i=1}^{K} E_{error}(i)$, and $E_{error}(i)$ is a function of weight vector W_i only. We can derive optimal weight vector to get minimum $E_{error}(i)$ as

$$E_{error}(i) = \sigma_x^2 \sum_{j=1}^{K} (W_i H_j - \delta_{i-j})^2 + \sigma_n^2 W_i W_i^T \quad (12)$$

To minimize $E_{error}(i)$, we take a derivative of the equation above. After some algebraic operations, the optimal weight vector W_i is written as

$$W_i^{opt} = \left[(\sum_{j=1}^{K} H_j H_j^T) + \frac{\sigma_n^2}{\sigma_x^2} I_{N\times N} \right]^{-1} H_i \quad (13)$$

Then, the maximum SNR_{out} is expressed as

$$\mathrm{SNR_{out}^{opt}} = \frac{\mathrm{K}}{\sum_{\mathrm{i=1}}^{\mathrm{K}} \sigma_{\mathrm{x}}^2 \sum_{\mathrm{j=1}}^{\mathrm{K}} (\mathrm{W_i H_j} - \delta_{\mathrm{i-j}})^2 + \sigma_{\mathrm{n}}^2 \mathrm{W_i W_i^T}} \quad (14)$$

where W_i, $1 \leq i \leq K$, are the optimal weight vectors. The optimal MMSE based nonmaximally decimated filterbank post-equalizer maximizes the SNR_{out} at the output of decoder/equalizer. It is shown that it outperforms the zero-forcing PR equalizer based system used in [3] and [5]. This improvement makes the filterbank based precoder structure perform successfully for the blind identification and equalization of unknown channels.

4. SIMULATIONS AND PERFORMANCE COMPARISONS

In this section, we perform unknown channel identification and equalization using the proposed algorithm for a few channel types.

The unknown linear time invariant channels, which are going to be investigated, are

(i) The channel $h = \frac{1}{9}[1\,,\,2\,,\,2.5\,,\,2\,,\,1]$ with four zeros on the unit circle as used in [3]. 500 BPSK symbols, $\{x(n)\}$ are generated and transmitted through the unknown channel. In this case, the order of unknown channel is 4. We select the critical values of $K = 5$, $N = K + L = 9$. $H_{part}(z)$ becomes a constant matrix.

Using the proposed blind channel identification algorithm, 100 and 500 Monte Carlo simulation runs are performed for an SNR_{in} of 20 dB. The estimated channel coefficients, $\hat{h}$, are found as [0.111 , 0.225 , 0.277 , 0.222 , 0.103], and [0.111 , 0.223 , 0.277 , 0.221 , 0.110]. It is observed that the blind identification is performed well. The coefficient h_0 is estimated directly which is not the case in [5].

After a successful channel identification step, the channel equalization is performed. The channel equalization is performed using the new proposed optimal MMSE based linear FIR combiner. The overall reconstruction signal to noise ratio after post-equalization is displayed in Fig. 3. It is shown that the optimal MMSE based FIR equalizer outperforms the zero-forcing equalizer. It is much more robust when SNR is low.

(ii) The channel with 7 zeros at $0.2; \pm1.5; \pm0.5j; 0,2783 \pm j * 0.3488$, $h = [1\,,\,-0.757\,,\,-1.690\,,\,1.473\,,\,-1.183\,,\,0.505\,,\,-0.175\,,\,0.022]$ in [5]. We could select any integer $K \geq 8; N \geq K+7$. For a simple demonstration we pick $(K, N) = (8, 15)$. In this case, $H_{part}(z)$ becomes a constant matrix. 500 and 1000 Monte Carlo simulations are run to blindly estimate the unknown channel. Assume that the SNR_{in} is 20 dB, the estimated channel coefficients, $\hat{h}$, are found as $[0.999\,,\,-0.754\,,\,-1.691\,,\,1.454\,,\,-1.170\,,\,0.520\,,\,-0.189\,,\,0.045]$ and $[0.999\,,\,-0.754\,,\,-1.691\,,\,1.468\,,\,-1.185\,,\,0.508\,,\,-0.176\,,\,0.021]$, respectively.

Using the identified channel coefficients, the optimum MMSE based FIR equalization is performed. The overall reconstruction SNR_{out} is displayed in Fig. 4. It is seen that the proposed optimal MMSE based post-equalizer is much more robust than the zero-forcing equalizer proposed in [5].

5. CONCLUSIONS

In this paper, an efficient nonmaximally decimated multirate filterbank structure is proposed for the blind identification of unknown transmission channels. An optimal minimum mean square error based linear equalizer using the blind channel identification scheme is investigated. Simulation results show that the performance and robustness of the proposed blind identification and equalization scheme are superior to the scheme proposed in [3] and [5].

References

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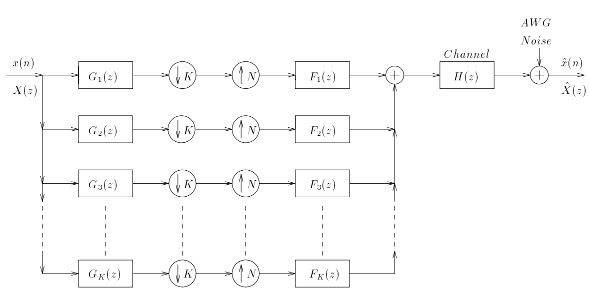

Figure 1: The proposed Nonmaximally Decimated Filterbank Based Precoder / Pre-equalizer structure for Blind Channel Identification and Equalization.

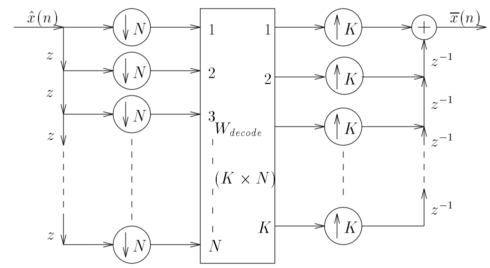

Figure 2: The proposed Nonmaximally Decimated Filterbank Based Decoder / Post-equalizer structure for Blind Channel Identification and Equalization.

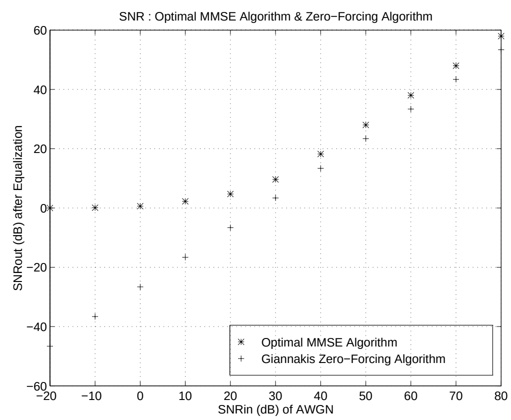

Figure 3: SNR_{out} performance of the proposed optimal MMSE based equalizer and zero-forcing equalizer used in [5] vs SNR_{in}. For (5,9) case of transmission channel given in [3].

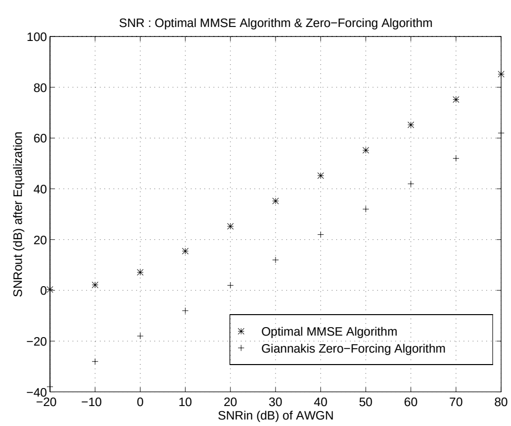

Figure 4: SNR_{out} performance of the proposed optimal MMSE based equalizer and zero-forcing equalizer used in [5] vs SNR_{in}. For (8,15) case of transmission channel given in [5].