FAST COMPUTATION OF EFFICIENT DECISION FEEDBACK EQUALIZERS FOR HIGH SPEED WIRELESS COMMUNICATIONS

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ABSTRACT

Decision feedback equalization (DFE) structures have recently been proposed for the efficient equalization of wireless channels with long postcursor response, which is a bottleneck problem for high speed communications over multipath channels with large delay spread. These structures are equivalent to the conventional DFE, but remove postcursor intersymbol interference (ISI) prior to feedforward filtering. We investigate the relationship between these structures and fast equalizer coefficient computation. Based on this relationship, we obtain a fast algorithm for computing optimal DFE settings which has significantly lower complexity than other known approaches for these high speed wireless channels. An example is given for data rates and channel profiles of the type considered for the proposed North American high definition television (HDTV) terrestrial broadcast mode.

1. INTRODUCTION

The problem of efficient equalizer design is an important practical issue for several types of next-generation high speed wireless communications and broadcast technologies, including broadband time division multiple access (TDMA) [8] and high definition television (HDTV) systems [9]. For example, in the proposed North American HDTV terrestrial transmission mode standard [7], transmission symbol rates of 10.76 MHz and delay spreads of 20 μsecs can result in sampled channel responses which span hundreds of symbols, requiring similarly complex equalizers. Typically, these channels have a long (and sparse) postcursor component, and may also include a short (but strong) precursor component. See Figure 1 which shows a typical terrestrial HDTV channel response (Channel D in [9]) which has these characteristics.

We focus on complexity reduction of the finite impulse response (FIR) DFE for this type of high speed wireless scenario. Here the conventional FIR DFE can require a long feedback filter (on the order of the channel length) to achieve the desired performance. Although infinite impulse response (IIR) DFE's have been proposed to handle long postcursor response in digital subscriber loops (DSL) (c.f. [4][10][6][1]), they are not applicable here because the multipath channel does not have a slowly decaying pulse tail. In [5], we proposed alternative realizations of the DFE, called the *partial feedback equalizer* (PFE) and the *complete feedback equalizer* (CFE) which cancel postcursor ISI *prior* to feedforward filtering. Since the feedforward filter spreads out the (initially sparse) postcursor response, the PFE and CFE can better exploit tap allocation strategies than the DFE (see [5]).

In this paper we address methods of efficiently computing the PFE and CFE settings using fast algorithms, motivated by the work in [2]. We present a systematic view of how postcursor ISI can be removed prior to feedforward filtering, and examine the relationship between this removal of postcursor ISI and fast equalizer coefficient computation We find that the PFE suggests a fast algorithm which has significant computational advantages for the high speed wireless channels considered here. Since the DFE settings can also be obtained in this way, our method gives the *lowest* complexity minimum mean square error (MMSE) FIR DFE computation that we know of for these types of channels.

This paper is organized as follows. The channel model is presented in Section 2. Section 3 briefly reviews the DFE structure. The PFE and CFE realizations are derived in Sections 4 and 5 respectively. Methods for computing the various equalizers and detailed complexity comparisons are developed in Section 6, where we also give explicit numbers for a typical high speed terrestrial HDTV broadcast scenario.

2. CHANNEL MODEL

The symbol-rate sampled baseband equivalent received signal which results from transmitting an uncoded linear modulation over a slowly varying (relative to symbol rate) multipath channel is modelled as

$$y_k = y(kT) = \sum_{n=0}^{L} c_n I_{k-n} + n_k. \tag{1}$$

where $\{I_n\}$ is the i.i.d. symbol sequence with variance σ_I^2, $\{c_n : 0 \le n \le L\}$ is the (equivalent) FIR channel impulse response, and $\{n_k\}$ is zero-mean complex white Gaussian noise with variance σ_n^2. This is in fact an appropriate model for the proposed HDTV terrestrial broadcast mode [7] which uses interleaved coded 8-VSB and seperate synchronization sequences.

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3. DFE STRUCTURE

The DFE is briefly described to set notation. The DFE output before slicing satisfies

$$\widehat{I}_k = \sum_{n=-K}^{0} f_n y_{k-n} + \sum_{n=1}^{L} b_{DFE,n} \tilde{I}_{k-n}, \qquad (2)$$

where $\{f_n\}$ are the feedforward filter coefficients, $\{b_{DFE,n}\}$ are the feedback filter coefficients, and $\left\{\widetilde{I}_n\right\}$ are the output decisions. This DFE parameterization satisfies the "key assumption" [3], i.e., the feedback filter length is greater than or equal to the channel length (we make the "key assumption" because reducing DFE complexity by truncating the feedback filter gives unacceptable performance degradation for wireless channels like Figure 1). Under this assumption (and assuming correct past decisions) the MMSE DFE has anticausal feedforward filter, and the feedback filter cancels all postcursor ISI:

$$b_{DFE,n} = -\sum_{m=-K}^{0} f_m c_{n-m}, \quad n = 1, ..., L. \qquad (3)$$

In practice K is chosen on the order of one to five times the channel precursor length, and for the channels of interest $K < L$ (due to the long channel postcursor length).

4. PFE STRUCTURE

The proposed PFE subtracts out postcursor ISI from the received sample y_k in a *time invariant* way prior to feedforward filtering:

$$\tilde{y}_k = y_k - \sum_{n=L_0}^{L} c_n \tilde{I}_{k-n}. \qquad (4)$$

Due to causality constraints it can be shown that $L_0 > K$, and for definiteness we choose $L_0 = K + 1$, in which case the residual postcursor ISI is completely cancelled by a feedback filter of length K (assuming correct past decisions). Hence we arrive at the following partial feedback equalizer (PFE):

$$\tilde{y}_k = y_k - \sum_{n=K+1}^{L} c_n \tilde{I}_{k-n} \qquad (5)$$

$$\widehat{I}_k = \sum_{n=-K}^{0} f_n \tilde{y}_{k-n} + \sum_{n=1}^{K} b_{PFE,n} \tilde{I}_{k-n}. \qquad (6)$$

Now assuming correct past decisions, the minimization of the mean-square error, $MSE = E\{|I_k - \widehat{I}_k|^2\}$, yields the same feedforward filters $\{f_n\}$ and equalizer outputs for the PFE and DFE. To see this, observe that if the PFE has feedforward filter $\{\bar{f}_n\}$, then the PFE output can be written as $\widehat{I}_k = \sum_{n=-K}^{0} \bar{f}_n y_{k-n} + \sum_{n=1}^{L} \bar{b}_n \tilde{I}_{k-n}$, for some $\{\bar{b}_n\}$. Hence $MSE_{PFE} \geq MSE_{DFE}$. Furthermore, if we choose $\bar{f}_n = f_n$ for all n and

$$b_{PFE,n} = -\sum_{m=n-K}^{0} f_m c_{n-m}, \quad n = 1, ..., K, \qquad (7)$$

then it is straightforward to show that $\bar{b}_n = b_{DFE,n}$, and consequently $MSE_{PFE} = MSE_{DFE}$. The implementation of the PFE is similar to the DFE in that the inputs to the PFE are generated by a tapped delay line (TDL) as shown in Figure 2.[1]

5. CFE STRUCTURE

The proposed CFE subtracts out postcursor ISI from the received sample y_k in a *time varying* way prior to feedforward filtering:

$$\tilde{y}_k^{(l)} = y_k - \sum_{n=L_k^{(l)}}^{L} c_n \tilde{I}_{k-n}, \qquad k = l, ..., l + K. \qquad (8)$$

The notation $\tilde{y}_k^{(l)}$ denotes the residual obtained from y_k by subtracting out all known postcursor at time l, i.e., before detecting I_l. Due to causality constraints it can be shown that $L_k^{(l)} > k - l$, $l \leq k \leq l + K$, and without loss of generality we choose $L_k^{(l)} = k - l + 1$, $l \leq k \leq l + K$, in which case there is no residual postcursor ISI and no additional feedback filter is required (assuming correct past decisions). Hence we arrive at the following complete feedback equalizer (CFE)*:*

$$\tilde{y}_k^{(l)} = y_k - \sum_{n=k-l+1}^{L} c_n \tilde{I}_{k-n}, \qquad k = l, ..., l + K, \qquad (9)$$

$$\widehat{I}_k = \sum_{n=-K}^{0} f_n \tilde{y}_{k-n}^{(k)}. \qquad (10)$$

Now assuming correct past decisions, the minimization of the mean-square error $E\{|I_k - \widehat{I}_k|^2\}$ yields the same feedforward filters $\{f_n\}$ and equalizer outputs for the CFE and DFE. The argument is similar to the one made in Section 4 relating the PFE and DFE. The implementation of the CFE is more complex than the DFE or PFE in that the inputs to the CFE are generated by a more general shift register with feedback (compared to a TDL) as shown in Figure 3.

6. INITIALIZATION AND UPDATE OF EQUALIZER COEFFICIENTS

For the high speed wireless communications considered here, the multipath channel is slowly varying relative to symbol rate. For this scenario, an appropriate choice for high performance DFE design is the nonadaptive MMSE DFE computation based on adaptive channel and noise power estimation as described in [2]. To employ this method the channel estimate is initialized using least squares (or possibly a blind identification algorithm), and tracked using LMS with updates clocked at some fraction of symbol rate.

It is natural to associate a version of this procedure with each of the DFE, PFE and CFE structures, tailored to the specific parameterizations. This viewpoint yields three methods for computing equalizer coefficients (based on the channel

[1] in Figure 2 and 3 we use $D-$ transform notation, and in particular $C_{K+1:L}(D) = \sum_{n=K+1}^{L} c_n D^n$

estimate), each with different complexity. Two of the methods ("DFE" and "CFE" methods) are known (albeit in a different context), while the third method ("PFE" method) is apparently new, and has some unique computational advantages for the equalization of channels with long postcursor response. Note that all the methods compute the (common) feedforward filter, so that all methods can be used to compute the MMSE FIR DFE settings if desired (the DFE and PFE feedback filters can be computed from (3) and (7), respectively). We briefly summarize the methods below, using the following notation. Let D denote the unit right-shift operator, and $D^i = D \circ \cdots \circ D$ (i- times). Hence $D^i[x_{1,...,}x_n]^T = [\mathbf{0}_i^T, x_1, ..., x_{n-i}]^T$ for $i \geq 0$ (and similarly for $i \leq 0$), where $\mathbf{0}_J$ is a $J \times 1$ vector of zeroes. Now let $\mathbf{c}_i = [c_i, ..., c_0]^T, 0 \leq i \leq L$, and $\mathbf{C}_{i,j} = [D^{-i}\mathbf{c}_i|\cdots|D^j\mathbf{c}_i]^T$. Also, for jointly wide-sense stationary random processes $\{\mathbf{x}_k\}$,$\{\mathbf{y}_k\}$, let $\mathbf{R}_{xy} = \frac{1}{2}E\left\{\mathbf{x}_k\mathbf{y}_k^H\right\}$, $\mathbf{R}_{yy} = \frac{1}{2}\mathrm{E}\left\{\mathbf{y}_k\mathbf{y}_k^H\right\}$ and $\mathbf{R}_{x|y} = \mathbf{R}_{xx} - \mathbf{R}_{xy}\mathbf{R}_{yy}^{-1}\mathbf{R}_{yx}$.

6.1. "DFE" Method

This is essentially the method proposed in [2]. Denote the vector of DFE feedforward filter inputs at time k by $\mathbf{y}_k = [y_{k+K}, ..., y_k]^T$, and let $\mathbf{X}_k = [I_{k+K}, ..., I_{k-L}]^T$ and $\mathbf{n}_k = [n_{k+K}, ..., n_k]^T$. In this notation $\mathbf{y}_k = \mathbf{C}_{L,K}\mathbf{X}_k + \mathbf{n}_k$, so that (using the matrix inversion lemma)

$$\left(\mathbf{R}_{I|y}\right)^{-1} = \frac{1}{\sigma_I^2}\mathbf{I}_{K+L+1} + \frac{1}{\sigma_n^2}\mathbf{C}_{L,K}^H\mathbf{C}_{L,K}, \tag{11}$$

where $\mathbf{I}_J$ is the $J \times J$ identity matrix. Now define the Cholesky factorization $\left(\mathbf{R}_{I|y}\right)^{-1} = \mathbf{LDL}^H$,where $\mathbf{L}$ is a lower triangular monic matrix and $\mathbf{D}$ =diag$(d_0, ..., d_{K+L})$. It is shown in [2] that the MMSE DFE feedback and feedforward filters are given by

$$\mathbf{b}_{DFE} = [\mathbf{0}_K^T, 1, -b_{DFE,1}, ..., -b_{DFE,L}]^H = \mathbf{L}\mathbf{e}_{K+1}, \tag{12}$$

$$\mathbf{f}^H = [f_{-K}, ..., f_0] = \frac{1}{d_K\sigma_n^2}\mathbf{e}_{K+1}^T\mathbf{L}^{-1}\mathbf{C}_{L,K}^H, \tag{13}$$

where $\mathbf{e}_J$ is the unit vector with J -th component equal to one. The computation in this method is dominated by the Cholesky factorization of the $K + L + 1$-dimensional matrix $\left(\mathbf{R}_{I|y}\right)^{-1}$, which is a Toeplitz matrix, and can be factored in $O((K + L + 1)^2)$ operations using the Schur algorithm.

6.2. "PFE" Method

Here we adapt the method in [2] to the PFE. Denote the vector of PFE feedforward filter inputs at time k by $\tilde{\mathbf{y}}_k = [\tilde{y}_{k+K}, ..., \tilde{y}_k]^T$, and let $\tilde{\mathbf{X}}_k = [I_{k+K}, ..., I_{k-K}]^T$. In this notation $\tilde{\mathbf{y}}_k = \mathbf{C}_{K,K}\tilde{\mathbf{X}}_k + \mathbf{n}_k$, so that

$$\left(\mathbf{R}_{\tilde{I}|\tilde{y}}\right)^{-1} = \frac{1}{\sigma_I^2}\mathbf{I}_{2K+1} + \frac{1}{\sigma_n^2}\mathbf{C}_{K,K}^H\mathbf{C}_{K,K}. \tag{14}$$

Now define the Cholesky factorization $\left(\mathbf{R}_{\tilde{I}|\tilde{y}}\right)^{-1} = \tilde{\mathbf{L}}\tilde{\mathbf{D}}\tilde{\mathbf{L}}^H$,where $\tilde{\mathbf{L}}$ is a lower triangular monic matrix and $\tilde{\mathbf{D}}$ =diag$(\tilde{d}_0, ..., \tilde{d}_{2K})$. Then similarly to [2] the MMSE PFE feedback and feedforward filters are given by

$$\mathbf{b}_{PFE} = [\mathbf{0}_K^T, 1, -b_{PFE,1}, ..., -b_{PFE,K}]^H = \tilde{\mathbf{L}}\mathbf{e}_{K+1}, \tag{15}$$

$$\mathbf{f}^H = [f_{-K}, ..., f_0] = \frac{1}{\tilde{d}_K\sigma_n^2}\mathbf{e}_{K+1}^T\tilde{\mathbf{L}}^{-1}\mathbf{C}_{K,K}^H. \tag{16}$$

The computation in this method is dominated by the Cholesky factorization of the $2K + 1$-dimensional matrix $\left(\mathbf{R}_{\tilde{I}|\tilde{y}}\right)^{-1}$, which is a Toeplitz matrix, and can be factored in $O((2K + 1)^2)$ operations with the Schur algorithm.

6.3. "CFE" Method

Here we solve the normal equations for the CFE feedforward filter. Denote the vector of CFE feedforward filter inputs at time k by $\overline{\mathbf{y}}_k = [\tilde{y}_{k+K}^{(k)}, ..., \tilde{y}_k^{(k)}]^T$, and let $\overline{\mathbf{X}}_k = [I_{k+K}, ..., I_k]^T$. In this notation $\overline{\mathbf{y}}_k = \mathbf{C}_{K,0}\overline{\mathbf{X}}_k + \mathbf{n}_k$,so that

$$\mathbf{R}_{\overline{y}\overline{y}} = \sigma_I^2\mathbf{C}_{K,0}\mathbf{C}_{K,0}^H + \sigma_n^2\mathbf{I}_{K+1}. \tag{17}$$

The MMSE CFE feedforward filter is given by

$$\mathbf{f}^H = [f_{-K}, ..., f_0] = \sigma_I^2\mathbf{e}_{K+1}^T\mathbf{C}_{K,0}^H\mathbf{R}_{\overline{y}\overline{y}}^{-1}. \tag{18}$$

The computation in this method is dominated by the inversion of the $K + 1$ -dimensional matrix $\mathbf{R}_{\overline{y}\overline{y}}$, which is not a structured matrix, and can only be inverted in $O((K + 1)^3)$ operations.

6.4. Discussion and Example

The three methods described above for MMSE equalizer computation have different complexities, and the detailed breakdown is shown in the Table. Note that the matrix inversion method and the backsubstitution method refer to variants of the fast algorithms derived in [2] for the DFE, applied here also to the PFE (only the standard matrix inversion method is applicable to the CFE). Also shown are the actual numbers for an equalizer designed for the HDTV terrestrial broadcast channel in Figure 1. Here we used 64 QAM modulation with symbol rate of 5.38 Mhz and raised cosine pulse shaping with 11.5% excess bandwidth. The channel length was $L = 126$, and the feedforward filter length was $K = 40$, which was the smallest feedforward filter length which yielded a raw target symbol error rate of .001 at 25 dB receiver input SNR. The numbers bear out the potentially significant computational advantages of the proposed "PFE" method when $K < L$, as occurs in the case of high speed signalling over multipath channels with large delay spread.

7. REFERENCES

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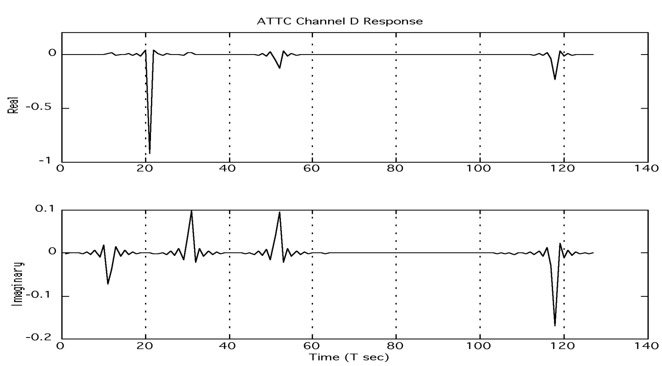

Figure 1: ATTC channel D

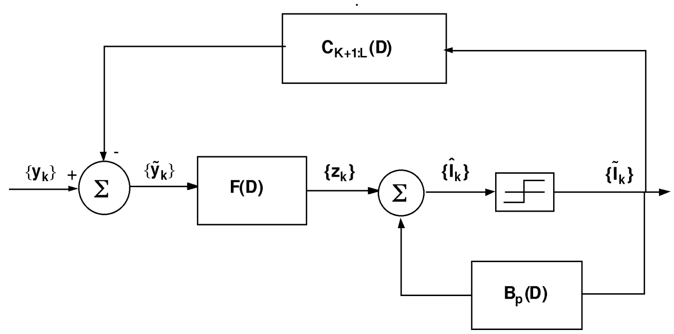

Figure 2: Symbol-spaced PFE structure

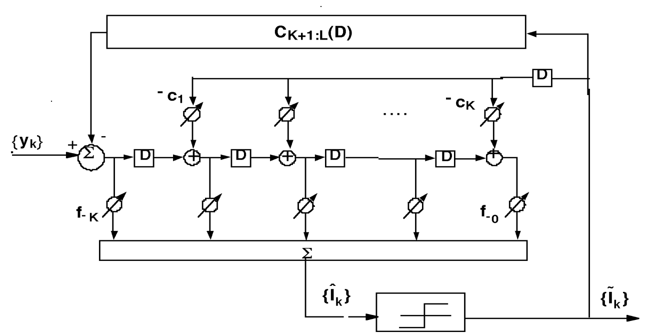

Figure 3: Symbol-spaced CFE structure

	Complex Multiplies	Complex Adds
DFE		
Matrix Inversion Method	$2K^2 + L^2 + 17K + 6L + 7LK + 9 = 55801$	$2K^2 + L^2 + 10K + 5L + 4LK + 7 = 40273$
Back Substitution Method	$K^2 + 14K + 2L + 6LK + 5 = 32657$	$K^2 + L + 4K + 3LK + 1 = 17007$
PFE		
Matrix Inversion Method	$10K^2 + 23K + 9 = 16929$	$7K^2 + 15K + 7 = 11807$
Back Substitution Method	$7K^2 + 16K + 5 = 11845$	$4K^2 + 5K + 1 = 6601$
CFE		
Matrix Inversion Method	$2K^3 + 6K^2 + 7K + 4 = 137884$	$K^3 + 2K^2 + K = 67240$