# A COMPARATIVE PERFORMANCE EVALUATION OF DMT (OFDM) AND DWMT(DSBMT) BASED DSL COMMUNICATIONS SYSTEMS FOR SINGLE AND MULTITONE INTERFERENCE

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## ABSTRACT

Multicarrier modulation techniques have been proposed in the digital subscriber line (DSL) applications. In this paper, the performance of DMT (OFDM) and DWMT (DSBMT) techniques for single and multitone interference are investigated. It is shown that a DMT system is sensitive to the location of narrow band interference. DMT technique needs additional narrow band interference canceller before forward FFT transform for performance improvements. In DSBMT technique, due to a limited spectral overlap between its subcarriers, single (multi)- tone interference could effect only a few subchannels which correspond to these interferences. DSBMT has a superior performance than DMT.

## 1. INTRODUCTION

In the field of digital subscriber line (DSL) communications, multicarrier modulation systems such as Orthogonal Frequency Division Multiplexing (OFDM) techniques namely Discrete MultiTone (DMT) and Discrete Wavelet Multicarrier Transceiver (DWMT) or Discrete Subband Multicarrier Transceiver (DSBMT) are employed. DMT uses Discrete Fourier Transform as its modulation basis [1,2]. DMT has been adopted as an ANSI standard for ADSL applications [3]. Discrete Wavelet Multicarrier Transceiver (DWMT) or Discrete Subband Multicarrier Transceiver (DSBMT) was proposed in VDSL application [4]. The basic structure of a multicarrier modulation based system is displayed in Fig. 1. The modulation filters used in the transceiver form an orthogonal basis function set. This kind of synthesis/analysis discrete orthogonal basis function design have been investigated in the theory of multirate filter bank [7,8].

## 2. PERFORMANCE OF DMT (OFDM) BASED SYSTEM FOR SINGLE AND MULTITONE INTERFERENCE

In ADSL application scenarios, the transmitted signal frequency range may be up to 1 or 2 MHz. In VDSL application, the signal frequency range is even higher. There will be some narrow band interference such as AM radio stations and other single and multitone interference sources. It is naturally assumed that a DMT based system can shut down one subchannel corresponding to that narrow band or single tone interference. Will this kind of technique be effective? We examined this problem carefully in this section.

It is well known that IFFT/FFT keep their orthogonality only to their N tones (or N points) on the unit circle in Z plane. The modulation basis functions spectrally overlap. There is a -13 db between the passband bin and the adjacent bin. Fig. 2 displays the frequency response of FFT basis functions $g_{15}(n), g_{16}(n), g_{17}(n), g_{18}(n)$ which correspond to around $\frac{\pi}{4}$ of 128 point FFT. If the single-tone interference is perfectly located on one of these N points on the unit circle, then the N size IFFT/FFT could decompose the single tone interference into only one corresponding subchannel or DFT bin. That is what one usually assumes. Whenever the narrow band or single tone interference is not perfectly located on one of these N frequency points, say a little bit away from DFT bin points, or the sampling frequency is drafting a little bit, then the single tone interference energy spreads into many subchannels. The amount of interference leakage depends on the location and the strength of the narrow band interference. It is shown that this kind of interference will significantly degrade the DMT performance.

In a DMT transceiver system, the received signal is the composite of desired signal, additive white Gaussian noise and single (multi)-tone interference. We assume that a single-tone interference at the frequency $\omega_{st}$ with random phase $\phi_{st}$ which is uniformly distributed from 0 to $2\pi$. The energy of signal to single-tone interference ratio is defined as Signal to Interference Ratio ($SIR$). Let's assume the magnitude of the single-tone interference is $A_{st}$, and the transmitted signal energy is normalized to 1. Then, the sampled discrete single-tone interference signal is

$$I_i^{st} = A_{st} cos(\omega_{st} i + \phi_{st}) \tag{1}$$

The sampled additive white Gaussian noise is assumed to have a variance of $\sigma_n^2$. Let's denote $V_k = [v_0, v_1, ..., v_{N-1}]$ as the noise sequence. Assume that a $N_f + 1$ tap TEQ pre-equalizer is used to reduce the duration of channel impulse response. The coefficients of the pre-equalizer is $W = [w_0, w_1, ..., w_{N_f}]$. Then the interference at the output of the pre-equalizer is

$$z_k = \sum_i A_{st} cos(\omega_{st} i + \phi_{st}) w_{k-i} \quad (2)$$

The noise component at the output of the pre-equalizer is

$$zv_k = \sum_i v_i w_{k-i} \quad (3)$$

Then the correlation between $zv_k$ and $zv_{k-l}$ is

$$\mathbf{E}[zv_k zv_{k-l}] = \sigma_n^2 (\sum_i w_i w_{i+l}) \quad (4)$$

and the correlation between $z_k$ and $z_{k-l}$ can be derived as below

$$\mathbf{E}[z_k z_{k-l}] = \frac{A_{st}^2}{2} \sum_m cos(\omega_{st} m) \sum_{\hat{k}} w_{\hat{k}} w_{\hat{k}-(l-m)} \quad (5)$$

where $\mathbf{E}$ stands for the expectation operation. It can be calculated easily because the auto-correlation of weight coefficients of the TEQ pre-equalizer is used in last equation. It is function of $l$.

Let's form a single-tone interference vector $\mathbf{z_{st}}$ and a noise component vector $\mathbf{zv}$ with size $N$. The interference plus noise vector is denoted as $\hat{\mathbf{z}}$. Then, the interference plus noise component after $FFT$ is written as

$$\mathbf{Z} = [Z_0, Z_1, ..., Z_{N-1}] \quad (6)$$

Where each component $Z_m$ is calculated as

$$Z_m = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \hat{z}_k cos(\frac{2\pi km}{N}) - j \sum_{k=0}^{N-1} \hat{z}_k sin(\frac{2\pi km}{N}) \quad (7)$$

Here $1 \leq m \leq \frac{N}{2}$. The real and imaginary parts of $Z_m$ is denoted as $Z_m^R$ and $Z_m^I$ respectively. It can be shown that the variance of the output at the real and imaginary subchannel can be derived as

$$\begin{aligned} \mathbf{E}[R_m^2] &= \mathbf{E}[(Z_m^R)^2](W_m^R)^2 - \mathbf{E}[2Z_m^R Z_m^I]W_m^R W_m^I \\ &\quad + \mathbf{E}[(Z_m^I)^2](W_m^I)^2 \\ \mathbf{E}[R_m^2] &= \mathbf{E}[(Z_m^R)^2](W_m^R)^2 + \mathbf{E}[2Z_m^R Z_m^I]W_m^R W_m^I \\ &\quad + \mathbf{E}[(Z_m^I)^2](W_m^I)^2 \quad (8) \end{aligned}$$

where $W_m^R$ and $W_m^I$ are the real and imaginary components of one tap complex frequency domain equalizer after the $FFT$. The signal to noise and interference ratio in each real and imaginary subchannel can be calculated using definitions $SNIR_m^R = \frac{1}{\mathbf{E}[R_m^2]}$ and $SNIR_m^I = \frac{1}{\mathbf{E}[I_m^2]}$, respectively. Let's define $SNIR$ as $[SNIR^R, SNIR^I]$. The theoretical $SNIR$ in DMT based system using Eq. 8 for AWG 26 loop plant with length of 9 kft for a signal tone interference and AWGN environment is displayed in Fig. 3. The simulation result of $SNIR$ is also plotted in the figure. The analysis and simulation results fit well. Here the single tone interference frequency $\omega_{st}$ is at $\frac{\pi}{4}$ which means that the single-tone locates exactly at the bin 65 of the 512 FFT, the interference is decomposed only in subchannel 65.

If the single tone interference frequency $\omega_{st}$ is shifted by a small amount at $\frac{\pi}{4} + 0.0001$, The interference leaks in all the other subchannels in Fig. 3. It is not sufficient to shut down only subchannel 65, the overall DMT system performance degrades severely.

As the narrow band interference is not an ideal single tone interference in a practical scenario, we considered the received interference with some bandwidth. Let's assume that the power spectrum density of this narrow band interference is flat with the center frequency at $\omega_{sto}$ and bandwidth $NB_w$. The interference at different frequency is assumed to be independent. The overall interference energy is fixed. In this case, the interference leakage component could be derived as a contribution from all these independent single tone interferences with different frequencies.

The $SNIR$ performance of DMT based system for a narrow band interference is investigated on a AWG 26 9kft loop plant. The $SIR$ used are 40 dB, 60 dB and 80 dB with center frequency at $\omega_{sto} = \pi/4$ and $NB_w = 0.01\pi$. Fig.4 displays the theoretical evaluation as well as simulation results. It is clear that the simulation results match the theoretical ones.

Several narrow band interference scenarios are evaluated. Fig.5 displays performance sensitivity for interference bandwidth when the center frequency $\omega_{sto}$ is at $\frac{\pi}{4}$. It is observed that whenever the bandwidth is decreased, the $SNIR$ performance approaches the ideal case where no interference leakage in other subchannels except subchannel 65.

## 3. PERFORMANCE OF DWMT (DSBMT) BASED SYSTEM FOR A SINGLE AND MULTITONE INTERFERENCE

Discrete Wavelet Multicarrier Transceiver (DWMT) or Discrete Subband Multicarrier Transceiver ( DSBMT) have been proposed in digital subscriber line applications. It is a better multicarrier modulation scheme compared to OFDM (DMT) due to its well localized subcarriers in frequency. In order to decrease the spectral overlapping of subcarriers in DMT, a Discrete Wavelet Multitone Transceiver uses better stopband properties for its subcarriers. Fig. 6 shows the frequency response of subcarriers 15, 16, 17 and 18 of a 64 equal-band cosine modulated filterbank.

As it is displayed in Fig.1, the transmitted supersymbol $y(n)$, which is the synthesis filter bank output, is transmitted through a highly attenuated channel $C(n)$. The received signal is the composite of desired signal, additive white noise and the single (multi)-tone interference. In order to reconstruct the transmitted signal, an adaptive linear combiner must be implemented at the output of the analysis filter bank. These synthesis and analysis filter banks are maximally decimated. The linear combiner will effectively equalize the channel, cancel the intersymbol interference and interchannel interference. Fig. 7 displays the block diagram of linear combiner after the analysis filter bank [4]. The computation complexity of this adaptive linear combiner depends on the performance of the side lobe of multicarrier filter banks. The filterbank used in our implementation have less than -80 dB stopband side lobe properties. We need only to eliminate the interchannel interference from one adjacent subcarrier.

In our simulation, a 64 equal band synthesis and analysis cosine modulated filter bank is designed. These filter banks have very good stopband properties. In the simulation, the multitone interference with $\omega_0 = \frac{\pi}{4} - 0.02$, $\omega_1 = \frac{\pi}{4} - 0.02 + 1e-3$ and $\omega_0 = \frac{\pi}{4} - 0.02 - 1e-3$ is injected at the receiver, with $SIR$ of 0 dB. After the initial training, the signal to noise and interference ratio is displayed in Fig. 8. The adaptive linear combiner works fine except subcarrier 16 where multitone interference locate. Subchannel 16 can be shut down. All other subchannels works fine. The over all transmission doesn't suffer severe performance loss in the presence of strong single tone interference. In another multitone interference scenario, multitone interferences are at $\omega_0 = \frac{\pi}{4}$, $\omega_1 = \frac{\pi}{4} + 1e-3$ and $\omega_2 = \frac{\pi}{4} - 1e-3$, The system performance is displayed in Fig. 9. We see that all the subchannels work fine except subchannels 16 and 17. It results from the fact that these three interference tones hit both subchannel 16 and 17. Multitone interference are decomposed only into these two subchannels.

## 4. CONCLUSIONS

In this paper, the performance evaluation of competing DSL communications techniques for single and multitone interference are presented. This study highlights the sensitivity of DMT performance to a single tone (narrow band) interference. In order to eliminate single (multi)-tone interference, DMT based system needs additional single (multi)-tone interference canceller before the forward FFT transform. The DSBMT technique outperforms DMT for these interference environments. Its robustness comes from its better localized orthogonal subcarriers in frequency.

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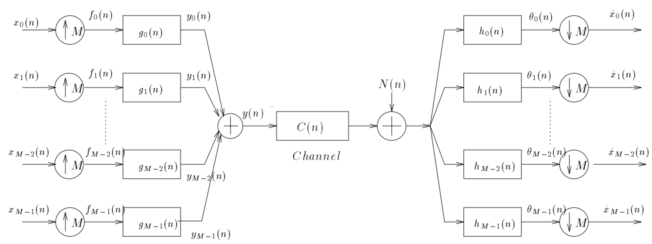


Figure 1: Basic Structure of a Multicarrier based digital transceiver.

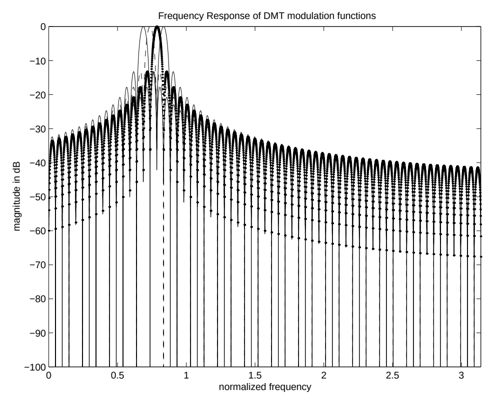


Figure 2: Frequency Response of Subcarrier 15, 16, 17 and 18 of 128 point DFT basis functions.

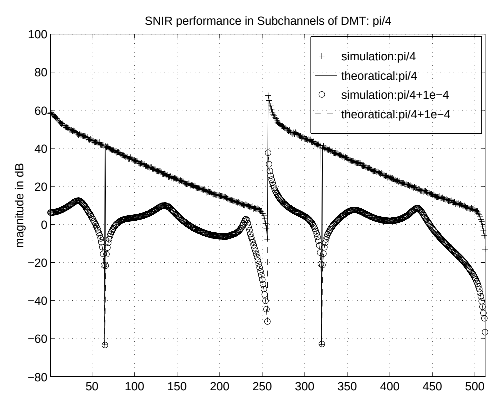


Figure 3: The sensitivity of SNIR for each subchannel of DMT based system for a single-tone interference and AWGN scenario with $\omega_{st} = \frac{\pi}{4}$ and $\omega_{st} = \frac{\pi}{4} + 0.0001$ on AWG 26, 9 Kft loop plant, $SNIR = 0dB$ .

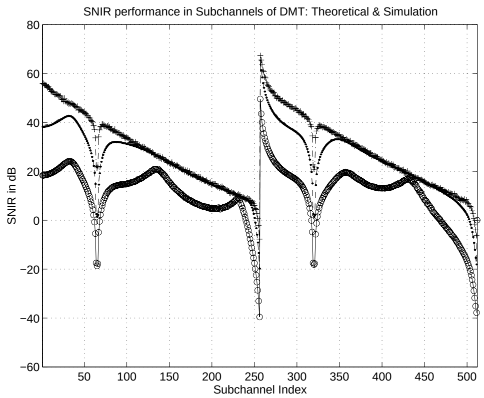


Figure 4: The theoretical and simulation SNIR performance of each subchannels of DMT based system with fixed center frequency $\omega_{sto} = \frac{\pi}{4}$, NB bandwidth is $0.01\pi$,$SIR = 40dB$, $SIR = 60dB$, and $SIR = 80dB$, respectively.

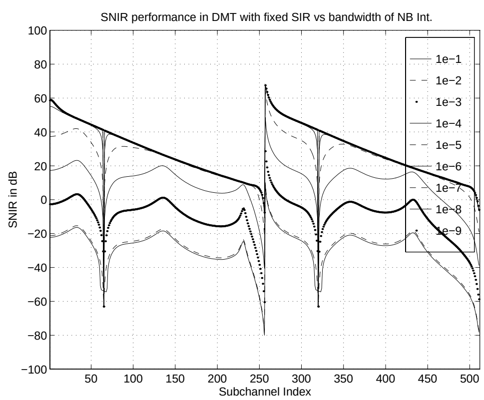


Figure 5: The performance of SNIR in each subchannels of DMT based system in different NB bandwidth with fixed SIR and center frequency $\omega_{sto} = \frac{\pi}{4}$.

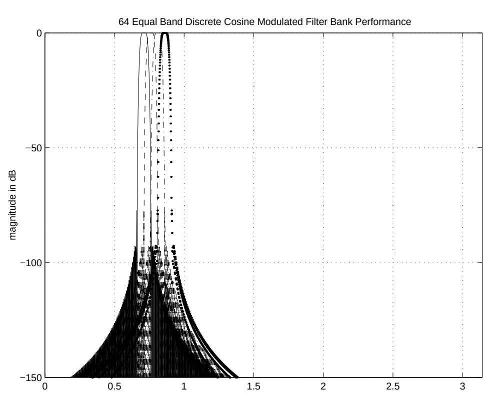


Figure 6: Frequency Response of Subcarrier 15, 16, 17 and 18 of a 64 Equal Band Discrete Cosine Modulated Filter Bank.

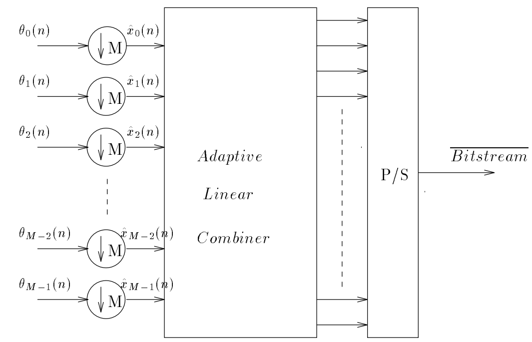


Figure 7: Adaptive linear combiner after the analysis filter bank at the receiver of DSBMT.

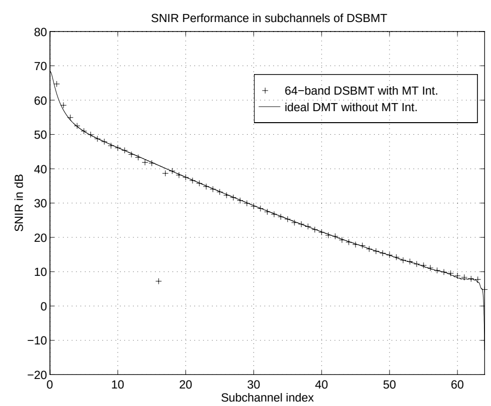


Figure 8: DWMT (DSBMT) Multicarrier Transceiver Performance in Multitone Interference Environment on AWG 26 9kft loop plant.

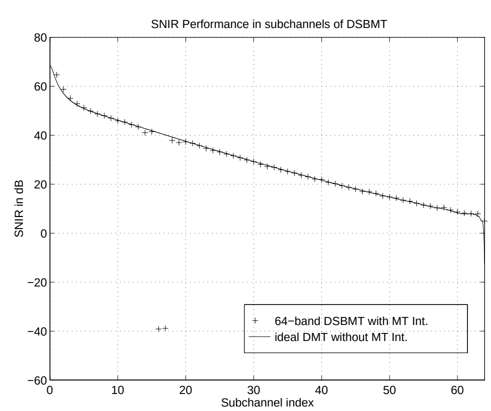


Figure 9: DWMT (DSBMT) Multicarrier Transceiver Performance in Multitone Interference Environment on AWG 26 9kft loop plant.