DIRECTION FINDING FOR UNSTRUCTURED EMITTERS IN THE PRESENCE OF STRUCTURED INTERFERERS

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ABSTRACT

This paper addresses the problem of direction finding for unstructured emitters in environments that contain interference signals with exploitable properties. The CA-JML algorithm, presented here, offers dramatic angle estimation improvement in environments that contain exploitable interference. The performance improvement is equivalent to the removal of the interferers from the environment. The complexity of the algorithm, however, is comparable to MUSIC. Even when structured interferers are not present, the CA-JML algorithm can reduce the angle error bias exhibited in MUSIC in challenging environments. The CA-JML algorithm also admits a simple relaxation technique as each emitter is localized, which reduces the probability of angle estimation error due to the presence of ambiguous peaks in the angle objective function.

1. INTRODUCTION

A common signal processing task of sonar, radar or sesmic sensor arrays is to detect and localize narrow band emitters impinging on the array. The MUSIC algorithm, [6], achieves this goal by exploiting the angles of arrival of the received wavefronts. The algorithm does not need knowledge of the signal structure, but uses a calibrated array to estimate the angles of arrival. However, when signals have known and exploitable properties, such as constant modulus, burstiness, or known constellations, it is possible to exploit their structure to obtain linear copy weights and direction finding (DF). DF obtained by property exploitation, or Copy-Aided DF, yields inherently superior angle estimation than conventional superresolution techniques. This is because the angle estimation adheres to a lower Cramer-Rao-Bound (CRB), (equivalent to a signal model that assumes the signal of interest is known), and does not suffer from DF error bias due to the presence of interferers having nearly the same angle of arrival as the signal of interest (SOI) [1, 2, 4, 3]. Despite these advantages, there is still a need to DF signals of unknown or un-exploitable structure. These signals typically occur in environments that contain interferers of exploitable structure. The goal of this paper is to introduce a new algorithm, referred to as Copy-Aided Joint Maximum Likelihood (CA-JML), that effectively eliminates the presence of structured interferers and allows the DF and copy of any unstructured emitters that remain. The CA-JML algorithm is derived as a generalization of the Copy-Aided DF paradigm, in that it assumes that a weight subspace containing all the SOIs has been obtained. The traditional Copy-Aided DF situation occurs as a special case of a rank 1 weight subspace.

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2. DIRECTION FINDING FOR UNSTRUCTURED SIGNALS

2.1. CA-JML Signal Model and Derivation

The CA-JML algorithm can be formulated using maximum-likelihood estimation, from a signal model that assumes a known signal subspace, but makes no assumption on the structure of that signal subspace. The signal model can be described mathematically as follows:

$$\boldsymbol{x}(n) \quad = \quad \boldsymbol{a}_g s(n) + \boldsymbol{i}(n), \tag{1}$$

where $\boldsymbol{a}_g$ is the received aperture vector due to the signal of interest $s(n)$, at sample number n , and $\boldsymbol{i}(n)$ is the received interference vector, which includes environmental and receiver noise, and might also include additional interferers. The interference $\boldsymbol{i}(n)$ is modeled as a complex circular Gaussian random vector of zero mean with unknown covariance $\boldsymbol{R_{ii}}$, and the received aperture is modeled by

$$\boldsymbol{a}_g \quad \equiv \quad \boldsymbol{a}(\theta) g, \tag{2}$$

where $\boldsymbol{a}(\theta)$ is the steering vector[1] as a function of angle of arrival (AOA) θ, and g is an unknown complex gain constant. For brevity we refer to $\boldsymbol{a}(\theta)$ as $\boldsymbol{a}$ with an implicit dependency on the AOA.

[1] A model of the steering vector that mitigates against multipath, polarization diversity and other impairments is developed in [1]

After substitution of (2) into (1), maximum likelihood estimation yields the objective function [3, 1],

$$\rho_{ML} \quad \equiv \quad \ln\left(1 + \frac{|\boldsymbol{w}^H \boldsymbol{a}|^2 \left(\boldsymbol{a}^H \boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}}^{-1} \boldsymbol{a}\right)^{-1}}{R_{ss} - \boldsymbol{R}_{\boldsymbol{x}s}^H \boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}}^{-1} \boldsymbol{R}_{\boldsymbol{x}s}}\right), \tag{3}$$

where $\boldsymbol{w} \equiv \boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}}^{-1} \boldsymbol{R}_{\boldsymbol{x}s}$, $\boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}} \equiv \langle \boldsymbol{x}(n)\boldsymbol{x}(n)^H \rangle_n$, $\boldsymbol{R}_{\boldsymbol{x}s} \equiv \langle \boldsymbol{x}(n)s^*(n)\rangle_n$, and $R_{ss} \equiv \langle s(n)s^*(n)\rangle_n$. This can be written in a more amenable form as follows. Define the whitened signal subspace weights by $\boldsymbol{w}_x \equiv \boldsymbol{R}_{\boldsymbol{x}}^{-H} \frac{\boldsymbol{R}_{\boldsymbol{x}s}}{R_s}$, where $\boldsymbol{R}_{\boldsymbol{x}}^{-H}$ is the Hermitian inverse of the Cholesky factor of $\boldsymbol{R}_{\boldsymbol{x}\boldsymbol{x}}$ and $Rs \equiv \sqrt{R_{ss}}$. Also define the whitened aperture vector, $\boldsymbol{a}_x \equiv \boldsymbol{R}_{\boldsymbol{x}}^{-H} \boldsymbol{a}$. Substituting these quantities in (3) yields

$$\rho_{ML} \quad \equiv \quad \ln\left(1 + \frac{\boldsymbol{w}_x^H P(\boldsymbol{a}_x) \boldsymbol{w}_x}{1 - \boldsymbol{w}_x^H \boldsymbol{w}_x}\right), \tag{4}$$

where $P(\boldsymbol{B})$ is the projection operator defined by $P(\boldsymbol{B}) \equiv \boldsymbol{B}(\boldsymbol{B}^H \boldsymbol{B})^{-1} \boldsymbol{B}^H$ and $P_\perp(\boldsymbol{B}) \equiv \boldsymbol{I} - P(\boldsymbol{B})$.

Equation (4) is the form of the maximum likelihood function used by the copy aided DF paradigm. It assumes knowledge of the copy weights, encapsulated here in the vector $\boldsymbol{w}_x$. The copy weights can be found using a variety of techniques. The CA-JML algorithm, however, assumes that $\boldsymbol{w}_x$, is contained in the range space of a subspace matrix $\boldsymbol{W}_x$. This assumption is consistent with the environment typically assumed for DF using the MUSIC algorithm. In mathematical terms, it implies that we can write $\boldsymbol{w}_x \equiv \boldsymbol{W}_x \boldsymbol{v}$, where $\boldsymbol{W}_x$ is an $M_{sensor} \times N_{sigs}$ subspace matrix, and $\boldsymbol{v}$ is an $N_{sigs} \times 1$ vector of unknown scrambling gains. A technique for obtaining $\boldsymbol{W}_x$ for unstructured signals is provided in Section 3. After substituting $\boldsymbol{W}_x \boldsymbol{v}$ into (4) we obtain the CA-JML spectrum as the following eigenvalue problem: $S_C \equiv \cdots$

$$\max_{\boldsymbol{u}:\boldsymbol{u}^H\boldsymbol{u}=1} \boldsymbol{u}^H \boldsymbol{Q}_w^H P(\boldsymbol{a}_x) \boldsymbol{Q}_w \boldsymbol{u} = \boldsymbol{q}_{\boldsymbol{a}_x}^H P(\boldsymbol{W}_x) \boldsymbol{q}_{\boldsymbol{a}_x}, \tag{5}$$

where $\boldsymbol{q}_{\boldsymbol{a}_x} \equiv \frac{\boldsymbol{a}_x}{\|\boldsymbol{a}_x\|}$ is the unit normalized $\boldsymbol{a}_x$ and $\boldsymbol{Q}_w$ is an orthonormal matrix spanning the column space of $\boldsymbol{W}_x$. The CA-JML spectrum is a function of the angle of arrival θ, and has its maxima at the DOAs of the signals captured in the signal weight subspace matrix $\boldsymbol{W}_x$.

2.2. Advantages of CA-JML

Because the CA-JML spectrum is a simple generalization of the copy-aided DF spectrum, it offers an architectural advantage to systems which would employ both copy-aided and copy unaided DF. The DF spectra for both copy-aided and unaided DF can be dealt with using the same CA-JML algorithm.

An additional advantage of the CA-JML spectrum is the ability to reduce the complexity of the spectrum after the DOA of an unstructured signal has been estimated. The spectrum is modified by removing $\boldsymbol{w}_x$ from the weight subspace $\boldsymbol{W}_x$. In this situation it should be noted that the CA-JML spectrum will contain multiple peaks, each peak corresponding to a genuine signal within the signal weight subspace. Mathematically, the cancellation of a signal can be accomplished by using the optimal $\boldsymbol{v}$ determined previously from the eigenvalue problem. Therefore, if $\boldsymbol{v}_1$ and $\boldsymbol{\theta}_1$ are the optimal scrambling gains and DOA for signal 1, then the complexity of the CA-JML spectrum can be reduced by forcing the search into the orthogonal complement of the $\boldsymbol{v}_1$ weights. This exploits an additional property of the weight subspace that constrains the true "unscrambled" copy weights to be an orthonormal mixing of the CA-JML weight subspace vector $\boldsymbol{W}_x$,

$$\boldsymbol{W}_{copy} = \boldsymbol{R}_{\boldsymbol{x}}^{-1} \boldsymbol{W}_x \boldsymbol{V}, \tag{6}$$

where $\boldsymbol{V}$ is orthonormal, $\boldsymbol{V}^H \boldsymbol{V} = \boldsymbol{I}$. The CA-JML weights obtained in Section 3 have this orthonormal-combining property.

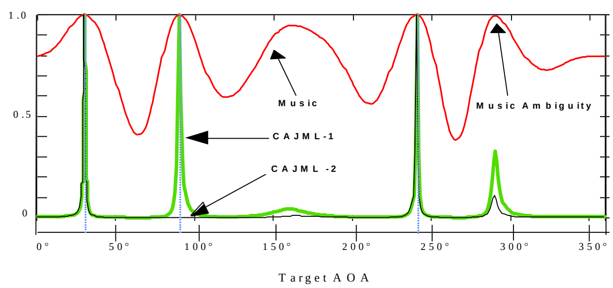

Figure 1: CA-JML and Music Spectra

A comparison between CA-JML and MUSIC in environments that do not contain structured emitters shows some distinct advantages for the CA-JML algorithm. The primary mathematical difference between MUSIC and CA-JML is that CA-JML adds an inverse data covariance matrix whitening to the aperture and the subspace weights that MUSIC does not employ. This whitening causes sharper spectral peaks, reduces false maxima, or spectral ambiguities, and also reduces DF error bias in difficult environments. This can be seen in Figure 1, where the MUSIC spectrum is seen to exhibit large ambiguities not present in the CA-JML spectrum. Note also that the spectral peak at 90^o is removed in the secondary spectrum CAJML-2. The computer simulated environment used to generate Figure 1 contains three emitters arriving at angles of $30°$, $90°$ and $240°$. The emitters are received at 30, 20 and 25 dB signal to white noise power ratio (SWNR) respectively, and the noise interference is spatially colored, circularly symmetric, complex Gaussian. A six element, isotropic array is used to DF each emitter.

If $\boldsymbol{V}_1^\perp$ is an orthonormal basis for the vector subspace orthogonal to $\boldsymbol{v}_1$, then a new signal weight subspace can be generated by the formula, $\boldsymbol{W}_{x_2} \equiv \boldsymbol{W}_x \boldsymbol{V}_1^\perp$. By substituting $\boldsymbol{W}_{x_2}$ for $\boldsymbol{W}_x$ in (5), a new spectrum is generated, which has cancelled signal 1 from the spectrum.

The ability to cancel peaks after they have been identified improves the performance of the DF search strategy as Figure 2 indicates. After the SOI-1 peak is found, it is removed from the spectrum and the modified spectrum is plotted. The modified spectrum does not contain the strong ambiguity that would prevent the DF of SOI-2 if only the initial spectrum is used.

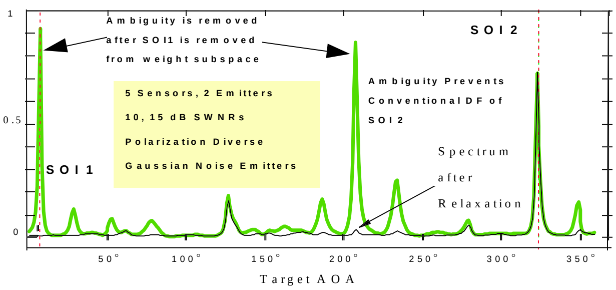

Figure 2: Removal of Ambiguities after Relaxation Step

In environments where MUSIC exhibits angle estimation bias, CA-JML adheres more closely to the Deterministic Cramer Rao Bound [7]. This can be seen in Figure 3, where two emitters are within 6^o of one another. In this environment CA-JML clearly outperforms the MUSIC algorithm when measured in terms of RMS angle error.

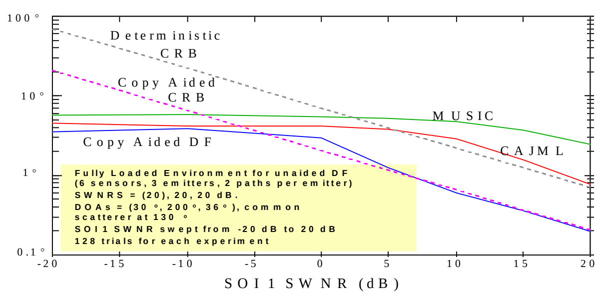

Figure 3: RMS DF Error Performance for Nearly Co-Located Emitters

3. CA-JML WEIGHTS AND THE REMOVAL OF STRUCTURED INTERFERERS

A key to improved performance for the CA-JML algorithm is the calculation of the weight subspace. If the weight subspace can be calculated from deterministic parameters then it can be shown that the CA-JML DF performance will adhere to the Copy-Aided CRB, which is equivalent to a signal model with a known SOI [5]. In most cases, however, the weight subspace will be derived from stochastic parameters.

A general model for SOI subspace estimation is

$$\boldsymbol{X} = \boldsymbol{S}_K \boldsymbol{A}_K^H + \boldsymbol{S}_U \boldsymbol{A}_U^H + \boldsymbol{J}, \qquad (7)$$

where $\boldsymbol{X} = [\boldsymbol{x}(1), \boldsymbol{x}(2), \cdots \boldsymbol{x}(N)]^H$ is the Hermitian transpose of a block of N data vector samples received in an N_{sens} array, $\boldsymbol{S}_U$ is an $N \times M_U$ matrix whose columns contain the conjugate of the M_U unknown, unstructured, emitter waveforms, $\boldsymbol{S}_K$ is an $N \times M_K$ matrix of M_K known and conjugated emitter waveforms, $\boldsymbol{A}_K$ is the unknown $N_{sens} \times M_K$ received aperture matrix excited by the known emitters, $\boldsymbol{A}_U$ is the unknown $N_{sens} \times M_U$ received aperture matrix for the unknown and unstructured emitters and $\boldsymbol{J}$ is the Hermitian transpose of the received interference. The columns of the aperture matrices $\boldsymbol{A}_U$ and $\boldsymbol{A}_K$ contain the steering vectors for each signal. $\boldsymbol{J}$ may be considered as due to purely to receiver or environmental noise, in which case it is often modeled as white, circularly symmetric, complex Gaussian noise, or it may contain interference waveforms. This model yields the maximum likelihood estimation function,

$$\begin{aligned} \rho_{copy} \quad \equiv \quad & Tr\left\{ \boldsymbol{R}_{\boldsymbol{ii}}^{-1} (\boldsymbol{X} - \boldsymbol{S}_K \boldsymbol{A}_K^H - \boldsymbol{S}_U \boldsymbol{A}_U^H)^H \cdots \right. \\ & \left. (\boldsymbol{X} - \boldsymbol{S}_K \boldsymbol{A}_K^H - \boldsymbol{S}_U \boldsymbol{A}_U^H) \right\}, \qquad (8) \end{aligned}$$

where $\boldsymbol{R}_{\boldsymbol{ii}}$ is the known or measured interference autocorrelation matrix for $\boldsymbol{J}$. $\boldsymbol{R}_{\boldsymbol{ii}}$ can be measured in nearby channels or in different time blocks than the signals of interest. Any signal waveforms included in $\boldsymbol{R}_{\boldsymbol{ii}}$ will be canceled from the angle estimator. If all signals need to be localized, then $\boldsymbol{R}_{\boldsymbol{ii}}$ can be set to the identity matrix or to an estimate of the background noise covariance.

After minimizing over $\boldsymbol{A}_K$, $\boldsymbol{A}_U$ and $\boldsymbol{S}_U$, the CA-JML weight subspace can be obtained from

$$\boldsymbol{W}_x = \boldsymbol{R}_{\boldsymbol{x}}^{-H} \boldsymbol{R}_{\boldsymbol{i}}^H \hat{\boldsymbol{A}}_{Us\boldsymbol{i}}, \qquad (9)$$

where $\boldsymbol{R}_{\boldsymbol{x}}$ is the Cholesky factor obtained from the QR decomposition of $\boldsymbol{X}$, $\boldsymbol{R}_{\boldsymbol{i}}$ is the Cholesky factor of $\boldsymbol{R}_{\boldsymbol{ii}}$, $\hat{\boldsymbol{A}}_{Us\boldsymbol{i}} = \boldsymbol{Z}\sqrt{\boldsymbol{\Lambda}}$, and $\boldsymbol{Z}$ is the dominant eigenspace associated with the M_U largest eigenvalues of $\tilde{\boldsymbol{R}}$. $\tilde{\boldsymbol{R}}$ is defined by,

$$\tilde{\boldsymbol{R}} \equiv \boldsymbol{R}_{\boldsymbol{i}}^{-H} \boldsymbol{X}^H P_{\perp}(\boldsymbol{S}_K) \boldsymbol{X} \boldsymbol{R}_{\boldsymbol{i}}^{-1} - \alpha \boldsymbol{I}, \qquad (10)$$

where $\boldsymbol{I}$ is the $N_{sens} \times N_{sens}$ identity matrix and $\alpha = 1$. The matrix $\boldsymbol{\Lambda}$ is an $M_U \times M_U$ diagonal matrix and has the M_U largest eigenvalues on its diagonal. When $\boldsymbol{R}_{\boldsymbol{ii}}$ is known only approximately, then α is set to the smallest eigenvalue of $\boldsymbol{R}_{\boldsymbol{i}}^{-H} \boldsymbol{X}^H P_{\perp}(\boldsymbol{S}_K) \boldsymbol{X} \boldsymbol{R}_{\boldsymbol{i}}^{-1}$, which should be near unity in an underloaded array.

If we assume that the emitter waveforms are uncorrelated, then for large N, $\tilde{\boldsymbol{R}}$ assumes the form, $\tilde{\boldsymbol{R}} \approx \boldsymbol{A}_{U\boldsymbol{i}} \boldsymbol{R}_{\boldsymbol{s}_U \boldsymbol{s}_U} \boldsymbol{A}_{U\boldsymbol{i}}^H$, where $\boldsymbol{A}_{U\boldsymbol{i}} \equiv \boldsymbol{R}_{\boldsymbol{i}}^{-H} \boldsymbol{A}_U$ and $\boldsymbol{R}_{\boldsymbol{s}_U \boldsymbol{s}_U} \equiv \boldsymbol{S}_U^H \boldsymbol{S}_U$. This asymptotic form yields the important result that

$$\hat{\boldsymbol{A}}_{Us\boldsymbol{i}} \approx \boldsymbol{A}_{U\boldsymbol{i}} \boldsymbol{R}_{\boldsymbol{s}_U} \boldsymbol{V}, \qquad (11)$$

where $\boldsymbol{V}$ is an $M_U \times M_U$ orthonormal matrix and $\boldsymbol{R}_{\boldsymbol{s}_U}$ is the Cholesky factor of $\boldsymbol{R}_{\boldsymbol{s}_U \boldsymbol{s}_U}$, which is diagonal when the signals are uncorrelated. It follows that the CA-JML weight subspace is just an orthonormal scrambling of the "true" copy weights as in (6) and consequently it is possible to remove each emitter from the DF spectrum as it is localized.

The signals in $\boldsymbol{S}_K$ are typically obtained by exploiting known properties of the emitter waveforms. If they are constant modulus signals, then a multi-target constant modulus algorithm can be employed [4]. If the signals are localized in time (i.e. bursty), or frequency then this too can be exploited [3]. In some cases signals are estimated using non-linear techniques such as demod/remod data directed techniques [2]. For these signals the formula in (10) is appropriate, requiring the cross correlation between the known signals and the data, and the auto-correlation of the known signals. The non-linear techniques hold the promise of being able to copy and DF more signals then the number of sensors, but are more computationally intensive. If on the other hand $\boldsymbol{S}_K$ is obtained by the application of linear copy weights, so that $\boldsymbol{S}_k = \boldsymbol{X}\boldsymbol{W}_K$, then (10) can be simplified to,

$$\tilde{\boldsymbol{R}} \equiv \boldsymbol{R}_{\boldsymbol{i}}^{-H} \boldsymbol{R}_{\boldsymbol{x}}^{H} P_{\perp}(\boldsymbol{R}_{\boldsymbol{x}} \boldsymbol{W}_k) \boldsymbol{R}_{\boldsymbol{x}} \boldsymbol{R}_{\boldsymbol{i}}^{-1} - \alpha \boldsymbol{I}. \qquad (12)$$

The dominant subspace of $\tilde{\boldsymbol{R}}$ can be effectively tracked using any of a wide variety of subspace tracking techniques such as [8]. The only additional calculations required over that of MUSIC is the calculation of $P_{\perp}(\boldsymbol{R}_{\boldsymbol{x}} \boldsymbol{W}_k)$.

The performance of CA-JML in a mixed environment is demonstrated in Figure 4. This computer generated environment contains 7 emitters received by an 8 sensor array. The 3 weak SOI emitters arrive at 14^o, 95^o and 240^o, and are received at a signal to white noise power ratio (SWNR) of 5 dB each. The other 4 strong interference emitters are FM signals, received at a 30 dB SWNR, with AOAs of 20^o, 160^o, 210^o, and 320^o. The MUSIC spectrum can not DF the weak emitters and has no spectral peaks at their directions of arrival (DOA). The 4 FM signals are estimated using a multi-target CMA algorithm. The copy weights from the CMA algorithm are used in (12) and the dominant subspace is extracted. From this, the CA-JML spectrum is plotted in Figure 4. The CA-JML spectrum has spectral peaks at each of the weak emitter DOAs and nulls at the interference DOAs.

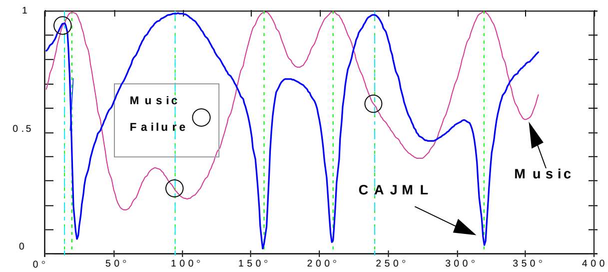

Figure 4: CA-JML Finds Weak Emitter DOAs

4. CONCLUSION

Three advantages of the CA-JML algorithm are demonstrated in this paper. The first is the ability of the CA-JML algorithm to remove peaks from its DF spectrum at emitter DOAs that have already been identified. The second advantage is smaller ambiguities and smaller DF error bias than the MUSIC algorithm. The third advantage is the ability to cancel signals that have exploitable structure, consequently improving performance, and the loading limits of the algorithm. The CA-JML algorithm offers a natural synthesis between the copy and DF of structured signals, and the copy and DF of unstructured signals in mixed environments.

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