MULTIWEIGHT OPTIMIZATION IN OBE ALGORITHMS FOR IMPROVED TRACKING AND ADAPTIVE IDENTIFICATION

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ABSTRACT

Optimal Bounding Ellipsoid (OBE) algorithms offer an attractive alternative to traditional least squares methods for identifying linear-in-parameters signal and system models due to their low computational efficiency, superior tracking ability, and selective updating that permits processor sharing among tasks. These benefits are further enhanced by *multiweight optimization* (MWO) which yields improved per-point parameter convergence. This paper introduces the MWO process and describes advances in its implementation including the incorporation of a forgetting factor for improved tracking, a new method for efficient weight computation, and extensions to volume-minimizing OBE algorithms. Simulation studies illustrate the results.

1. INTRODUCTION

Optimal bounding ellipsoid (OBE) identification algorithms are used to identify *linear-in-parameters* models of the form

$$y_n = \boldsymbol{\theta}_*^T \boldsymbol{x}_n + \varepsilon_{n*} \tag{1}$$

in which $\boldsymbol{\theta}_* \in \Re^m$ is the unknown "true" parameter vector to be identified; $\{\boldsymbol{x}_n\}$ is a sequence of measurable vectors of dimension m; and $\{\varepsilon_{n*}\}$ is an unknown error sequence. OBE algorithms are based on the premise that, for each n, the model error has a known pointwise energy bound, $\varepsilon_{n*}^2 \leq \gamma_n^2$. Given data on times $t \in [1, n]$, these bounds imply an *exact feasibility set*, say Ω_n, of estimates for $\boldsymbol{\theta}_*$ whose elements are consistent with the bounds and the observations. OBE algorithms work with an hyperellipsoidal set, say $\bar{\Omega}_n \supset \Omega_n$, hence $\boldsymbol{\theta}_* \in \bar{\Omega}$. The observations are scrutinized with respect to their ability to "shrink" $\bar{\Omega}_n$, hence to tightly bound Ω_n. At time n (e.g., [1]),

$$\bar{\Omega}_n \stackrel{\text{def}}{=} \left\{\boldsymbol{\theta} : (\boldsymbol{\theta} - \boldsymbol{\theta}_n)^T \boldsymbol{C}_n (\boldsymbol{\theta} - \boldsymbol{\theta}_n) \leq \kappa_n\right\} \tag{2}$$

in which $\boldsymbol{C}_n$ is the weighted *covariance* matrix of the observations, $\boldsymbol{C}_n = \sum_{t=1}^n q_{t,n} \boldsymbol{x}_t \boldsymbol{x}_t^T$, κ_n is the scalar

$$\kappa_n = \boldsymbol{\theta}_n^T \boldsymbol{C}_n \boldsymbol{\theta}_n + \sum_{t=1}^{n} q_{t,n}(\gamma_t^2 - y_t^2), \tag{3}$$

and $\boldsymbol{\theta}_n$, the center of $\bar{\Omega}_n$, is a particular weighted least-square-error estimator of $\boldsymbol{\theta}_*$, $\boldsymbol{\theta}_n = \boldsymbol{P}_n \boldsymbol{c}_n$, with $\boldsymbol{P}_n \stackrel{\text{def}}{=} \boldsymbol{C}_n^{-1}$ and $\boldsymbol{c}_n \stackrel{\text{def}}{=} \sum_{t=1}^n q_{t,n} y_t \boldsymbol{x}_t$. The weighting sequence at time n, $\{q_{t,n}\}_{t=1}^n$, is chosen to optimally diminish some set measure of the hyperellipsoid (see Section 2).

OBE algorithms make selective use of incoming data in updating the ellipsoid and central estimator. Frequently, the observations at time n contain no innovation in the sense that they cannot be used to reduce $\bar{\Omega}_n$. This is manifest in the failure to find valid weights, and the effort of updating can be avoided. Depending on the properties of the sequence $\{\varepsilon_{n*}\}$, OBE algorithms often update only 10% of the time or less.

All published OBE algorithms can be manipulated into the formal framework above [1]. In no case, however, is there an attempt to *reoptimize* any of the "previous" weights at time n in light of the new measurements $\boldsymbol{x}_n$ and y_n. All optimization in existing OBE algorithms is accomplished by manipulating the *current* weight only. The *globally* optimal solution at time n would optimize *all* weights $\{q_{t,n}\}_{t=1}^n$, in light of *all* known information $\{(\boldsymbol{x}_t, y_t)\}_{t=1}^n$.

2. OBE WITH MULTIWEIGHT OPTIMIZATION

Generalities. The paper [2] reports a first attempt to develop OBE algorithms that "revisit" $K \geq 1$ past weights at each time, so that the identification can more fully exploit information in the evolving observation stream. In these *OBE algorithms with multiweight optimization* (OBE-MWO), the ellipsoid is optimally diminished with respect to the current and past K observations, conditioned upon extant information at $n - K - 1$. At time n, OBE-MWO reoptimizes the time block $n-K, n-K+1, \ldots n-1$ by making additive adjustments to the corresponding K past weights[1], subject to the constraint that revised ("accumulated") weights remain nonnegative. Accordingly, the time-varying weights are

$$q_{t,n} = \alpha_n \times \left\{ \begin{array}{ll} \sum_{i=0}^{K} \lambda_t^i, & 0 \leq t \leq n-K \\ \sum_{i=0}^{n-t} \lambda_t^i, & n-K < t \leq n \end{array} \right. , \tag{4}$$

with the constraint $q_{t,n} \geq 0$ for any t and n. λ_t^i for $i > 0$ denotes the ith additive adjustment to the weight at time t, λ_n^0 is the original weight computation at time n. This

[1] As amended by a forgetting factor.

Table 1: SUMMARY OF VECTOR AND MATRIX NOTATION.

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be $N \times M$ matrices, and $\boldsymbol{a}$ be an N-vector.

- $\boldsymbol{A}(i,j)$ denotes the (i,j) element of $\boldsymbol{A}$ and $\boldsymbol{a}(i)$ the ith element of $\boldsymbol{a}$.
- $\mathcal{D}(\boldsymbol{a})$ is the diagonal matrix with ith diagonal element $\boldsymbol{a}_(i)$.
- Let $\boldsymbol{A}$ be $N \times N$. $\searrow(\boldsymbol{A})$ is the diagonal matrix formed by setting all off-diagonal elements of $\boldsymbol{A}$ to zero.
- $\boldsymbol{A}(*,i)$ is the N-vector comprising the ith column of $\boldsymbol{A}$.
- The *Hadamard product*, $\boldsymbol{A} \circ \boldsymbol{B}$, is the $N \times M$ matrix with (i,j) element $\boldsymbol{A}(i,j)\boldsymbol{B}(i,j)$ [3].

Quantity	Notation
Regressor matrix	$\boldsymbol{X}_n = \left[\begin{array}{ccc} \boldsymbol{x}_{n-K} & \cdots & \boldsymbol{x}_n \end{array}\right]$
Output vector	$\boldsymbol{y}_n^T = \left[\begin{array}{ccc} y_{n-K} & \cdots & y_n \end{array}\right]$
Weight vector	$\boldsymbol{\lambda}_n^T = \left[\begin{array}{ccc} \lambda_{n-K+1}^{k-1} & \cdots & \lambda_n^0 \end{array}\right]$
Weight matrix	$\boldsymbol{\Lambda}_n = \mathcal{D}(\boldsymbol{\lambda}_n)$
Bound vector	$\boldsymbol{\gamma}_n^T = \left[\begin{array}{ccc} \gamma_{n-K} & \cdots & \gamma_n \end{array}\right]$
Bound matrix	$\boldsymbol{\Gamma}_n = \mathcal{D}(\boldsymbol{\gamma}_n)$
Innovation vector	$\boldsymbol{\varepsilon}_n = \boldsymbol{y}_n - \boldsymbol{X}_n^T \boldsymbol{\theta}_{n-1}$
Covar.-weighted regressor energy matrix	$\boldsymbol{G}_n = \boldsymbol{X}_n^T \boldsymbol{P}_{n-1} \boldsymbol{X}_n$

computation is generalized from that in [2] by the inclusion of a time-invariant sequence of forgetting factors $\{\alpha_t\}_{t=1}^n$.

OBE-MWO characteristically checks whether incoming data are sufficiently innovative to diminish the set $\bar{\Omega}_n$. This effectively checks for the existence of an optimal $\lambda_n^0 > 0$. If the check fails, the past weights are already optimal in light of the new measurement and no updating is required. If the observation at time n is informative, then, once the optimal weights $\{\lambda_{n-i}^i\}_{i=0}^K$ are found, $\boldsymbol{P}_n$, $\boldsymbol{\theta}_n$, and κ_n are updated using recursions developed in[2] [2]:

$$
\begin{array}{rcl}
\alpha_n \boldsymbol{P}_n &=& \boldsymbol{P}_{n-1} - \boldsymbol{P}_{n-1}\boldsymbol{X}_n \boldsymbol{H}_n^{-1} \boldsymbol{\Lambda}_n \boldsymbol{X}_n^T \boldsymbol{P}_{n-1} \\
\boldsymbol{\theta}_n &=& \boldsymbol{\theta}_{n-1} + \boldsymbol{P}_n \boldsymbol{X}_n \boldsymbol{\Lambda}_n \boldsymbol{\varepsilon}_n \\
\kappa_n &=& \alpha_n \kappa_{n-1} + \boldsymbol{\gamma}_n^T \boldsymbol{\Lambda}_n \boldsymbol{\gamma}_n - \alpha_n \boldsymbol{\varepsilon}_n^T \boldsymbol{H}_n^{-1} \boldsymbol{\Lambda}_n \boldsymbol{\varepsilon}_n,
\end{array} \qquad (5)
$$

where $\boldsymbol{H}_n = \alpha_n \boldsymbol{I} + \boldsymbol{\Lambda}_n \boldsymbol{G}_n$, and other notation is defined in Table 1.

The general steps of an OBE-MWO algorithm are shown in Table 2. The computational details are dependent upon the optimization criterion (discussed below). In any case, the algorithm must assure that each reoptimized weight remains nonnegative. This is done using a pair of weight-accumulation vectors of dimension $K+1$, $\underline{\boldsymbol{\lambda}}_n^*$ and $\bar{\boldsymbol{\lambda}}_n^*$ (underbar denotes *a priori*, overbar *a posteriori*). Prior to the

Table 2: OBE-MWO ALGORITHM (GENERAL STEPS).

I. Initialization:

1. $\boldsymbol{\theta}_K = \boldsymbol{0}$, $\kappa_K = \mu$ and $\boldsymbol{P}_K = \frac{1}{\mu^2}\boldsymbol{I}$, μ small.
2. $\bar{\boldsymbol{\lambda}}_K^* = \boldsymbol{0}$.

II. Recursion:

For $n = K+1, K+2, \ldots$
    Form $\underline{\boldsymbol{\lambda}}_n^*$ from $\bar{\boldsymbol{\lambda}}_{n-1}^*$
    If current K observations are innovative,
        determine optimal weight vector, $\boldsymbol{\lambda}_n$
        [optimization criterion dependent].
    Otherwise, next n.
    If $\bar{\boldsymbol{\lambda}}_n^*(i) \geq 0$ for all i, update:
        $\boldsymbol{\varepsilon}_n = \boldsymbol{y}_n - \boldsymbol{X}_n^T \boldsymbol{\theta}_{n-1}^T$
        $\boldsymbol{G}_n = \boldsymbol{X}_n^T \boldsymbol{P}_{n-1} \boldsymbol{X}_n$
        Update $\boldsymbol{P}_n$, $\boldsymbol{\theta}_n$, κ_n using eqs. (5).
    Otherwise, next n.
Next n.

update at time n, $\underline{\boldsymbol{\lambda}}_n^* = [\bar{\boldsymbol{\lambda}}_{n-1}^*(n-K+1) \cdots \bar{\boldsymbol{\lambda}}_{n-1}^*(n-1)\ 0]$, whereas the *a posteriori* value is $\bar{\boldsymbol{\lambda}}_n^* = \underline{\boldsymbol{\lambda}}_n^* + \boldsymbol{\lambda}_n$.

Minimization of κ_n: QOBE-MWO. The defining matrix for the hyperellipsoid at time n is [see (2)] $\boldsymbol{P}_n^{-1}/\kappa_n$. The determinant $\det\{\kappa_n \boldsymbol{P}_n\}$ is proportional to the square of the volume of the ellipsoid, and is most often minimized as the OBE optimization criterion. Minimization of the parameter κ_n has been used in [4], and more recently by Gollamudi *et al.* [5, 6] in the *quasi-OBE* (QOBE) algorithm which provides interesting interpretations of κ_n minimization. Because of the relative algebraic simplicity of the QOBE, generalized κ_n minimization was used in developing the initial OBE-MWO algorithm in [2]. Let us refer to this version of the method as *QOBE-MWO*.

To find κ_n-optimal weights, Joachim *et al.* [2] have shown that[2]

$$
\boldsymbol{K}_n \stackrel{\text{def}}{=} \frac{\partial \kappa_n}{\partial \boldsymbol{\Lambda}_n} = \boldsymbol{\Gamma}_n^2 - \alpha_n \left[\mathcal{D}(\boldsymbol{H}_n^{-T} \boldsymbol{\varepsilon}_n)\right]^2. \qquad (6)
$$

$\boldsymbol{K}_n = \partial\kappa_n / \partial \boldsymbol{\Lambda}_n$ is the diagonal matrix with ith diagonal element is $\partial \kappa_n / \partial \lambda_{n-i}^i$. A solution for $\boldsymbol{\Lambda}_n = \mathcal{D}(\boldsymbol{\lambda}_n)$ is sought by setting $\boldsymbol{K}_n \equiv 0$, yielding

$$
\begin{array}{rcl}
\boldsymbol{\lambda}_n &=& \left[\boldsymbol{G}_n^{-1}\left(\alpha_n \boldsymbol{\varepsilon}_n - \boldsymbol{\gamma}_n \circ \boldsymbol{s}_n\right)\right] \circ \left(\boldsymbol{g}_n^T \circ \boldsymbol{s}_n\right) \\
&=& (\boldsymbol{G}_n \boldsymbol{S}_n \boldsymbol{\Gamma}_n)^{-1}(\alpha_n \boldsymbol{\varepsilon}_n - \boldsymbol{S}_n \boldsymbol{\gamma}_n)
\end{array} \qquad (7)
$$

in which $\boldsymbol{s}_n$ is the $K \times 1$ "sign" vector with $\boldsymbol{s}_n(i) = +1$ if the ith element of $\boldsymbol{H}_n^{-T}\boldsymbol{\varepsilon}_n$ is positive, and -1 otherwise; and the matrix $\boldsymbol{S}_n \stackrel{\text{def}}{=} \mathcal{D}(\boldsymbol{s}_n)$. Either equation in (7) is the desired solution for the κ_n-optimal weight vector. When $K=0$, this yields the optimal scalar weight for QOBE [5, 6]. Discussion of the solution appears below.

It remains to fill in details of the algorithm of Table 2 that are specific to κ_n optimization surrounding the computation of the optimal weights. As in conventional OBE

[2] Result modified to include forgetting factors.

algorithms, QOBE-MWO admits a shortcut to the determination of whether an optimal weight *exists*. The check for innovation at time n in the conventional QOBE algorithm ($K = 0$) is $|\varepsilon_n| > \gamma_n$. With MWO ($K > 0$), satisfaction of this simple test is still necessary for any further computation to be required on the window $n-K, \ldots, n$. Indeed, if there is no innovation in the observation at time n, then the past K weights are already optimal. The computational cost of the method is therefore drastically reduced by recourse to the simple scalar check used in QOBE prior to further optimization. As formulated above, the computational complexity of QOBE-MWO is significantly worse *per update* than that of QOBE. However, these selective algorithms tend to incorporate so few data that, even with the additional burden at times of update, the overall complexity of QOBE-MWO remains $\mathcal{O}(m)$, the complexity of the simple checking step. Further, there is empirical evidence that reoptimization may result in a significant reduction in the number of updates (Section 3) with respect to QOBE which is already sparse. Finally, further work has begun to yield promising results for more efficient updating. These results will be included in future papers.

Volume. A solution for a volume minimizing OBE-MWO is considerably more difficult, but we report recent progress here. Let $\boldsymbol{B}_n \stackrel{\text{def}}{=} \boldsymbol{P}_n \kappa_n$, and denote the "volume ratio" between consecutive iterations by $V_n \stackrel{\text{def}}{=} \det \boldsymbol{B}_n / \det \boldsymbol{B}_{n-1}$. Following a development similar to that in [7], it can be shown that the $\boldsymbol{\Lambda}_n = \mathcal{D}(\boldsymbol{\lambda}_n)$ that minimizes V_n solves

$$0 = m\boldsymbol{\Gamma}_n^2 - m\left[\mathcal{D}(\boldsymbol{H}_n^{-T}\varepsilon_n)\right]^2 - \searrow\!(\boldsymbol{G}_n\boldsymbol{H}_n^{-T})\kappa_n. \quad (8)$$

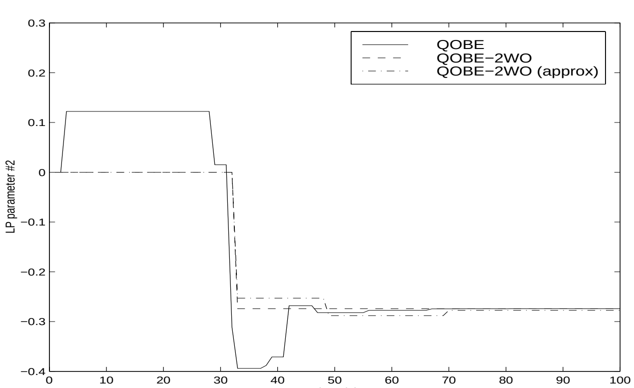

Figure 1: Convergence performance of QOBE, QOBE-2WO and QOBE-2WO (approx.) for the initial simulation study of Sec. 3. Only the second parameter is plotted. The numbers of updates are 18, 4, and 6, respectively.

Unlike the QOBE-MWO algorithm, details of the optimization remain open pending the search for a more efficient weight computation and existence check. We therefore are unable to make meaningful statements about efficiency for the volume case. It is possible to use the results above in a "brute force" mode in order to study the performance of the volume-based algorithm in tracking and related identification tasks. While a closed form solution to (8) remains an open issue, the following approximation method has been found to provide a reasonable solution.

A Useful Approximation. For simplicity the forgetting factor is omitted here. In either optimization case, the recursions (5) require the expensive computation of the term $\boldsymbol{H}_n^{-1}\boldsymbol{\Lambda}_n$. In turn, this term appears in (8) in the quantity κ_n, so that the sought weight matrix is deeply embedded in the right side. It is useful to expand the product $\boldsymbol{H}_n^{-1}\boldsymbol{\Lambda}_n$ as an infinite series $\boldsymbol{H}_n^{-1}\boldsymbol{\Lambda}_n = \sum_{i=0}^{\infty}(-1)^i(\boldsymbol{G}_n\boldsymbol{\Lambda}_n)^i$ which can then be approximated by a small number of terms.[3] For example,

$$\boldsymbol{H}_n^{-1}\boldsymbol{\Lambda}_n \approx \boldsymbol{\Lambda}_n - \boldsymbol{\Lambda}_n\boldsymbol{G}_n\boldsymbol{\Lambda}_n. \quad (9)$$

The corresponding approximation for κ_n is

$$\kappa_n \approx \kappa_{n-1} + \boldsymbol{\gamma}_n^T\boldsymbol{\Lambda}_n\boldsymbol{\gamma}_n - \boldsymbol{\varepsilon}_n^T\boldsymbol{\Lambda}_n\boldsymbol{\varepsilon}_n - \boldsymbol{\varepsilon}_n^T\boldsymbol{\Lambda}_n\boldsymbol{G}_n\boldsymbol{\Lambda}_n\boldsymbol{\varepsilon}_n.$$

Minimizing κ_n quickly yields the weights (with corresponding κ_n at times of update)

$$\begin{aligned} \boldsymbol{\lambda}_n &= (2\boldsymbol{E}_n\boldsymbol{G}_n\boldsymbol{E}_n)^{-1}(\boldsymbol{\varepsilon}_n \circ \boldsymbol{\varepsilon}_n - \boldsymbol{\gamma}_n \circ \boldsymbol{\gamma}_n) \quad (10)\\ \kappa_n &= \kappa_{n-1} - \boldsymbol{\lambda}_n^T\boldsymbol{E}_n\boldsymbol{G}_n\boldsymbol{E}_n\boldsymbol{\lambda}_n \quad (11) \end{aligned}$$

where $\boldsymbol{E}_n = \mathcal{D}(\boldsymbol{\varepsilon}_n)$.

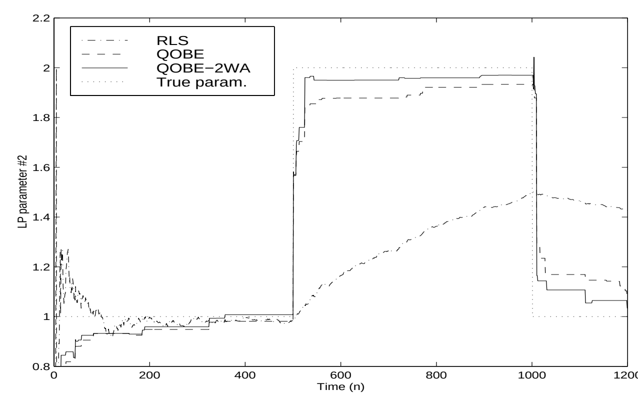

Figure 2: Comparison of performance of RLS, QOBE, and QOBE-2WO in tracking the time-varying system of Sec. 3. Only the second parameter is plotted. The numbers of updates are 1200, 46, and 48, respectively.

3. SIMULATION STUDIES

To discover the benefits of MWO, we investigate the performance of the QOBE-MWO algorithm with exact and ap-

[3] Note that the purpose of this development is not to reduce the computational complexity of the inverse, since K is usually small (1 to 3), but rather to express κ_n and the volume in a way that facilitates the derivation of an approximate closed-form solution.

proximated solutions compared with that of QOBE. Comparisons with RLS are also made in order to illustrate the superior tracking capabilities of the OBE methods.

Initially, we seek to identify an ARX model [8] of form (1) with $\boldsymbol{\theta}_*^T = \begin{bmatrix} 0.73 & -0.27 \end{bmatrix}$ and $\boldsymbol{x}_n^T = \begin{bmatrix} y_{n-1} & w_n \end{bmatrix}$ in which both $\{w_n\}$ and $\{\varepsilon_{n*}\}$ are bounded by $[-1, +1]$. Figure 1 shows the convergence behavior of the parameter estimator. The exact QOBE-MWO algorithm shows the best convergence, while the approximated version converges slightly less quickly, but uses a reduced number of points with respect to QOBE (quantified in caption), and is of lower complexity than the exact version.

In a second experiment, the true parameters are abruptly changed from $\boldsymbol{\theta}_*^T = \begin{bmatrix} -0.2 & 1.0 \end{bmatrix}$ to $\boldsymbol{\theta}_*^T = \begin{bmatrix} -0.8 & 2.0 \end{bmatrix}$ then back again. The superior tracking of QOBE-MWO compared to RLS and the conventional QOBE algorithm is evident in Fig. 2. This is due to the ability of the algorithm to quickly converge to the correct parameter estimator. Again, the higher computational cost (i.e., over all $K+1$ data) is offset by the small number of points used.

Finally, Fig. 3 illustrates the effectiveness of QOBE-MWO in spectral estimation of a speech sound. We blindly estimate the 14 linear-prediction parameters [9] of the phoneme /ɪ/ and subsequently reconstruct its spectrum using a 256-point frame. The superior spectral approximation using QOBE-MWO is evident. The reference spectrum (light series of dots) is provided by the "covariance" method conventionally used in speech analysis [9]. This experiment demonstrates another interesting (empirical) property of QOBE-MWO with respect to QOBE. Note that QOBE updates only one time on the frame. This is presumably due to overestimated bounds which cause the QOBE algorithm to stop taking points [7]. The QOBE-MWO is more aggressive in finding innovation in the data, even in the presence of these "loose" error bounds.

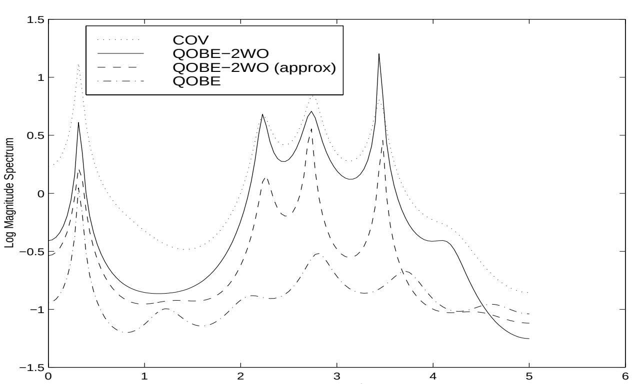

Figure 3: Spectral estimates of a 256-point speech segment (phoneme /ɪ/) using OBE-2WO (exact and approximate) and QOBE. The reference spectrum is provided by the batch "covariance" method used in speech analysis.

4. CONCLUSION

This paper has introduced the concept of MWO for OBE algorithms. Simulation studies verify the expected results that MWO benefits performance with respect to conventional OBE, and even further improves upon the well-known superiority of OBE algorithms over RLS. Moreover, the computational complexity increase *per update* required by MWO with respect to OBE is offset by significantly less-frequent updating. Further, future research will likely yield more efficient computational procedures for MWO. The paper has focused principally upon a κ-optimization (QOBE-type) MWO algorithm while providing initial results on a volume minimization algorithm.

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