

MYOPIC DECONVOLUTION COMBINING KALMAN FILTER AND TRACKING CONTROL

P. Sarri, G. Thomas, E. Sekko, P. Neveux

Laboratoire d'Automatique Industrielle
I.N.S.A de Lyon, Bât 303
20, Avenue Albert Einstein
69621 Villeurbanne, France

☎ (33) 04.72.43.81.98 Fax : (33) 04.72.43.85.35

✉ sarri@lasecu.insa-lyon.fr

ABSTRACT

In this paper, we propose a deconvolution method based on discrete-time optimal control. By combining Kalman filtering with optimal control, we state the problem in terms of tracking problem. This leads to solve a set of recurrent equations, including in particular a matrix Riccati equation. We present a method that transforms the solution of these recurrent equations in that of a linear system of equations. Once the linear system has been set up, the deconvolution procedure becomes very fast, and permits on-line deconvolution. It is also possible to use the discrete impulsional response, and perform blind deconvolution. This technique include a L_2 or H_∞ optimal filter. Numerical examples illustrate the robustness of the procedure.

1. INTRODUCTION

This paper presents a deconvolution method for discrete-time linear system [4][5][6]. We propose to restore a distorted signal when the recorded output signal is noisy. We combine L_2 or H_∞ filter with optimal control, in order to achieve tracking control to the filtered output. Consider a discrete-time linear system in which noises are added to the input and output. The noised system model is given by a state equation

$$\begin{aligned} x_{k+1} &= A_d x_k + B_d u_k + B_d w_k \\ y_k^m &= C_d x_k + v_k \end{aligned} \quad (1)$$

G. Thomas: Ecole Centrale de Lyon, M.I.S/L.A.G.E.P, 36 rue Guy de Collongue 69131 ECULLY, FRANCE. ✉ samoht@cc.ec-lyon.fr.

E. Sekko: Université Claude Bernard Lyon I C.P.E Lyon L.A.G.E.P 43, Bd du 11 Novembre 1918 69622 Villeurbanne. ✉ sekko@lagep.cpe.fr

P. Neveux: Université Claude Bernard Lyon I C.P.E Lyon L.A.G.E.P 43, Bd du 11 Novembre 1918 69622 Villeurbanne. ✉ neveux@lagep.cpe.fr

When using a Kalman or H_∞ discrete filter [1][3][4] the state and output estimation are given by

$$\begin{aligned} \hat{x}_{k+1} &= (A_d - KC_d)\hat{x}_k + B_d u_k + K y_k^m \\ y_k^e &= C_d \hat{x}_k \end{aligned} \quad (2)$$

The equation of the system model is

$$\begin{aligned} x_{k+1} &= A_d x_k + B_d u_k \\ y_k &= C_d x_k \end{aligned} \quad (3)$$

then we form the augmented system

$$\begin{aligned} X_{k+1} &= A X_k + B_1 u_k + B_2 y_k^m \\ e_k &= C X_k \end{aligned} \quad (4)$$

where

$$X_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}, \quad A = \begin{bmatrix} A_d & 0 \\ 0 & A_d - KC_d \end{bmatrix}$$

$$B_1 = \begin{bmatrix} B_d \\ B_d \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ K \end{bmatrix}, \quad C = [C_d \quad -C_d]$$

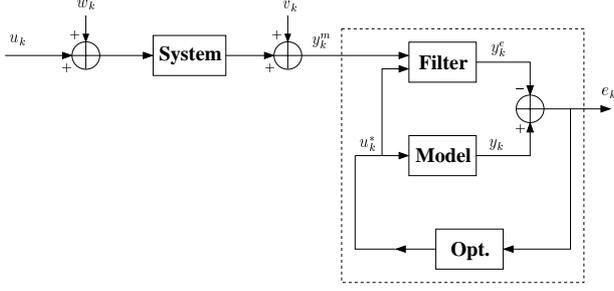
The purpose, is to determine the optimal input u_k^* which minimizes the following criterion

$$J = \sum_{k=0}^{N-1} (e_k^2 + \alpha u_k^2) + e_N^2, \quad e_k = y_k - y_k^e \quad (5)$$

which can be expressed by

$$J = \sum_{k=0}^{N-1} (X_k^T Q X_k + \alpha u_k^2) + X_N^T Q X_N, \quad Q = C^T C. \quad (6)$$

This configuration is illustrated at Figure 1.



u_k : input signal.
 y_k^m : recorded signal.
 w_k : gaussian noise added to the input.
 v_k : gaussian noise added to the output.

Fig 1 : Optimal deconvolution configuration.

2. SOLUTION OF THE PROBLEM

The solution is given by the following recurrent equations [4]

$$u_k^* = -SB_1^T H_{k+1} A X_k + S(A^T - A^T H_{k+1} B_1 S B_1^T)^{-1} \theta_k$$

where

$$\begin{aligned}
 S &= (\alpha + B_1^T H_{k+1} B_1)^{-1} \\
 H_k &= A^T H_{k+1} A - A^T H_{k+1} B_1 S B_1^T H_{k+1} A + Q \\
 \theta_k &= (A^T - A^T H_{k+1} B_1 S B_1^T)(\theta_{k+1} - H_{k+1} B_2 y_k^m) \\
 X_{k+1} &= (A - B_1 S B_1^T H_{k+1} A) X_k + B_2 y_k^m \\
 &\quad + B_1^T (A^T - A^T H_{k+1} B_1 S B_1^T)^{-1} \theta_k
 \end{aligned}$$

H_k and θ_k must be solved backwards with the initial conditions $H_N = Q$ and $\theta_N = 0$. It corresponds to the discrete version of the optimal tracking problem applied in the context of deconvolution as shown in Figure 1.

3. FAST DECONVOLUTION

The following method used an N -dimensional linear system equations, N being the number of samples. As in the previous section this method is based on optimal control theory. The solution of that linear system represents the deconvolved input. The discrete state equation of the augmented system (4), can be developed as follows

$$X_k = A^k X_0 + A^{k-1} B_1 u_0 + \dots + B_1 u_{k-1} + A^{k-1} B_2 y_0 + \dots + B_2 y_{k-1}. \quad (7)$$

In this form the discrete state vector will only depend on the initial state X_0 and the time step k . Using the equation (7), we replace X_k in the criterion J . Now, the criterion J only depends on the initial state X_0 and the inputs u_k $k = 0, 1, \dots, N-1$. The matrix Q is non-negative and α is

strictly positive. The criterion J is quadratic. The condition of optimality is

$$\frac{\partial J}{\partial u_k} = 0, \quad k = 0, 1, \dots, N-1. \quad (8)$$

This condition lead to a N -dimensional linear system in which the unknowns are

$$u_k, \quad k = 0, 1, \dots, N-1.$$

Let $U = (u_0 \ u_1 \ \dots \ u_{N-1})^T$ be the unknown vector, and $Y_m = (y_0^m \ y_1^m \ \dots \ y_{N-1}^m)^T$ the recorded vector. The linear system to solve is

$$[\alpha I_{N,N} + \Phi]U = -\Pi Y_m - \lambda \quad (9)$$

where

$$\begin{aligned}
 \Phi &= \begin{bmatrix} B_1^T S_{N-1} B_1 & \dots & B_1^T A^{(N-1)T} S_0 B_1 \\ \vdots & \ddots & \vdots \\ B_1^T S_0 A^{N-1} B_1 & \dots & B_1^T S_0 B_1 \end{bmatrix}, \\
 \Pi &= \begin{bmatrix} B_1^T S_{N-1} B_2 & \dots & B_1^T A^{(N-1)T} S_0 B_2 \\ \vdots & \ddots & \vdots \\ B_1^T S_0 A^{N-1} B_2 & \dots & B_1^T S_0 B_2 \end{bmatrix}, \\
 \lambda &= \begin{bmatrix} B_1^T S_{N-1} A X_0 \\ \vdots \\ B_1^T S_0 A^N X_0 \end{bmatrix}
 \end{aligned}$$

and

$$S_n = Q + A^T Q A + \dots + A^{nT} Q A^n$$

$I_{N,N}$ is the identity matrix of dimension N , and A^{nT} denotes the transpose of the matrix A^n .

The resolution of the linear system (9) gives the same solution as the one given in section 2.

Remarks

- The linear system (9) is nonsingular by construction and by the assumption made over the matrix Q and the parameter α .
- Once the linear system is built and stored in memory, it is easy to make fast on-line deconvolution, i.e. to treat fast sequences of output signals.

4. FAST DECONVOLUTION USING IMPULSE RESPONSE

In practice it is not always possible to obtain the discrete state equation of any process or sensor. It is easier to obtain the discrete impulse response. We propose to use the discrete impulse response h_i , in order to describe the system

by a state equation in which the order is equal to the number of samples N . Let $H = [h_0 \ h_1 \ \dots \ h_{N-1} \ h_N]$ be the vector corresponding to the discrete impulsional response. In that case, the state equation of the system is

$$\begin{aligned} x_{k+1} &= A_d x_k + B_d u_k \\ y_k &= C_d x_k + D_d u_k \end{aligned} \quad (10)$$

where

$$A_d = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \dots & \dots & \dots & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$C_d = [h_N \ h_{N-1} \ \dots \ h_2 \ h_1], \quad D_d = h_0.$$

As in section 3, but by use of the discrete impulse response, we can get a linear system of equations of dimension N . In this way, after putting in memory the matrix Φ , Π and the vector λ , it is possible to perform fast deconvolution. Including an optimal filter, the numerical resolution of a linear system of dimension N (N can be until 1000 samples) takes just a second or a few.

5. MYOPIC DECONVOLUTION

The technique proposed in the previous section can be applied to myopic deconvolution. The use of a pair of input/output of the process, enables us to restore a degraded input signal corresponding to a measured output signal. Thus, one can say that the identification procedure is embedded in the deconvolution procedure. As in some identification method, one must take care to choose a pair of measured signal that are rich enough in harmonic in order to guarantee a realistic solution. The idea consists of a transformation of the convolution relation [2].

$$y = h * u \equiv y * u_1 = y_1 * u \quad \text{with} \quad y_1 = h * u_1.$$

Therefore, the impulse response h is replaced by the couple I/O (u_1, y_1) . Then to achieve the deconvolution, one just have to apply the method of the previous section.

6. ILLUSTRATIVE EXAMPLE

This example is based on a real process that measures hydrocarbon rate [1]. Figure 2 shows the output recorded signal to be restored. The method is first tested with the impulse response h_i ($i = 0, \dots, N$) shown in Figure 3. The Figure 4 shows the result obtained.

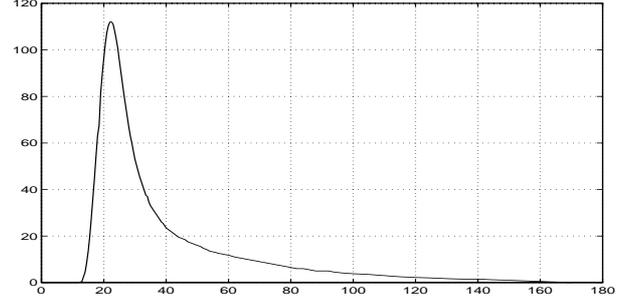


Fig 2 : Output signal.

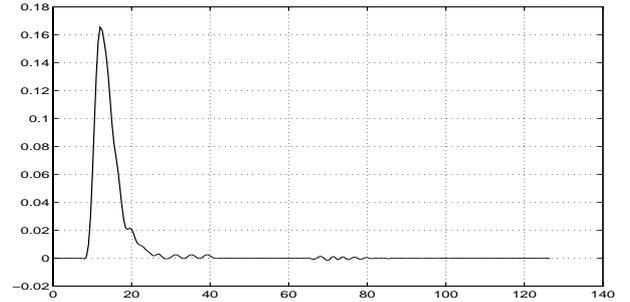


Fig 3 : Impulsional response of the process.

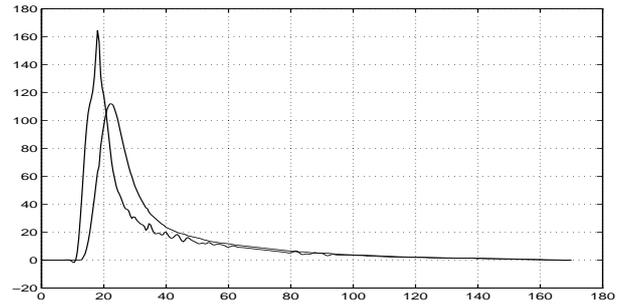


Fig 4 : Restored signal.

The Figures 5 and 6 show the results by use of a pair of I/O signals to perform blind deconvolution.

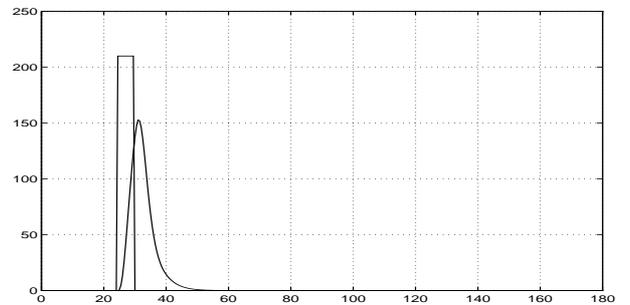


Fig 5 : Pair of I/O for blind deconvolution.

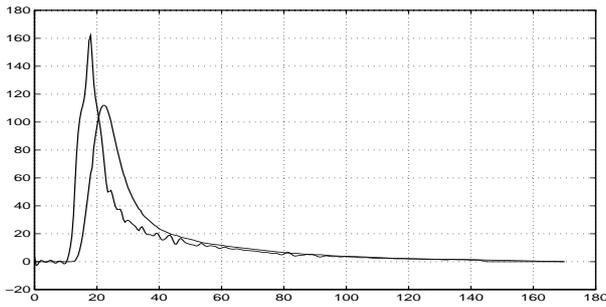


Fig 6 : Restored signal.

Robustness of the method

The Figures 7,8,9 show the results obtained when the output is corrupted by noise.

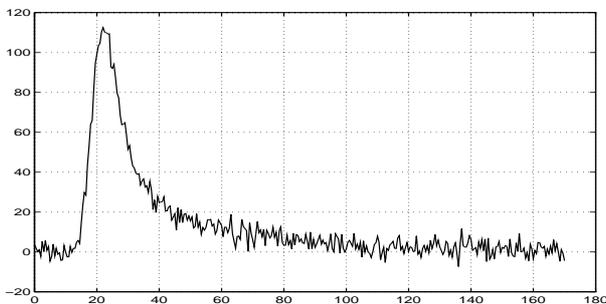


Fig 7 : Noisy output signal of the process SNR=42dB.

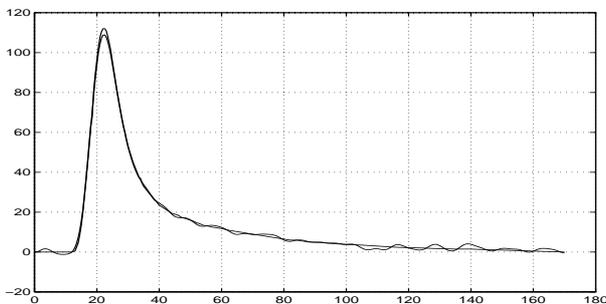


Fig 8 : Filtered / Noise free output signal $var = 10^{-6}$.

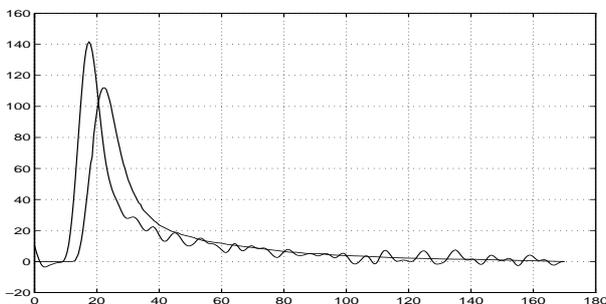


Fig 9 : Restored signal with $\alpha = 0.001$.

7. CONCLUSION

We proposed a fast on-line deconvolution method, based on optimal control and kalman filtering. In discrete-time we have transformed the optimal control problem into the resolution of an N -dimensional linear system equations, N being the number of samples. This configuration offers three advantages confirmed by numerical results. The first, is the possibility of fast deconvolution by solving a static linear system of equations. The second advantage offers the possibility to use the impulse response of the plant. In fact, the transformation of the impulse response in discrete state equations, can not be used to solve the recurrent equations related in section 2. These equations include a matrix Riccati recurrent equation of dimension $N \times N$. Therefore, after the linear system of equations has been set up, one just have to store it in memory and then perform fast deconvolution. The third advantage allows us to replace the information given by the impulse response by the one given by a pair of Input/Output of the process. With these advantages, the method is fast, robust and is easy to work with.

8. REFERENCES

- [1] G. THOMAS. Amélioration des performances d'un analyseur de $c_n h_m$. *Rapport de fin de contrat CETIAT-LAGEP*.
- [2] G. THOMAS, C. DUSSAUSSOIS, F. BURET AND PH. AURIOL. Deconvolution without system model or a new blind deconvolution. *IEEE International Conference on Acoustics, Speech and Signal processing*, march 1985.
- [3] I. YAESH, U. SHAKED. A transfer function approach to the problems of discrete-time systems: h_∞ -optimal linear control and filtering. *IEEE Transactions On Automatic Control*, 36:1264–1271, 1991.
- [4] R. BELLMAN. Dynamic programming and lagrange multipliers. *Proc Nat. Acad. Sc., USA*, 42:767–769, 1956.
- [5] R. E. BELLMAN, S. E. DREYFUS. Applied dynamic programming. *princeton University Press, Princeton*, 1962.
- [6] T. BASAR. A dynamic games approach to controller design: disturbance rejection in discrete-time. *Proc IEEE. Decision and Control, Tampa, FL*, pages 407–414, 1989.