# A GLOBALLY CONVERGENT APPROACH FOR BLIND MIMO ADAPTIVE DECONVOLUTION

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## ABSTRACT

We address the deconvolution of MIMO linear mixtures. The approach is based on the construction of a hierarchical family of composite criteria involving CM criterion and second order statistics constraint. Altougth, the criteria are based on fourth order statistics, we give a complete proof of convergence of this structure. We show that each cost function leads to the restoration of one single source. Moreover the approach is naturally robust with respect to the channels order estimation. An adaptive alorithm is derived for the simultaneous estimation of all sources.

**Keywords:** Blind Adaptive Sources Separation, Constant Modulus Criterion, Multiples Input/Multiples Outputs systems.

## 1. INTRODUCTION

The so-called signal separation problem of MIMO (Multiple Inputs / Multiple Outputs) linear convolutives mixtures arises in a wide variety of signal processing and communications applications. It is a crucial issue in wireless multi-users digital communication systems when the different users share parts of the same frequency band and are received on an omni-directional antenna. The aim is to separate the incoming digital signals, arriving from different or possibly the same directions on multiple antennas and to equalize the contribution of each user in order to restore the transmitting signals. The restoration of multiple input signals is also required when cross-polarization of two orthogonally polarized sources occurs due to multipath propagation with finite delay spread ([2]). In this case, we know that the two sources are temporally and spatially mixed.

In most existing systems, a training sequence known by the receiver, is periodically sent by the transmitter in order to identify the unknown channel. However, the use of such training sequences has some important known drawbacks. The training sequence requires a non-neglectable amount of the data rate and there is some applications where the training scenario is not feasible ([4]).

In order to restore the input signal blindly (*i.e.* without training) many contributions have been developed (see [9], and other) mainly based on instantaneous mixtures. In the convolutive case, very few results exist. One of the first contribution which extend the equalizations tools to the more general problem of MIMO deconvolution is addressed in [2] and more recently in [3] (see also [8]). The approach consists in an iterative subtraction of the estimated signal from the mixture. In [6], a solution based on the same criterion and additive constraint is proposed in the specific case of instantaneous mixtures. A similar idea was independently proposed by [10] and [8] in the convolutive case. Unfortunately, only partial performances results are given and there is no guarantee on the proposed composite criteria to avoid undesirable convergence settings.

Based on our works on CM behavior ([1],[2]), we propose a new design for blind mixtures deconvolution of MIMO channels. We show that it is possible to construct a set of composite criteria based on sparing constraints in order to guarantee that all local minima of each criterion achieve perfectly a single source restoration. Efficient stochastic adaptive algorithm is presented. The complete convergence proofs are provided.

## 2. PROBLEM FORMULATION

We consider the MIMO linear convolutive mixture, where the $L$-dimensional observation writes as:

$$y(n) = [\, H(z)\, ]\, s(n) \tag{1}$$

we suppose that $s(n) = (s_1(n), .., s_P(n))^\top$ is the signal of interest of dimension $P$. The components $s_k(n)$ (referred as the k$^{th}$ source) are i.i.d., zero mean, sub-Gaussian and mutually independent (**A-1**). We denote $\sigma_k^2$ the variance of each source $s_k(n)$. The channel transfer function $H(z) = (h_1(z), ..., h_P(z))$ is a polynomial matrix of dimension $L \times P$ (with $L > P$) such that $Rg\,(H(z)) = P$ for all $z \in \mathbb{C}$ (**A-2**). Each column $(h_k(z))_{k=1,P}$ is a $L$-dimensional function $h_k(z) = (h_k^{(1)}(z), ..., h_k^{(L)}(z))^\top$ where $h_k^{(j)}(z) = \sum_{p=0}^{Q_{k,j}} h_k^j(p)$ (**A-3**). The degree of $h_k(z)$ is defined by $Q_k = \max_{j=1,P} Q_{k,j}$. Finally, we suppose that $H(z)$ is a column reduced matrix, *i.e.*, $Rg\,(h_1(Q_1), .., h_1(Q_P)) = P$ (**A-4**).

By collecting the observation $y(n)$ in the regressor vector $Y_N(n) = (y(n)^\top, y(n-1)^\top, ..., y(n-(N+1))^\top)^\top$ of dimension $NL$, we get,

$$Y_N(n) = \sum_{l=1}^{P} \mathcal{T}(\mathrm{h}_l)\; S_l(n) \tag{2}$$

where $\mathcal{T}(h_l)$ is the Sylvester convolution matrix of dimension $NL \times (N + Q_k)$, between the source $s_l(n)$ and the $L$-dimensional sensors (see, [1]). $S_l(n) = (s_l(n), ..., s_l(n - (N+Q_k+1)))^\top$ is a vector of dimension $N + Q_k$ which contains the contributions of the source $s_l(n)$. We assume that

$N \geq \sum_{k=1}^{P} Q_k$ (**A-5**).

Since the mixture is linear, restoring M input signals $s_k$ (with $M \leq P$) turns to estimate $M$ vectors $(g_k)_{1\leq k \leq M}$ of length $NL$ such as,

$$f_k^{\star\top} \stackrel{def}{=} g_k^{\star\top}\mathcal{T}(\mathrm{H}) = (f_{|1|,k}|...|f_{|l|,k}|...|f_{|P|,k})^\top = u_k$$

where $u_k$ is a canonical vector of of length $K \stackrel{def}{=} NP + \sum_{k=1}^{P} Q_k$ selecting the $k^{th}$ source (up to a scaled arbitrary permutation) with arbitrary delay. Under the key identifiability assumptions **A-2,4,5** the convolution matrix $\mathcal{T}(\mathrm{H})$ is full-column rank (see [7], for example). Thus, all global impulse responses $f_k^\top = g_k^\top \mathcal{T}(\mathrm{H})$ are achievable, in particular those leading to $f_{|l|,k} = \delta^p_{|l|,k}$ *i.e.* $(0...010..0)$ for $l = k$ and $f_{|l|,k} = 0_{N+Q_l}$ for $l \neq k$, corresponding to the solution $u_k$.

## 3. HIERARCHICAL CRITERIA

We investigate the estimation of the 'equalizers vectors' $g_k^\star$. Under identifiability conditions, we know, see Appendix, that 'separating' solutions of the CM criterion $\Phi_c(g_k) = E[\big(|v_k(n)|^2 - r\big)^2]$, (where $v_k(n) = g_k^\top Y_N(n)$ and $r$ an arbitrary dispersion constant) are described by:

$$f_1^\star = \pm\sqrt{r/r_l}\,(0...0|...|\underbrace{0..010..0}_{l}|...|0...0)$$

where $r_l \stackrel{def}{=} E[s_l^4]/E[s_l^2]$ is the so-called constant dispersion of the source number $l$. We denote $\mathcal{F}_1^\star$ the subset of dimension $2^N \sum_k 2^{Q_k}$ of all separating vector $f_1^\star$ and $\bar{\mathcal{F}}_1$ the subset of all other extrema. Note that $\Phi_c$ leads to perfect restoration of one source, the drawback is the absence of guarantee concerning multiple selection of the same source. This motivates the following sections.

### 3.1. Criteria

We suggest to introduce a set of criteria $(\Phi_k)_{1\leq k\leq M}$, according to the following procedure:

$$\Phi_k((g_s)_{s=1,k}) \stackrel{def}{=} \Phi_c(g_k) + \beta_k \sum_{j=1}^{k-1}\sum_{m=-T}^{+T} \mathcal{E}_{k,j}(m)$$

we take the convention $\Phi_1((g_s)_{s=1}) \stackrel{def}{=} \Phi_c(g_1)$ for $k = 1$. $\Phi_c(g_k)$ denotes herein the CM criterion. The criterion $\mathcal{E}_{k,j}(m) = |E[v_k(n)v_j(n-m)]|^2$ is a quadratic function of $g_k$ and $g_j$ based on second order statistics. Basically, it corresponds to a decorrelation constraint between all outputs $v_k(n)$, for $k = 1, ..., M$ and over all possible delays $m$ corresponding to $T = N + \max_k(Q_k)$ . $\beta_k$ is a positive constant, introduced to control the level of the constraint. It will be characterized in the sequel.

This set of criteria is based on a 'hierarchical' principle: (**a**) Each criterion $\Phi_j(g_1, ..., g_j)$ share $j-1$ parameter with $\Phi_{j-1}(g_1, ..., g_{j-1})$. (**b**) From $\Phi_{j-1} \to \Phi_j$ a constraint is added. Note that according to the whiteness assumptions **A-1**, it is not necessary to know $Q_k$ accurately, an overestimation of $Q_k$ is sufficient.

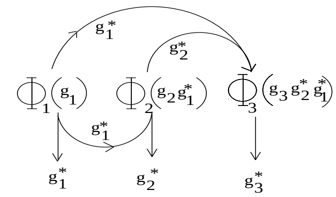


**Figure 1.** Connections between $\Phi_k$ for $k = 1, 2, 3$.

### 3.2. Extrema Analysis

An analytical analysis of $(\Phi_k)_{1\leq k\leq M}$ extrema is proposed in this subsection, in the real case for sake of simplicity.

Let us point out that the extrema solutions of the criteria $\Phi_2$ depend of the extrema solutions of $\Phi_1$ and so on, until $\Phi_M$. Thus, to exhibit the solutions of $\Phi_k$ we have to 'plug' all previous extrema solutions of $(\Phi_s)_{s<k}$ in $\Phi_k$.

In the sequel, we deal first with the separating solutions. Note that if we introduce the solutions $(\mathcal{F}_j^\star)_{j<k}$ in $\Phi_k$, the characterization of the stationary points of criterion $\Phi_k$ is easily derivable, see next proposal (and Appendix).

**Proposal 1** *For $f_j^\star \in \mathcal{F}_j^\star$ $(1 \leq j \leq k-1)$, the extrema $f_k \in \mathcal{F}_k$ of $\Phi_k$ are solutions of the equation:*

$$2P_s\Delta_s(f_k)\, f_k + r\beta_k \sum_{l\in I_{k-1}} \sum_{\epsilon=1}^{N+Q_l} \frac{\sigma_l^4}{r_l} f_{|l|,k}(\epsilon)\ \delta_\epsilon = 0_K \qquad (3)$$

*where $I_{k-1}$ is a subset of dimension $k-1$ which containt the subscripts of all sources selected by criteria $(\Phi_s)_{s<k}$. $\delta_\epsilon$ denotes a canonical vector of length $K$ with 1 on the $(\epsilon+1)^{th}$ entry and $0_k$ a null vector of same dimension.*

The previous equation stems from a straightforward gradient derivation of criterion $\Phi_k$ expressed in term of the global impulse response $f_k = \mathcal{T}(\mathrm{H})^\top g_k$, see [5] for details.

Moreover, one can give, the expression of the separating solution (for which there is only one non-zeros components) of $\Phi_k$, see next proposal.

**Proposal 2** *The separating solution $f_k^\star \in \mathcal{F}_k^\star$ are described by:*

$$f_k^\star = \begin{cases} \pm\sqrt{\frac{r}{r_l}\,(1 - \frac{\beta_k}{2\rho_l})}\; u_l & for\, l \in I_{k-1}\ and, \\ \pm\sqrt{\frac{r}{r_l}}\; u_l & elsewhere \end{cases}$$

According to the previous result, one can remark that it is always possible to select the level constraint $\beta_k$ in order to avoid separating solutions given by previous criteria $(\Phi_s)_{s<k}$. We get the simple necessary condition,

$$\beta_k \geq 2\,\rho_l \qquad (4)$$

where we recall that $\rho_l = E[s_l^4]/E[s_l^2]^2$. Actually, for preserving from multiple selection of the same source, we only have to verify that each criterion has a level constraint such as $\beta_k \geq 2\max_{m=1,P}\rho_m$. Note that at 'each step ' $k$ the dimension of the separating set $\mathcal{F}_k^\star$ is reduced by $2^{N+Q_j}$, where we recall that $Q_j$ is the degree of the channel 'excited' by the $j^{th}$ source supposed selected at step $k-1$. The iterative 'plugging' scheme is illustrated in Figure 1, with $M = 3$.

From lack of space, we omit the analytical solution of the 'non-separating' solution belonging to $\bar{\mathcal{F}}_k$ (see, [5]). However, let us point out that it will be shown in the next section, that the corresponding vector are necessarely unstable attractors ! We show moreover that the separating solutions belonging to $\mathcal{F}_k^\star$ are the only global minima.

### 3.3. Stability

We investigate the stability of the extrema $f_k$ by analyzing the sign definiteness of the Hessian matrix of each criterion $\Phi_k$. The result is summarized in the next proposition.

**Proposal 3** *The extrema of criterion $\Phi_k$ corresponding to global minima must verify the condition:*

$$\mathcal{T}(H)\, \Psi_k((f_s)_{s=1,k})\, \mathcal{T}(H)^\top \geq 0 \tag{5}$$

*with $\Psi_k(.) \stackrel{def}{=} 2\Psi_c(f_k) + \beta_k\, \Psi_\epsilon(f_1, ..., f_{k-1})$ where $\Psi_c(.)$ and $\Psi_\epsilon(.)$ are symmetric matrices of dimension $K \times K$ defined respectively by,*

$$\Psi_c(f_k) = \mathcal{P}_s \Delta_s(f_k) + 6\, \mathcal{P}_s f_k f_k^\top \mathcal{P}_s + \mathcal{P}_s \mathcal{K}_s \; diag(f_k f_k^\top) \text{ and,}$$

$$\Psi_\epsilon((f_s)_{s<k}) = \sum_{m=-T}^{+T} \sum_{j<k} J_s^{(m)}\, f_j f_j^\top\, J_s^{(m)\top}$$

*where $J_s^{(m)}$ is a bloc-diagonal matrix of dimension $K \times K$ such that $J_s^{(m)} = I_P \otimes (\sigma_1^2 J_{m,1}, ..., \sigma_P^2 J_{m,P})$ with $J_{m,k}$ a Jordan matrix of dimension $(N + Q_k) \times (N + Q_k)$ defined as $(J_{m,k})_{a,b} = 1$ if $a - b = m$ and $0$ elsewhere.*

Let us point out that, according to the particular 'plugging' design introduced above, the only two situations that must be addressed are $\mathcal{F}_k^\star \cup (\mathcal{F}_j^\star)_{j \leq k}$ and $\mathcal{F}_k \cup (\mathcal{F}_j^\star)_{j\leq k}$ for the subset stability analysis of each criterion.

Let us consider first the case where all extrema $f_j$ are belong $(\mathcal{F}_j^\star)_{j \leq k}$. From the hypothesis of sub-Gaussian input signals (*i.e.* $\rho_k \leq 3$), it is straightforward to check that $\Psi_k$ is a diagonal matrix with positive terms. Indeed, for $i \neq l$ we get $\Psi_c(f_k^\star)_{|i|} = \sigma_i^2(3 - \rho_l)\frac{r}{r_l} I_{N+Q_i}$. For the bloc $l$, the diagonal components of the bloc-matrix $\Psi_c(f_k^\star)_{|l|}$ are equal to $\sigma_l^2(3-\rho_l)\frac{r}{r_l}$ or $2\sigma_l^2\, r$. $\Psi_\epsilon$ is a diagonal matrix for which the non-zeros components take the form $\sigma_j^4 r / r_j$. Thus, the inequality (5) is always verified and the correspondings points $f_k^\star$ are global minima.

In the second case, we consider the extrema point $f_j^\star \in (\mathcal{F}_j^\star)_{j<k}$ and $f_k \in \bar{\mathcal{F}}_k$. $\Psi_\epsilon$ is a positive diagonal matrix and $\Psi_c$ is a sparse symmetric matrix. The analysis of the sign of $\Psi_c$ is not straightforward excepted in the trivial case $f_k = 0$, which is maximum since $\Psi_k(0) = -r\mathcal{P}_s$. For the other extrema $f_k \in \bar{\mathcal{F}}_k$ the non diagonal contributions of $\Psi_c$ are of the form $(\Psi_c(f_k))_{ij} = \pm 6\, \sigma_i^2 \sigma_j^2 \underline{\omega}_i \underline{\omega}_j$ or $0$, for which some terms $\underline{\omega}_j$ are equal to $\omega_j$ (see Lemma 2). We can also verify that the diagonal contributions are positive. (From lack of space it is impossible to give here all the details of the differents contributions, see [5]). Due to the particular form of $\Psi_c$ we can derive an analysis of the quadratic form associated to $\Psi_k$, the result is given in the next proposal.

**Proposal 4** *If $f_k \in \bar{\mathcal{F}}_k$, for any vector $x$ of dimension $K$, we have the following decomposition,*

$$x^\top \Psi_k(f_k, (f_s^\star)_{s<k})\, x = \sum_k \lambda_k x_k^2 + \sum_{i,j} x_{ij}^\top B_{ij} x_{ij}$$

*where the $\lambda_k$ denotes positives terms and $B_{ij}$ a matrix of dimension $2 \times 2$ of negative determinant $|B_{ij}| = 4\,\sigma_i^4 \sigma_j^4 \underline{\omega}_i^2 \underline{\omega}_j^2 (\rho_i \rho_j - 9)$. The notation $x_{ij}$ introduced above denotes the vector $(x_i x_j)$ of dimension $2$ extracted from $x$.*

Thus we can easily verify that there is some vector $\bar{x}$ such that $\bar{x}^\top \Psi_k \bar{x} = \sum_k \lambda_k x_k^2 \geq 0$ and vectors $\underline{x}$ such that $\underline{x}^\top \Psi_k\, \underline{x} = \sum_{i,j} x_{ij}^\top B_{ij} x_{ij} \leq 0$ which proof that $f_k$ is a saddle point. See Figure 2 for the location of the extrema.

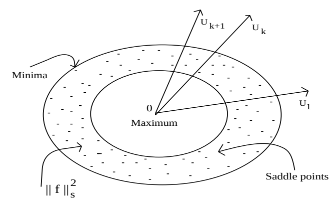


**Figure 2.** Location of the extrema.

## 4. ALGORITHM

An adaptive algorithm for minimizing the criteria $(\Phi_k)_{1 \leq k \leq M}$ stems naturally from the design introduced in Sect 3.1.

For each function $\Phi_k((g_s)_{s=1,k})$, we propose to derive a simple gradient descent algorithm according to the scheme:

$$g_k^{(t+1)} \leftarrow g_k^{(t)} - \mu\, \nabla_{g_k} \Phi_k \left( g_k^{(t)} / (g_s^{(t+1)})_{s<k} \right) \tag{6}$$

with $\mu$ a small positive step-size. The gradient is given by the expression,

$$\nabla_{g_k} \Phi_k(g_k) = \nabla_{g_k} \Phi_c(g_k) + \beta_k \sum_{j=1}^{k-1} \sum_{m=-T}^{+T} \nabla_{g_k} \mathcal{E}_{k,j}(m) \tag{7}$$

We get, $\nabla_{g_k} \Phi_c(g_k) = 4 \left( |v_k(n)|^2 - r \right) v_k(n) Y_N^*(n)$ and, $\nabla_{g_k} \mathcal{E}_{k,j}(m) = 2\, (g_k^\top \hat{R}_Y(m)\, g_j)\, \hat{R}_Y^*(m)\, g_j^*$ with $v_k(n) = g_k^\top Y_N(n)$ and where $\hat{R}_Y(m)$ denotes the estimation of covariance matrix $E\left[ Y_N(n) Y_N^*(n-m)^\top \right]$. The matrix $\hat{R}_Y(m)$ may be estimated by a recursive procedure, for instance. Actually, the important point herein is to notice that the asymptotic convergence points (in mean) of the algorithm are exactly the extrema of the criteria $(\Phi_k)_{1\leq k \leq M}$ if the estimator $\hat{R}_Y$ is unbiased. For more details see [5]. Therefore, we know that (6) converge to the desired separating points.

## 5. SIMULATIONS

Since the convergence proof is established, we give just an illustrative example in this section (see [5] for extensive simulations). The simulations were performed with a channel $H(z)$ driven by two BPSK sequence $(\pm 1)$. The zeros location of $h_{ij}(z)$ are display in Table I.

| Zeros locations of the channel | | |
|---|---|---|
| $h_{11}(z)$ | -22.9968 | -0.4007 |
| $h_{21}(z)$ | -0.0597- 0.6147i | -0.0597+ 0.6147i |
| $h_{31}(z)$ | -0.2605- 1.1492i | -0.2605+ 1.1492i |
| $h_{12}(z)$ | -0.3139- 0.8746i | -0.3139+ 0.8746i |
| $h_{22}(z)$ | -0.6007- 0.8939i | -0.6007+ 0.8939i |
| $h_{32}(z)$ | -0.1137- 0.4419i | -0.1137+ 0.4419i |

Table I

We select $N=4$. One may easily check that $H(z)$ verifies the identifiability conditions. The step size is selected to $\mu = 0.01$ for the two updating equations. The receivers $g_1$ and $g_2$ are initialized respectively with $\delta_3$ and $\delta_{10}$. We take $\beta_1 = \beta_2 = 2$. The covariance matrices are estimated recursively, with a forgetting factor $\alpha = 0.98$.

We plot, for several realizations, the trajectories of the residual ISI concerning the estimation of each source (see Figure 3). At convergence ($t = 3000$), we plot the global impulse response $f_1^\star$ and $f_2^\star$ (see Figure 4). In this case, the associated receivers $g_1^\star$ and $g_2^\star$ achieve respectively the restoration of $s_1$ and $s_2$ with delays $\nu=4$ and $\nu=6$.

## A APPENDIX

In this appendix, we give the main results concerning the derivation of $\Phi_c$ extrema points. In Lemma 1, we give the gradient equation characterization. The analytical expression of the extrema is derived in Lemma 2. A location of the extrema, in terms of global impulse response space, is given in Lemma 3.

**Lemma 1** *The gradient the CM in term of the global impulse response $f = \mathcal{T}(\mathrm{H})^\top g$ writes as:*

$$\nabla_g \Phi_c(g) = 4\mathcal{T}(H)\, \mathcal{P}_s\, \Delta_s(f) f \tag{8}$$

*where $\Delta_s(f) = (3\|f\|_s^2 - r)I + \mathcal{K}_s\, diag(ff^\top)$ is a diagonal matrix where $\mathrm{diag}(A)$ denotes the matrix extracted from $A$ with the same diagonal entries and 0 elsewhere. Herein $\|f_k\|_s^2$ is understood as,*

$$\|f_k\|_s^2 \overset{def}{=} \sum_{j=1}^{P} \sigma_j^2\, \|f_{|j|,k}\|_2^2 \tag{9}$$

*where $\|f_{|j|,k}\|_2^2 = \sum_{p=1}^{N+Q_j} f_{|j|,k}(p)^2$. $\mathcal{P}_s$ and $\mathcal{K}_s$ denotes two bloc-diagonal matrix of dimension $K \times K$ defined as $\mathcal{K}_s = I_P \otimes (\mathcal{K}_1, ..., \mathcal{K}_P)$ with $\mathcal{K}_i = (\rho_i - 3)\, I_{N+Q_i}$ and $\rho_i = E[s_i^4]/E[s_i^2]^2$. As the same way as $\mathcal{K}_s$, we have $\mathcal{P}_s = I_P \otimes (\mathcal{P}_1, ..., \mathcal{P}_P)$ with $\mathcal{P}_i = \sigma_i^2\, I_{N+Q_i}$.*

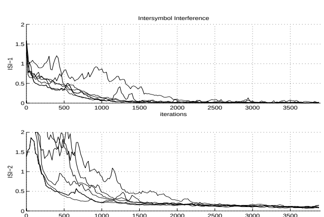

**Figure 3.** ISI versus iterations number.

**Lemma 2** *If we denotes $f_c = (f_{|1|}^{c\top}|...|f_{|l|}^{c\top}|...|f_{|P|}^{c\top})^\top$, the extrema points of $\Phi_c$ (i.e. the solutions of the equation $\mathcal{P}_s\, \Delta_s(f) f = 0$), then we have*

$$f_{|l|}^c = \begin{cases} \sum_{p \subseteq I_l} \pm\omega_l\, \delta_{|l|}^p \quad with\, \omega_l^2 = \dfrac{r}{\sigma_l^2(\rho_l - 3) + 3\sum_k \nu_k \frac{\sigma_l^2(\rho_l-3)}{\sigma_k^2(\rho_k-3)}} \\ or\ 0_{N+Q_l} \end{cases}$$

*where $I_l \overset{def}{=} \{0, 1, ..., N{+}Q_l{-}1\}$ and $\nu_k$ denotes the number of non-zeros components in each sub-vector $f_{|k|}^c$. Moreover, it is understood that $\delta_{|l|}^p$ and $0_{N+Ql}$ are respectively canonical vector (with non zero components at the $(p+1)^{th}$ entry) and null vector of dimension $N + Q_l$.*

*The notation $p \subseteq I_l$ is referred to a summation over all element of the subset $p$ included in $I_l$.*

**Lemma 3** *The set of all extrema points corresponding to the points $f_c \neq 0$ verify, $\frac{r}{3} \leq \|f_c\|_s^2 \leq min_l \frac{r}{r_l}$.*

## REFERENCES


[1] I. Fijalkow, A. Touzni, and J. Treichler, *Fractionally-Spaced Equalization using CMA: Robustness to channel noise and lack of disparity*, IEEE Tr. on SP special issue on Signal Processing for Advanced Communications, pp 268-271, January 1997..

[2] J.R. Treichler and M.G. Larimore, *New processing techniques based on the constant modulus adaptive algorithm*, IEEE Trans. Acoustics, Speech, Signal Processing, vol. ASSP-33, pp. 420-431, April 1985.

[3] J.K. Tugnait, *Blind spatio-temporal equalization and impulse response estimation for MIMO channels using a Godard cost function*, IEEE Tr. on SP special issue on SP, pp 268-271, January 1997.

[4] J. R. Treichler, I. Fijalkow, and C.R. Johnson, JR. *Fractionally spaced equalizers: How long should be ?*, in IEEE Signal Processing Mag., May 1996.

[5] A. Touzni, I. Fijalkow, J. R. Treichler, *Constant Modulus solutions for MIMO linear mixtures deconvolution*, to be submitted.

[6] L. Castedo, C.J. Escudero, and A. Dapena, *A blind Signal Separation Method for Multiuser Communications*

[7] A.Gorokhov, Ph. Loubaton, E.Moulines , *Second order blind equalization in multiple inputs multiple outputs FIR systems; a weighted squares approach*, in Proc. ICASSP 1996.

[8] A. Touzni, I. Fijalkow, *Blind adaptive equalization and simultaneous separation of convolutive mixtures*, International DSP conf. in GREECE, santorin, July 1997.

[9] J.F. Cardoso, B. Laheld, *Equivariant adaptive source separation*, IEEE Trans. SP, vol 44, pp. 3017-3030, Dec 1996.

[10] C.B. Papadias, A. Paulraj, *A constant Modulus algorithm for multi-user signal separation in presence of delay spread using antenna arrays*, to be published in SP Letters.


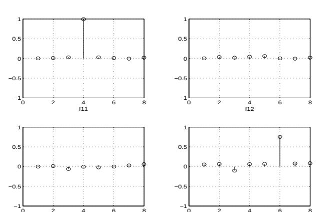

**Figure 4.** $f_1$ and $f_2$ at convergence.