FRACTIONALLY-SPACED EQUALIZATION OF TIME-VARYING MOBILE COMMUNICATIONS

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ABSTRACT

The improved convergence speed and tracking properties of fractionally-spaced equalizers are analyzed. We consider in particular the effect of a frequency offset between the transmitter baud rate and the receiver sampling clock that induces important time-variations. We show that a fractionally-spaced equalizer can handle the intersymbol interferences (ISI) induced when the propagation channel doesn't introduce too much ISI.

Keywords : Fractionally-spaced equalization, tracking, sampling frequency offset.

1. INTRODUCTION

Recent gathering and categorization of wideband communications received on a mobile vehicle have shown some difficulties due to a small frequency offset between baud and sampling clocks, [2] [1]. The resulting timing error, yielding to a "pulse shape drift" phenomenon, is bearable for most cases of mild channel time-variations. However, when the channel is difficult to estimate/equalize, the drift may prevent from a good estimation of the channel.

In this paper our goal is to analyze the tracking capabilities of the **fractionally-spaced equalizer** (FSE) when a sampling frequency offset occurs. Our study is limited to baseband signals after demodulation (supposed correct) has been performed. The frequencies of the receiver and the transmitter clocks differ one from the other by a small amount, possibly due to electronic defficiency or to a frequency estimation error. The offset is assumed to have a given value, we are not trying here to further remove it.

At first, we consider the case of a white gaussian additive channel noise (i.e., no channel dispersion) so that the equalizer's only goal is to track the pulse drift. This can be done by a short rapidly varying equalizer at the cost of a high residual variance. Afterwards, we add the effect of propagation, i.e., slowly time-varying measured channel dispersion caracteristics. The fast convergence properties of FSEs are measured. The importance of channel disparity, defined as the effective diversity [3], is enhanced. The question is: how robust is the fractionally-spaced equalizer to the system time-variations ?

[1] The authors would like to thank Applied Signal Technology Inc. (Sunnyvale, CA, USA) and Cornell University - Blind Equalization Research Group (CU-BERG) for providing the data files. These data files should soon be avaliable at `http://spib.rice.edu/spib.html`.

2. PROBLEM SETTING

Let us consider the received baseband data signal $y(t)$ described as

$$y(t) = \sum_k s_k h(t - kT) + w(t) \tag{1}$$

$h(t)$ modelizes the convolution of the pulse shaping filter, the communication channel $c(t)$ and the receiver matched filter. The received signal is the result of the convolution between $h(t)$ and the transmitted data symbols s_k. $w(t)$ is an additive, white Gaussian noise. s_k is drawn from an alphabet of M values with equal probability. The s_k represent an i.i.d sequence, with variance 1.

In order to study only the effect of the frequency sampling offset, we assume that the band-limited channel is ideal, i.e. the channel causes no ISI. If the received signal is sampled at a rate $frac1T + \tau$ where τ is the timing offset error, the expression becomes

$$y(nT+n\tau) = s_n h(n\tau) + \sum_{k\neq n} s_k h((n-k)T+n\tau) + w(nT+n\tau) \tag{2}$$

Even if the sequence $h(nT)$ satisfies the Nyquist condition, the values sampled with a frequency offset do not verify this condition. Moreover, (2) is no longer a convolution as it is when $\tau = 0$. It is as if we had a time-varying channel.

3. EFFECT OF A TIMING OFFSET

3.1. Effect on signal

To understand how the offset influences the received signal, we study numerically the evolution of the sampled $h(t)$ versus time for a given offset. To do so, we choose the raised cosine pulse shaping filter which is widely used in the practice. $h(t)$ is defined by

$$h(t) = sinc[\frac{t}{T}]\frac{cos[\pi\beta\frac{t}{T}]}{1-\frac{4\beta^2t^2}{T^2}} \qquad (3)$$

where β is the roll-off factor. At the instant $n(T+\tau)$, we can consider (2) as if we had an instantaneous convolution of the input s_k with $h((n-k)T+n\tau)$, denoted $h_n(k)$,

$$h_n(k) = sinc[(n-k)+n\frac{\tau}{T}]\frac{cos[\pi\beta((n-k)+n\frac{\tau}{T})]}{1-\frac{4\beta^2((n-k)+n\frac{\tau}{T})^2}{T^2}} \qquad (4)$$

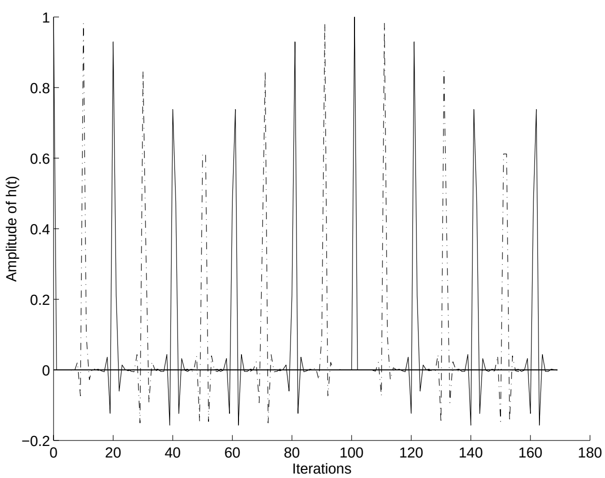

Figure 1: Evolution of $h_n(k)$ versus n, for $\frac{\tau}{T}=0.01$

Figure (1) represents the shape of $h_n(k)$ versus the instant n. The curves are drawn every $10T$. As long as $n\tau$ is small, the clock samples always around the peak of the cardinal sine function. But as times goes by, the value of $n\tau$ becomes more and more important and consequently the influence of the offset becomes effective. Because of the offset, we get a succession of deformed cardinal sines which are no more systematically centered around the desired value. The curves move away from the desired value by a distance of the order of $n\tau$. According to the sign of the offset, the shifting of the distorded cardinal sine is retarded or advanced versus the baud rate. We remark that the evolution of the pulse is periodical with period $\frac{T}{\tau}$ and affects mainly the amplitude of the main taps. This phenomenon is called timing drift or rolling.

For a large enough n, the rolling causes a jump of one bit or its repetition which distorts enormously the received signal and disturbes the remaining parts of the receiver.

Note that the effect of the sampling frequency offset remains the same when we sample with a fractional value $\frac{T+\tau}{L}$ with $L>1$.

3.2. The fractionally-spaced equalizer

In this paragraph, we want to know how the fractionally-spaced equalizer (FSE) is affected by a residual sampling frequency offset.

The temporal diversity ($L>1$) can be modelized by a multivariate system (see for instance [5]). In the time-varying context, we can extend this multichannel and equalizer system to:

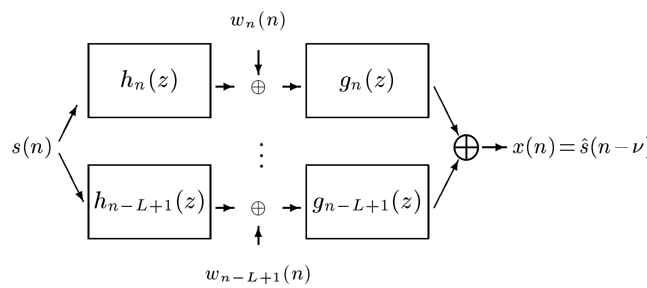

Figure 2: Multi-Channel / Equalizer

where each $h_n(z)$ is the instantaneous transfer function associated to the impulse response $h_n(k)$. $w_i(n) = w(LnT + i)$ is the sampled noise. The FSE processes the L-dimensionnal received signal by:

$$x(n) = \sum_{k=0}^{N-1} \mathbf{g}_k \mathbf{y}(n-k) = \mathbf{g}^\top \mathbf{Y}_N(n) \qquad (5)$$

where $\mathbf{g} = (\mathbf{g}_0^\top, ..\mathbf{g}_{N-1}^\top)^\top$ is the impulse response of the equalizer. The length of the FSE is equal to NL with N greater or equal to the channel degree Q. $Y_N(n)$ is the regression vector which contains the observations at the instants $n, n-1, ..., n-N+1$.

$$\mathbf{Y}_N(n) = \mathcal{T}(\mathbf{h}_n)S_{N+Q}(n) + W_N(n) \qquad (6)$$

where $S_{N+Q}(n) = (s(n), s(n-1), ..., s(n-N-Q+1))^\top$ and $\mathcal{T}(\mathbf{h}_n)$ is defined by

$$\begin{bmatrix} \mathbf{h}_n(-K) & .. & \mathbf{h}_n(K) & 0 & \cdots & 0 \\ 0 & \mathbf{h}_{n-1}(-K) & .. & \mathbf{h}_{n-1}(-K) & \cdots & 0 \\ & & \ddots & & & \\ 0 & \cdots & 0 & \mathbf{h}_{n-N+1}(-K) & .. & \mathbf{h}_{n-N+1}(K) \end{bmatrix} \qquad (7)$$

where $\mathbf{h}_n(i) = \mathbf{h}(i + n\tau + [n\frac{\tau}{T}])$ for $i = [-K, ...K]$. $\mathcal{T}(\mathbf{h}_n)$ is the matrix linked to the N transfer functions $\mathbf{h}_i(z)$ for $i = [n-N+1...n]$. This matrix depends on the time because of the presence of the timing offset.

In the case of an ideal channel, the impulse responses representing the pulse shaping and matched filter can be modelized by a small number of taps. The effective $2K$ taps are centered around a value that follows $n\tau$, we take $[n\tau]$ where $[x]$ represents the integer value of x.

If τ was zero, $\mathbf{h}_n(i) = \mathbf{h}_{n-1}(i), \forall n \geq 0$, so that $\mathcal{T}(\mathbf{h}_n)$ would be a bloc Toeplitz convolution matrix. In that case (i.e., stationary), even with a non-ideal channel, in absence of noise and under the channel identifiability condition (i.e., $h(z) \neq 0, \forall z$) and $N \geq Q$, $\mathcal{T}(\mathbf{h})$ is full column rank [5]. So that perfect equalization is achievable, i.e., there exists an equalizer $\mathbf{g}$ such as $\mathcal{T}(\mathbf{h})^\top \mathbf{g}$ is equal to a canonic vector. Besides, we know that an ill-condition of $\mathcal{T}(\mathbf{h})^\top \mathcal{T}(\mathbf{h})$ results in bad equalization performances, [3].

3.3. Offset tracking

Because of the good conditionning of $\mathcal{T}(\mathbf{h}_n)$ (in our simulations from (4)) at each instant, we can consider that there is an equalizer $\mathbf{g}_n$ realizing the correct inversion of $\mathcal{T}(\mathbf{h}_n)$,

$$\mathcal{T}(\mathbf{h}_n)^\top \mathbf{g}_n = \delta_\nu = [0..010..0]\top. \tag{8}$$

From a previous work, [1], we know that the matrix good condition is crucial for a good equalization when the channel is time-varying. It allows tracking with a faster convergence than when the matrix is ill-conditionned. The equalizer should therefore be able to correct the presence of the sampling frequency offset.

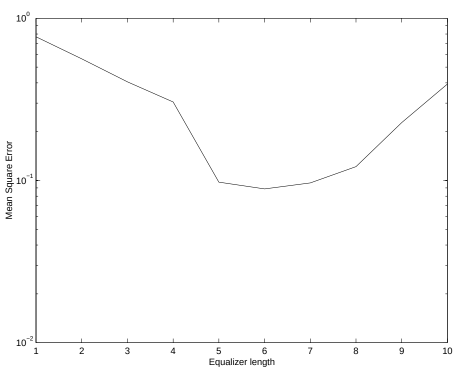

Figure 3: Mean square error versus N

The only remaining question is whether an adptive algorithm is quick enough to update the correct inversion at each instant? This implies a very large step-size to accelerate the convergence. Such a step-size induces a large residual jitter error. To illustrate the FSE performances, we simulate an FSE updated by an LMS algorithm. The received signal is sampled with a sampling frequency offset, $\frac{\tau}{T} = 10^{-2}$, which is larger than realistic values. The emitted data is an i.i.d and binary sequence. The channel is ideal, with an input signal to noise ratio, $SNR = 20dB$. The LMS is run with a step-size equal to 0.1. We have processed 20 realisations of a sequence of 500T, with different values of the equalizer length N.

As we can see on Figure 3, the performances of the LMS are correct as long as N is inside a very limited set of values. Below the limit of $N = 5$, the squared error is very important because the equalizer is unable to remove the ISI induced by the frequency offset. Above $N = 9$, the squared error becomes large because of the residual stochastic jitter due to the high value of the step-size and of the noise, see [4].

However the important result of this simulation is that we can bear a very high level of sampling frequency offset with an FSE with relevant length.

4. REAL CASE

In this section, we want to confront our study on FSEs with real data suffering from a sampling frequency offset. At first, we should note that realistic channels are not ideal. They induce ISI that causes extra degradation of the performances. But which is the limit value of the residual timing offset for which the equalizer performs correctly ? We try to evaluate empirically this limit by processing realistic channels.

4.1. Realistic signal

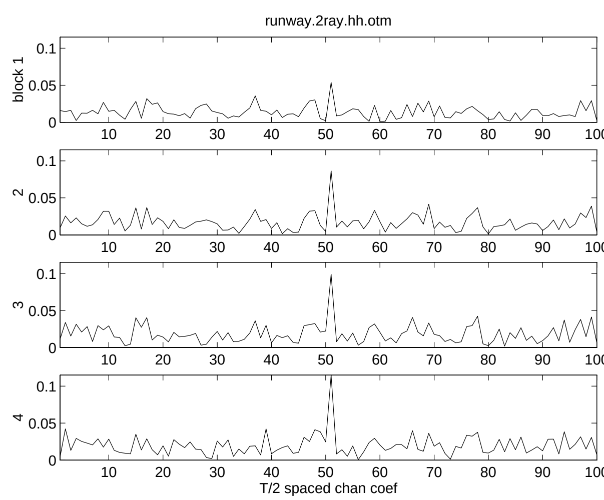

Figure 4: Evolution of the realistic channel runway.2ray.hh.otm

The realistic channels we study have been first analyzed by the Cornell University - Blind Equalization Research Group (CU-BERG) in collaboration with Applied Signal Technlogy, [2]. They are time-varying microwave channels estimated from data collected by an antenna put on a mobile vehicle in different environments. During the processinge of this data a problem of sampling frequency offset appeared on some snapshots. The frequency offset was evaluated to be comprised between $\frac{\tau}{T} = 10^{-5}$ and $\frac{\tau}{T} = 10^{-6}$. Figure 4 represents the evolution of a realistic mild time-varying channel versus time. We notice the similarity between the evolution of the numerically calculated pulse shapes in Figure 1 and of the realistic channel Figure 4. The amplitude of the realistic channel main taps may raise with time because of the frequency offset.

4.2. Fractionally-spaced equalization

We assume that the time-variation of the channel $c(t)$ by itself is very slow compared to the variations due to the frequency offset. This assumption seems to be reasonable with regard to Figure 4. The simulations are similar to that in the previous section but realized on the data from the file `runway.2ray.hh.otm` described in [2]. We want to know which values of equalizer length and step-size are adequate in order for the FSE to handle the channel dispersion combined with the receiver sampling frequency offset.

On Figure 5, we plot the equalizer input downsampled to one symbol per T, itis downsampled by a factor 8 from the real input. Obviously, the channel rolling induces ISI to be compensated by an equalizer.

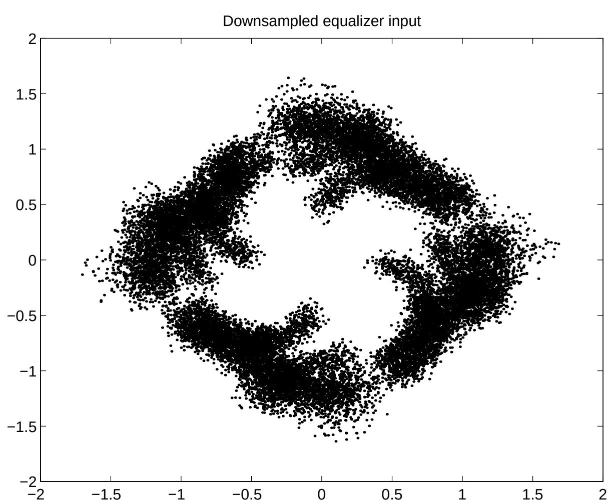

Figure 5: Downsampled equalizer input

On Figure 6, the equalizer output is displayed. Simulation has been run with a step-size μ equal to 0.01, and equalizer length $N = 8$. The ISI has been greatly reduced and the remaining ISI can be taken care of by any decision device.

Other simulations with $N = 8$ show that if we increase the step-size above $\mu = 0.02$, the resulting equalizer output shows a larger mean square error. It means that the algorithm jitter (i.e., stochastic jitter due to noise and non-zero updating term at each iteration) becomes important. On the other hand, a step-size smaller than $\mu = 0.003$ does not allow to follow the system time-variations. The range of step-size that provides with reasonable performances is much smaller than for simulated data.

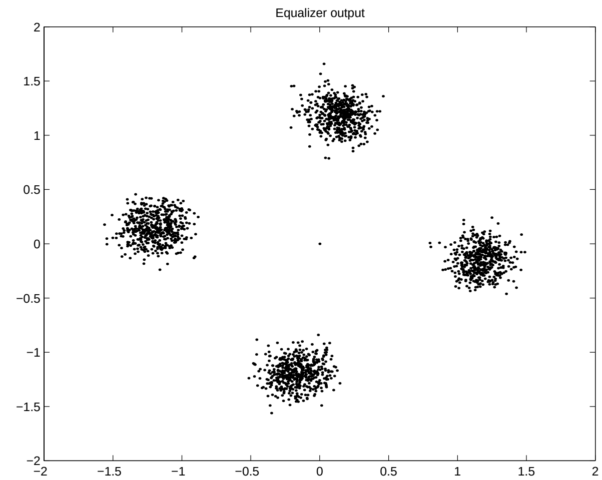

Figure 6: Equalizer output with $\mu = 0.01$

5. CONCLUSION

We have shown that FSEs allow to bear a quite high frequency offset between the transmitter baud rate and receiver sampling frequency. Even when there is some channel dispersion, good performances are acheived by the FSE on the training sequence.

The remaining question is now how to maintain these performances on the data for which the equalizer is not trained.

6. REFERENCES

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