OPTIMAL DOWNLINK POWER ASSIGNMENT FOR SMART ANTENNA SYSTEMS

Weidong Yang and Guanghan Xu

Department of Electrical and Computer Engineering, The University of Texas, Austin, TX 78712-1084, USA

ABSTRACT

Smart antenna systems have the potential to substantially increase the range of base stations and boost the SINRs of signals. In this paper, we study several criteria in downlink weighting vector design which is key to exploit the full potential of smart antenna systems, and give the optimal power assignment when the orientations of the weighting vectors are known. Simulation results have shown significant improvement offered by the proposed optimal power assignment method. In particular, we can equalize each user's downlink performance by significantly reducing the output power. Since the power amplifiers at the base station are the most expensive subsystems, this approach can lead to significant cost reduction for a base station.

1. INTRODUCTION

A smart antenna system can be used to extend the range of base station, reduce the cost of the base station, mitigate fading, and increase the system capacity and performance. Research on uplink, i.e., the link originating from a terminal to a base station, has been quite active [1, 2, 7, 5]. Not so many research results have been presented for downlink, i.e., the link originating from a base station to multiple terminals [6]. In [6], the problem of maximizing the sum of bit rates from the base station to multiple terminals is studied. For wireless communications, however, the problem of maximizing the smallest SINR (Signal-to-Interference-plus-Noise-Ratio) among all links originating from the base station to the multiple terminals is more interesting. Due to the difficulty of acquiring the downlink spatial signature in an FDD (Frequency Division Duplex) scheme, in this paper, we will only consider a TDD (Time Division Duplex) scheme. However, even though the downlink spatial signatures of all the active terminals can be acquired from the uplink, the problem of finding the best set of downlink weighting vectors (DWV) is much more complicated than its uplink counterpart, mainly because the optimal weight design problem for uplink can be treated individually while the design of the DWVs for all the terminals, are intertwined and cannot be separated. In other words, the DWV design must be solved globally to obtain an optimal or near optimal solution.

In this paper, we first present an objective function that is appropriate for real communication applications, *i.e.,* in voice communications, it is proper to consider a worst case signal-to-interference-and-noise-ratio (**SINR**) or the minimum of the SINR's for all the individual terminals sharing the same carrier frequency and time slot. Here we limit our study to the single cell case, *i.e.,* we do not consider the interference from other neighboring cells[1]. We will study the optimal power assignment given the orientations of the DWVs.

We use the following convention throughout the paper: $E\{\cdot\}$ for expectation; $\{\cdot\}^T$ for transpose; $\{\cdot\}^\#$ for conjugate transpose ; $tr\{\cdot\}$ for trace; $\{\cdot\}^*$ for conjugate; $var\{\cdot\}$ for variance.

2. PROBLEM STATEMENT

We assume that an M-element antenna array is used for communications between the base station and N mobile terminals. A TDD scheme is used, the N terminals share a common time slot and carrier frequency, hence we can use a baseband expression for the signal emitted from the base station. The symbol sequences for different terminals are $s_i(t),\ i \in \mathcal{N}_s$(here we denote the set $\{1, \cdots, N\}$ as $\mathcal{N}_s$), which are uncorrelated and with variance $var\{s_i(t)\} = 1$. The signal emitted by the antenna array can be described as:

$$s(t) = \sum_{i=1}^{N} \mathbf{w}_i s_i(t),$$

where $\{\mathbf{w}_i, i \in \mathcal{N}_s\}$ are the DWVs. The spatial signature [3] for the i^{th} terminal is $\mathbf{a}_i,\ i \in \mathcal{N}_s$, which are assumed known [8]. Hence the signal received by each terminal can be expressed as

$$\mathbf{a}_i^T s(t) + n_i, \quad i \in \mathcal{N}_s,$$

where $n_i,\ i \in \mathcal{N}_s$ are the thermal noise on each terminal, and they are modeled as i.i.d. Gaussian noise with the same variation $var\{n_i\} = \sigma^2$. This is a reasonable assumption since the sensitivity of all the terminals is often similar. Suppose we have found a set of DWVs by some methods, *e.g.,* the conjugates of spatial signatures, then we need to scale them to achieve the best performance.

Define $A = [\mathbf{a}_1 \ \cdots \ \mathbf{a}_N]$, $W = [\mathbf{w}_1 \ \cdots \ \mathbf{w}_N]$. The SINR

[1] It is reasonable to assume that most of the co-channel interference is from the same cell in a smart antenna system

for the i^{th} terminal is defined as:

$$SINR_i = \frac{E\{|\mathbf{a}_i^T \mathbf{w}_i s_i(t)|^2\}}{E\{|\mathbf{a}_i^T \sum_{k=1,k\neq i}^{N} \mathbf{w}_k s_k(t) + n_i|^2\}} \quad (1)$$

$$= \frac{|\mathbf{a}_i^T \mathbf{w}_i|^2}{\sum_{k=1,k\neq i}^{N} |\mathbf{a}_i^T \mathbf{w}_k|^2 + \sigma^2} \quad (2)$$

The power that the antenna array can emit is limited, p is the maximum available power. Hence

$$E\{|s(t)|^2\} = \sum_{i=1}^{N} |\mathbf{w}_i|^2 \leq p. \quad (3)$$

For voice communications, the bit-rate requirement for each terminal is the same. To guarantee the service offered to each terminal, we want to make the smallest SINR be above a certain level.

Define $SINR_{min} = \min_{i \in \mathcal{N}_s} SINR_i$, then the objective is to maximize $SINR_{min}$. In this way, we wind up with the following DWV design problem:

$$\text{Maximize } SINR_{min} \quad \text{s.t.} \quad \sum_{i=1}^{N} |\mathbf{w}_i|^2 \leq p \quad (4)$$

where $SINR_i = \dfrac{|\mathbf{a}_i^T \mathbf{w}_i|^2}{\sum_{k=1,k\neq i}^{N} |\mathbf{a}_i^T \mathbf{w}_k|^2 + \sigma^2}$, $i \in \mathcal{N}_s$.

3. OPTIMAL POWER ASSIGNMENT

We have the following lemmas,

Lemma 1 *If $\sum_{i=1}^{N} |\mathbf{w}_i|^2 < p$, then we can scale $\{\mathbf{w}_i\}$ proportionally to improve $SINR_{min}$.*

Lemma 2 *If $SINR_i$, $i \in \mathcal{N}_s$ are not the same, then we can scale $\{\mathbf{w}_i\}$ to improve $SINR_{min}$.*

Lemma 3 *If $\{\mathbf{a}_i\}$ are linear independent, then the DWVs should fall into the subspace spanned by the conjugates of the spatial signatures,* i.e., *$W = \mathbf{A}^* \Phi$, where Φ is a nonsingular square matrix. The performance of $\{\mathbf{w}_i\}$ is the same as that of $\{c_i \mathbf{w}_i\}$, where $|c_i| = 1$ for $1 \leq i \leq N$.*

From the above, we can have the following objective function:

$$\text{Maximize } SINR \quad \text{s.t.} \quad \sum_{i=1}^{N} |\mathbf{w}_i|^2 = p \quad (5)$$

where $SINR = SINR_1 = \cdots = SINR_N$. To facilitate the following discussion, we express each DWV $\mathbf{w}_i$ as $k_i \mathbf{v}_i$, where k_i, $i \in \mathcal{N}_s$ are positive numbers, $\mathbf{v}_i$ is called the orientation of $\mathbf{w}_i$, $i \in \mathcal{N}_s$. Consequently, from $SINR_i = SINR$, $i \in \mathcal{N}_s$, we have

$$\frac{|\mathbf{a}_i^T \mathbf{v}_i|^2 k_i^2}{\sum_{j=1,j\neq i}^{N} |\mathbf{a}_i^T \mathbf{v}_j|^2 k_j^2 + \sigma^2} = SINR_i \quad (6)$$

From the power constraint, we have

$$\sum_{i=1}^{N} |\mathbf{v}_i|^2 k_i^2 = p \ . \quad (7)$$

Let $\mathbf{R}$, an $N \times N$ matrix, be defined as follows:

$$R_{ij} = \begin{cases} \frac{|\mathbf{a}_i^T \mathbf{v}_j|^2}{|\mathbf{a}_i^T \mathbf{v}_i|^2} & i \neq j \\ 0 & i = j \end{cases}$$

and two $N \times 1$ vectors be defined as follows:

$$\mathbf{h} = [\frac{\sigma^2}{|\mathbf{a}_1^T \mathbf{v}_1|^2} \ \cdots \ \frac{\sigma^2}{|\mathbf{a}_N^T \mathbf{v}_N|^2}]^T, \ \ \mathbf{g} = [|\mathbf{v}_1|^2 \ \cdots \ |\mathbf{v}_N|^2]^T.$$

Apparently, for any solution of any practical meaning, a DWV should not be a zero vector, and the DWV for s_i should not be orthogonal to the spatial signature of s_i; consequently every element of h and g is positive. Let $y_i = k_i^2$, $i \in \mathcal{N}_s$, $y_{N+1} = 1$, and $\mathbf{y} = [y_1 \ \cdots \ y_N \ y_{N+1}]^T$. $\mathbf{y}$ is a $(N+1) \times 1$ vector. Then from equations (6) and (7), we have

$$\mathbf{C}\mathbf{y} = SINR \ \ \mathbf{B}\mathbf{y} \quad (8)$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_{N\times N} & \mathbf{0}_{N\times 1} \\ \mathbf{g}^T & -p \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{R} & \mathbf{h} \\ \mathbf{0}_{1\times N} & 0 \end{bmatrix}$$

$\mathbf{C}$ is nonsingular, then we have

$$\mathbf{D}\mathbf{y} = \frac{1}{SINR}\mathbf{y}, \quad \mathbf{D} = \mathbf{C}^{-1}\mathbf{B} = \begin{bmatrix} \mathbf{R} & \mathbf{h} \\ \frac{\mathbf{g}^T\mathbf{R}}{p} & \frac{\mathbf{g}^T\mathbf{h}}{p} \end{bmatrix} \quad (9)$$

which is a non-negative matrix (refer to P. 528, [4]). If a positive vector $\mathbf{y}$ (refer to P. 528, [4]) is the eigenvector of the matrix $\mathbf{D}$, then it can be scaled so that the last element of $\mathbf{y}$ will be 1.

Theorem 1

1. *For a non-negative matrix, the eigenvalue of the largest norm is positive, and its corresponding eigenvector can be chosen to be non-negative (refer to P. 543 [4])*
2. *For a non-negative matrix $\mathbf{D}$, the non-negative eigenvector corresponding to the eigenvalue of the largest norm is positive (refer to 1.4.).*
3. *For a set of spatial signature and a set of the orientation of the weighting vectors, there is only one solution to equation (8) (refer to 1.5.).*

Hence for a set of spatial signatures, and DWVs, there always exists a solution to equation (8); and in this case, the SINR margin equals the reciprocal of the largest eigenvalue of matrix D. When the orientation of each DWV is fixed, Equation (9) actually gives the optimal assignment of powers to each DWV. Equation (9) can be used to improve the result in an iterative algorithm to find better weighting vectors. The theorem also gives us an objective function upon which we can directly use the optimization techniques to find the optimal solution. Summarize the above discussions, we have

Theorem 2

1. *For a set of spatial signatures, and a set of DWVs, whose orientations are fixed, the power assignment scheme which leads to equal SINR margins exists and is unique, and the SINR margins are the reciprocal of the the largest eigenvalue of D.*
2. *The optimal SINR margin ($SINR_{optimal}$) is achieved when*

$$SINR_{optimal} = \frac{1}{\min\limits_{\mathbf{V} \in \mathbf{C}^{M \times N}} \text{the largest eigenvalue of } \mathbf{D}.}$$

4. SIMULATION RESULTS

Here we will compare the power assignment scheme and another scheme which gives equal power to the DWVs of each terminal's. We will consider two cases. The first case has fewer terminals, the second has more terminals than the first case. The number of antenna elements is 8.

In the first example, as shown in Table 1, we consider a case with 4 terminals. Their angles of arrival are 0 degree, 76 degrees, 152 degrees and 228 degrees, respectively. And the relative magnitudes of their spatial signatures are [1 2 2 4].

In the second example, the number of terminals is 30, and the number of antennas is 8. The SINR shown in Figure 2 is for the despread signal. It can be seen that when the new scheme offers a SINR of 10 dB, 12 terminals' SINR is below 10 dB. To boost their SINRs, the power emitted from the base station should be 2.72 times (4.34dB more) of the power used by the new scheme.

5. CONCLUSION

In this paper, we have studied the downlink weighting vector design for smart antenna systems. Given the orientation of downlink weight vectors, the optimal power assignment method has been proposed.

A APPENDIX

1.1. Proof for Lemma 1

Suppose that $\{\mathbf{w}_i \; i \in \mathcal{N}_s\}$ is a set of weighting vectors and $\sum\limits_{i=1}^{N} |\mathbf{w}_i^{'}|^2 = p^{'} < p$, then let $\kappa = \sqrt{\frac{p}{p^{'}}}$, apparently $\kappa > 1$. Let $\mathbf{w}_i^{'} = \kappa \mathbf{w}_i \; i \in \mathcal{N}_s$, we have $SINR_i^{'} > SINR_i$.

1.2. Proof for Lemma 2

Suppose we have a set of weighting vectors, and with $SINR_i^{'}$, $i \in \mathcal{N}_s$, not all equal, and $SINR_{min}^{'} = \min\limits_{i=1}^{N} SINR_i^{'}$. Then we can sort the SINRs in an ascending order, and find an index $v \geq 1$, so

$$\begin{aligned} SINR_{min}^{'} &= SINR_1^{'} = \cdots = SINR_v^{'} \\ &< SINR_{v+1}^{'} \leq SINR_{v+2}^{'} \leq \cdots \leq SINR_N^{'} \end{aligned}$$

Let $p_1 = \sum\limits_{i=1}^{v} |\mathbf{w}_i|^2$, and $p_2 = \sum\limits_{i=v+1}^{N} |\mathbf{w}_i|^2$, then $p_1 + p_2 = p$(because the weighting vector for each signal should not be zero, $p_1 \neq 0$ and $p_2 \neq 0$). We can choose $\alpha(\alpha > 1)$ and $\beta(0 < \beta < 1)$, so $\alpha^2 p_1 + \beta^2 p_2 = p$, apparently, β is related to α by $\frac{p - \alpha^2(1-p_2)}{p_2}$ and β is a strictly decreasing function of α. let $\mathbf{w}_i^{''} = \alpha \mathbf{w}_i^{'}$, $i = 1, \cdots, v$, and $\mathbf{w}_i^{''} = \beta \mathbf{w}_i^{'}$, $i = v+1, \cdots, N$. $SINR_i^{''}(\alpha)$, $i = 1, \cdots, v$ are strictly increasing functions of α. $SINR_i^{''}(\alpha)$, $i = v+1, \cdots, N$, are strictly decreasing functions of α. It is obvious that $SINR_i^{''}(1) = SINR_i^{'}$. Hence we can choose ϵ, so when $0 < 1 - \alpha < \epsilon$, that $|SINR_i^{''}(1) - SINR_i^{''}(\alpha)| < \frac{SINR_{v+1}^{'} - SINR_v^{'}}{2}$, $i = 1, \cdots, N$, hence $SINR_i^{''}(\alpha) > SINR_{min}$.

1.3. Proof for Lemma 3

Suppose the conjugates of the spatial signatures are linearly independent, then the whole space C^M can be decomposed into the orthogonal direct sum of two spaces: the subspace S_{ss} spanned by the conjugates of spatial signatures, and another subspace S_{ssc} which is the complementary orthogonal space of S_{ss}. every weighting vector $\mathbf{w}_i$ can be expressed as the linear combination of basis vectors of S_{ss} and S_{ssc}. It is apparent that the projection of $\mathbf{w}_i$ into S_{ssc} makes no influence on the SINR except consuming power. Such projections should be made to be zero vectors.

1.4. Proof for Theorem 1.2

Proof: According to theorem 1, we can find a non-negative eigenvector $\mathbf{y}$ corresponding to ζ, the eigenvalue of the largest norm. First, the $(N+1)^{th}$ element of the vector $\mathbf{y}$ is nonzero. Suppose this is not true, then we can represent $\mathbf{y}$ as $\begin{bmatrix} \mathbf{b}^T & 0 \end{bmatrix}^T$ where $\mathbf{b}$ is a non-negative $N \times 1$ vector, and at least for one k, $1 \leq k \leq N$, $b_k > 0$. We have

$$\mathbf{D}\mathbf{y} = \zeta \mathbf{y}$$

Consider both sides of the equation, we have

$$\mathbf{R}\mathbf{b} = \zeta \mathbf{b} \;\; , \;\; \mathbf{g}^T \mathbf{R}\mathbf{b} = 0 \tag{10}$$

Now let

$$\mathbf{f} = \begin{bmatrix} \mathbf{g}^T & 1 \end{bmatrix}^T$$

which is a positive vector, then

$$\mathbf{f}^T \mathbf{D}\mathbf{y} = \zeta \mathbf{f}^T \mathbf{y} = \zeta \sum_{i=1}^{N} \mathbf{b}_i \geq \zeta g_k b_k > 0 \tag{11}$$

On the other hand,

$$\begin{aligned} \mathbf{f}^T \mathbf{D} \begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix} &= \begin{bmatrix} \mathbf{g} \\ 1 \end{bmatrix}^T \begin{bmatrix} \mathbf{R} & \mathbf{h} \\ \frac{\mathbf{g}^T \mathbf{R}}{p} & \frac{\mathbf{g}^T \mathbf{h}}{p} \end{bmatrix} \begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix} & (12) \\ &= (1 + \frac{1}{p}) \mathbf{g}^T \mathbf{R}\mathbf{b} & (13) \end{aligned}$$

Consider (11) and (13), then

$$\mathbf{g}^T \mathbf{R} > 0 \tag{14}$$

(10) and (14) lead to contradiction. Hence we have proved that the last element of the eigenvector is positive. Second, $\mathbf{y}$ is positive. Consider

$$\mathbf{y} = \frac{1}{\zeta} \mathbf{D}\mathbf{y}$$

Methods	User 1	User 2	User 3	User 4	Power
Method 1	6.4	11.3	11.9	14.7	2.5
Method 2	10.3	10.3	10.3	10.3	1

Table 1. 4-user case (1) equal power assignment, (2) optimal power assignment

Consider both sides of the above equation, we have for $1 \leq i \leq N$,

$$y_i = \frac{1}{\zeta}(\sum_{j=1}^{N} R_{ij} y_j + h_i y_{N+1}) \geq \frac{h_i y_{N+1}}{\zeta} > 0.$$

Consequently $y_i > 0$ for $1 \leq i \leq N$. In this way, we have proved that all the elements of $\mathbf{y}$ are positive. Hence for a set of spatial signature and a set of the orientations of the DWVs, there always is a solution to equation (8); and in this case, the $SINR_i$ equals the reciprocal of the largest eigenvalue of matrix $\mathbf{D}$.

1.5. Proof for Theorem 1.3

Suppose there are two solutions $\mathbf{y}$ and $\mathbf{y}'$, $\mathbf{y}, \mathbf{y}' > 0$, $\mathbf{y}^{'} \neq \mathbf{y}$ and

$$SINR^{'} \leq SINR \tag{15}$$

Then we have $\mathbf{w}_i^{'} = \sqrt{y_i^{'}}\, \mathbf{v}_i$, $\quad \mathbf{w}_i = \sqrt{y_i}\, \mathbf{v}_i$

We have $\mathbf{w}_i^{'} = k_i \mathbf{w}_i$ for $i \in \mathcal{N}_s$, and not all k_i are 1, consequently we can find a re-indexing $\{r_i\}$, so $k_{r_1} \geq \cdots \geq k_{r_N}$, $k_{r_1} > 1$ and

$$\begin{aligned} SINR_{r_1}^{'} &= \frac{|\mathbf{a}_{r_1}^T \mathbf{w}_{r_1}^{'}|^2}{\left(\sum_{i=2}^{N} |\mathbf{a}_{r_1}^T \mathbf{w}_{r_i}^{'}|^2 + \sigma^2\right)} \\ &= \frac{|\mathbf{a}_{r_1}^T \mathbf{w}_{r_1}|^2}{\left(\sum_{i=2}^{N} |\mathbf{a}_{r_1}^T \mathbf{w}_{r_i}|^2 \frac{k_{r_i}^2}{k_{r_1}^2} + \frac{\sigma^2}{k_{r_1}^2}\right)} \\ &> \frac{|\mathbf{a}_{r_1}^T \mathbf{w}_{r_1}|^2}{\left(\sum_{i=2}^{N} |\mathbf{a}_{r_1}^T \mathbf{w}_{r_i}|^2 + \sigma^2\right)} = SINR_{r_1}. \end{aligned}$$

Hence

$$SINR^{'} = SINR_{r_1}^{'} > SINR_{r_1} = SINR. \tag{16}$$

Equations (15) and (16) lead to contradiction. Hence there is only one solution to equation (8).

REFERENCES

[1] Philip Balaban and Jack Salz. Optimum diversity combining and equalization in digital data transmission with applications to cellular mobile radio–part 1: Theoretical considerations. *IEEE Trans. on Communications*, pages 885–894, May 1992.

[2] Luis Castedo. An adaptive beamforming technique based on cyclostatonary signal properties. *IEEE Trans. on Signal Processing*, pages 1890–1900, August 1995.

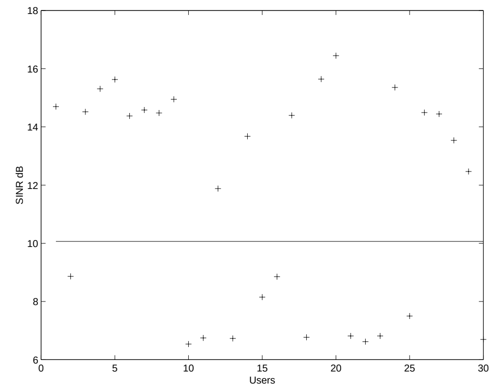

Figure 1. 30-user case

[3] Shiann-Shiun Jeng and et al. Experimental studies of spatial signature variation at 900 MHz for smart antenna systems. *Accepted by IEEE Trans. on Antenna and Propagation.*

[4] Peter Lancaster and Miron Tismenetsky. *The Theory of Matrices*. Academic Press, 1985.

[5] W.C. Lee and S.U. Pillai. Beamforming for efficient interference suppression in a mobile communication environment. In *29th Asilomar Conf. on Signals, Systems & Computers*, pages 771–776, October 30 to November 2, 1995.

[6] T. Murat, G. Xu, B. Evans, and H. Liu. Optimal weighting vectors for broadcast channels. In *International Conf. on Acoustics, Speech, and Signal Processing*, April 1997.

[7] M.K. Tsatsanis and G.B. Giannakis. Blind estimation of direct sequence spread spectrum signals in multipath. In *29th Asilomar Conf. on Signals, Systems & Computers*, pages 403–407, October 30 to November 2, 1995.

[8] G. Xu and H. Liu. Multiuser blind channel estimation and spatial channel pre-equalization. In *International Conf. on Acoustics, Speech, and Signal Processing*, volume 3, pages 1756–1759, May 1995.